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## Confronting Challenges, Overcoming Obstacles: A Conversation about Quantitative Literacy

Bernard L. Madison\*

Lynn A. Steen†

\*University of Arkansas, bmadison@uark.edu

†St. Olaf College, steen@stolaf.edu

# Confronting Challenges, Overcoming Obstacles: A Conversation about Quantitative Literacy

Bernard L. Madison and Lynn A. Steen

## **Abstract**

An edited transcript of the opening session of a workshop on quantitative literacy held Oct. 10-12, 2008 at Carleton College, Northfield, Minnesota. The workshop, which brought together interdisciplinary teams from two dozen colleges and universities, was sponsored by the Quantitative Inquiry, Reasoning, and Knowledge (QuIRK) Initiative at Carleton and the Washington-based Project Kaleidoscope. Two mathematicians in the forefront of quantitative literacy initiatives over the period 1997-2008, Lynn Arthur Steen and Bernard L. Madison, converse about attitudes, obstacles, changes and accomplishments. The conversation, structured as an interview, begins with the relationship between mathematics and quantitative literacy and moves through issues central to effective education in quantitative reasoning to the relationship of such reasoning to the US financial crises of 2008.

**KEYWORDS:** quantitative literacy, assessment

*An edited transcript of the opening session of a workshop on quantitative literacy held Oct. 10–12, 2008 at Carleton College, Northfield, Minnesota. The workshop, which brought together interdisciplinary teams from two dozen colleges and universities, was sponsored by the Quantitative Inquiry, Reasoning, and Knowledge (QuIRK) Initiative at Carleton and the Washington-based Project Kaleidoscope. Carleton economics professor Nathan Grawe, director of QuIRK, opened the workshop and explained that instead of a speech, the opening presentation would be an interview conducted among two of the founders of the QL movement.*

**Lynn Steen:** Thank you, it's a pleasure to be here. When Nathan first mentioned the idea of us opening this workshop I talked to Bernie and suggested that since we've been doing so much writing about QL it would be kind of dull to say it all over again. So I proposed to Bernie that I would simply pretend to be Tim Russert—to interview him and try to pin him down on some of the things he actually has written about QL to see if he stills believes them or if he's learned anything about how to get around some of the problems that have emerged.

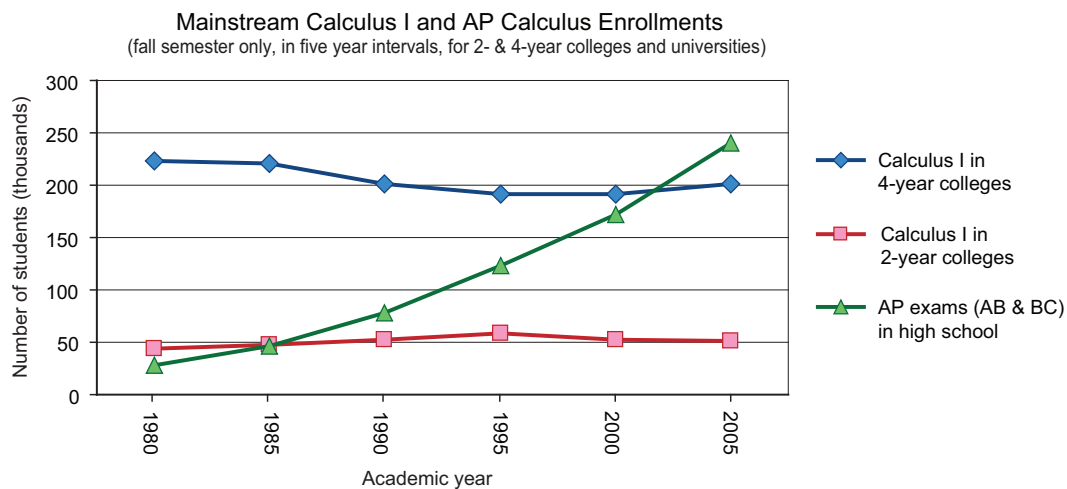
As a bit of a background for those of you who don't come from the mathematics community and may not have seen or ever heard of either one of us, I first got involved in QL a little over ten years ago when I was on the committee of the College Board that dealt with all their mathematics programs and exams. The parallel College Board committee that deals with their science exams came to the mathematicians and said, "We're worried that our science exams don't have enough quantitative material to adequately reflect the nature of modern science. Would you please tell us what quantitative literacy is required for modern science?" This query produced quite a bit of anxiety among mathematicians because we realized we couldn't answer that question with any evidence or coherence

Subsequently, historian Robert Orrill—at that time the Director of the Academic Affairs Office at the College Board—secured a grant from the Pew Charitable Trusts to start an initiative on quantitative literacy. That's where all our work on QL began. Orrill used the funds from the Pew grant to set up the National Council on Education and the Disciplines—better known as NCED. I recruited the assistance of Bernie who at that time was a visiting mathematician in the Washington office of the Mathematical Association of America (MAA) and looking for new projects to undertake. With Bernie's energetic leadership this project ran a national colloquium on QL at the National Academy of Sciences in Washington, a more recent Wingspread workshop on QL and teacher preparation, and the National Numeracy Network to which many of you belong. Upon returning to Arkansas, Bernie developed a QL course for journalism students that

has evolved into “Quantitative Reasoning in the Contemporary World” whose case studies are included in material for this workshop.

There will be little continuity to this interview. I haven’t any of Russert’s interview skills, but I do know where some of the soft spots are in the national campaign for QL. I haven’t told Bernie what my questions are: like the presidential debates, nobody but me knows what I’m going to ask him.

I’m going to begin with a common issue on campuses, namely, the relation between quantitative literacy requirements and the standard approach to mathematics requirements that centers on college algebra and calculus. Bernie, before becoming an advocate for QL you were chief reader for AP Calculus and helped reshape it into one of the best and most rapidly growing AP offerings. Here is a graph (Figure 1) that compares AP calculus enrollments with those in colleges. (The graph was created by David Bressoud at Macalester who is sitting right over there.) It displays data about the relationship of AP Calculus to the calculus that is taught in colleges and universities. The exponential graph shows what’s happening with AP Calculus while the two roughly horizontal lines show calculus enrollments in four- and two-year colleges. (The four-year enrollment has actually diminished from the 1980s.)



**Figure 1. Comparison of the number of students taking the Advanced Placement (AP) calculus exams with the number taking first-semester mainstream calculus during the fall term at five-year intervals, 1980-2005. (Source: “The Crisis of Calculus,” David M. Bressoud. In *Launchings from the CUPM Curriculum Guide*, April, 2007.<sup>1</sup>**

<sup>1</sup> [www.maa.org/columns/launchings/launchings\\_04\\_07.html](http://www.maa.org/columns/launchings/launchings_04_07.html) (accessed Dec 31 2008).

These data concern many people on college campuses—and not only mathematicians. One might think that since you were among the people who helped define AP calculus, you would be proud of this growth. But look at what you said in a talk about preparing for calculus (Madison 2006):

Understanding the concepts of calculus and how they are applicable to the world . . . is not accomplished by most calculus sequences, so the major general education value of calculus is missed. Thus calculus, which has been [the goal] of years of preparatory study, often is either blamed by failing students for their failures or seen as a disappointment for not living up to its advanced billing.

Some college curricula—business and premed, for example—benefit from [the focus on calculus] because it filters out students and presumably selects those most likely to succeed. [However,] it is no longer acceptable, social or otherwise, to be ignorant of mathematics. Therefore it is unacceptable to use . . . mathematics as a filter.

Could you help us understand where you really stand on this issue?

**Bernie Madison:** OK, your words come back to haunt you, right? However, I think that I still believe this. Calculus has occupied, especially in recent years, a privileged place in the mathematics curriculum, and, in fact, much of what we do in high school and pre-calculus college mathematics is to get ready for calculus. So calculus controls a great deal of what we do, but the calculus course at most institutions is a methods course, and the methods of calculus are not very close to quantitative reasoning. They are driven mostly by the needs of engineering and science students because they believe they need those methods. They probably don't need them anymore because technology will do much of the stuff that they spend years, and I spent years, learning in college mathematics.

So, I think what's going to happen—and I'm looking beyond this question—is that calculus has to change. That's what David Bressoud is looking at in some of the things he's doing. I think we have to emphasize the concepts of calculus because, quite frankly, the concepts of calculus are very important in quantitative reasoning: the notion of approximation, the notion of rate of change, the notion of accumulation. These are all concepts that are very important to one's ability to reason quantitatively in the world. So, calculus could be a very good QR course, but in most places unfortunately it is not. Did I answer the question?

**LS:** I'll give you a second chance. At many colleges—in fact, for a majority of students—it's not calculus alone that's the issue but the preparation leading up to it. In high school, it's Algebra 1 and Algebra 2; in college it's college algebra.

There are all sorts of national arguments going on about algebra so I want to stick with this point for one more question.

A few years ago you gave a talk at the Mathematical Sciences Research Institute in Berkeley with the title “Mathematical Proficiency for Citizenship” (Madison 2007). Here’s part of what you said:

What mathematics is critically important for informed participation in this highly quantified US society? This question has been considered since colonial times but has never been as difficult to answer as it is now at the beginning of the 21st century. A more difficult question is how can we design a developmental approach to achieve mastery of this mathematics and assessments to measure progress in this development. I do not pretend to have complete answers to these questions, but I am quite sure that our traditional introductory college mathematics courses and traditional assessments are inadequate responses.

All of our traditional courses—algebra, geometry, trigonometry—are sequenced and structured to prepare students for calculus. The QL initiative represented by folks in this room is just a kind of insurgency. So has calculus become the enemy of QL?

**BM:** Well, I still like that statement too. I am quite sure that traditional college mathematics courses and traditional assessments are inadequate responses to the needs of persons for participation in the highly quantified U.S. society. As a matter of explanation, mathematics never developed general education courses, not even around the turn of the 20th century when general education courses were being developed by most major mainline disciplines when the idea of a major had been introduced into the U.S. college curriculum. Prior to that, U.S. colleges offered the classical curriculum, which did not have general education as its focus. What we mathematicians did after the addition of majors was teach students the same stuff we taught them before in the classical curriculum, mostly classical algebra and geometry. I teach a course now that is the first real general education course in mathematics on our campus—and our campus is about 150 years old. Almost everything else has been classical, traditional mathematics. That new course has moved our whole program a little bit toward more reasonable general education. Everyday I encounter students who have had college algebra, but it’s totally inaccessible to them. They don’t recognize the need or the way to use algebra in reading the newspaper, for example.

I recently read an article by a science educator (Trefil 2008) from George Mason University who said that what we really need to do in science literacy is to educate students so that by the time they graduate they can read the newspaper. That’s in fact what I try to do. I use newspaper articles as class material, and when

we encounter something where algebra is needed the students don't recognize it. Even when, for example, they were trying to answer a question where the denominator changes and the numerator changes in a fraction, what did they do? They guessed and checked. Guess and check is a perfectly good strategy, but algebra would have solved the problem rather quickly, and these students should have been able to do that. So basically, whatever they're learning from these courses is not usable by them. I believe this is because they've never seen algebra except in very sanitized template exercises.

So I think I like this statement, and I am sticking to it.

**LS:** My next issue is a segue into the issue of leadership. If mathematics and its traditional courses aren't working, where should the leadership reside to bring about change? As you know, there is a lot of cross-disciplinary discussion on our campuses; the teams at this workshop are highly interdisciplinary—which is the general nature of the quantitative reasoning movement. However, that's not true on all campuses. In many situations the quantitative or mathematical requirement is just one or two courses in the math department. Most often the general education requirement is college algebra.

Several years ago you wrote an editorial about the challenge of QL addressed specifically to research mathematicians (Madison 2002). Here's what you said:

The sequence of mathematics courses from early high school through college calculus is linear and hurried, with no time to teach the mathematics in context, to help students develop the habits of mind necessary to interpret real-life situations in quantitative terms. . . . Many, if not most, students end up . . . learning mathematical skills that they are unable to use or to relate to their everyday lives. . . .

[Unfortunately,] QL has no specific place in most college degree programs. When it does, it often is mistakenly equated with mathematics, statistics, or other quantitative disciplines. However, the power of mathematics is its abstractness and its generality; QL is anchored in real-world data.

Mathematics should lead the effort to meet these challenges because of its centrality in college education, the size of its faculty, its traditions of teaching students from all disciplines, and its kinship with QL.

You and I know quite a few people—including some in this room—who argue fervently that mathematicians are the least likely people you would want to enlist in teaching quantitative literacy because their instinct is, as you write here, to move towards abstraction. On the other hand you conclude that mathematicians should lead the effort because mathematics is central in college

education, they have a large faculty, they meet students from other disciplines, and that mathematics has some link to QL. Are these organizational reasons sufficient to trump the fact that mathematical thinking is too abstract for QL?

**BM:** Probably not. Actually, this question came up at Wingspread in the summer of 2007, and I remember what [Carleton psychology professor] Neil Lutsky said. Neil can probably say it better than I can, but I think he said something to the effect that mathematicians play the same role in quantitative education as librarians (Lutsky 2008). We were initially taken aback by that, but then when we understood better what Neil meant, it wasn't quite as harsh as it first sounded.

Mathematics has a privileged place in the U.S. curriculum. Aside from English or English language arts, it's the only subject that is taught and tested in all grades 1 through 10 or 11 or sometimes 12. Then most colleges have mathematics requirements for most degree programs. So, mathematics has this chunk of the curriculum that other disciplines don't have. After all, mathematics is a quantitative discipline, and it should be based more in quantitative reasoning and quantitative processes than it is.

But it isn't. Right now, much of school mathematics and introductory college mathematics is based on techniques and methods and algorithms. It should be based more on reasoning, problem solving, the kinds of processes that are very important in developing quantitative reasoning.

So I don't know if we mathematicians should lead it or not. On my campus other faculty—psychologists and journalists and others—say, “it's up to you to do it.” You call us together and we will listen. They believe, as many people do, that mathematics has a leadership role to play here. I don't see it as leadership; I see it as partnership. Over the last several years I've changed my view on whether we mathematicians can lead in QL education. Rather, we should be partners with the other disciplines in developing quantitative reasoning programs on our campuses. So, I guess I've changed a little bit on this one.

**LS:** Let's move on to your experience in your own QL course. You've been teaching this course now for 4 or 5 years. (Copies of your casebook are available here at the workshop.) In the introduction to that course as well as in some earlier essays that you've written, you make reference to a quotation from educational historian Lawrence Cremin about the difference between “inert” and “liberating” literacy (Cremin 1988):

*Inert Literacy:* Level of verbal and numerate skills required to **comprehend** instructions, **perform** routine procedures, and **complete** tasks in a routine manner.

*Liberating Literacy:* Command of both the enabling skills needed to search out information and power of mind necessary to **critique** it, **reflect** upon it, and **apply** it in making decisions.

The boldfacing of verbs is my attempt at calling attention to the difference in skills required to comprehend, perform, and complete routine tasks versus to critique, reflect, and apply information in making decisions.

Based on your experience teaching a QL course, in which of these action verbs do your students grow the most (or least)? In which did they emerge strongest (or weakest?) What can you say about how your students match up to these literacy goals?

**BM:** Well, that's a tough one. Actually, I want to point out that comprehension is one thing, but comprehending instructions is something else. If you look at the development of numeracy (or quantitative literacy) in the western world it began in Great Britain about 1950 with the first appearance of the word numeracy. Subsequently, its meaning went through phases. In the first phase, 1950–1980, the original definitions were quite ambitious, and they fit with Cremin's definition of liberating literacy pretty well. In fact, that's what they meant for it to be. But then in 1982 the Cockroft Report (Cockroft 1982) moved numeracy into a mathematical phase where it meant applications of mathematics to everyday things—and that's very, very different. Numeracy is sometimes called functional mathematics, but it's very difficult to decide on all the applications of mathematics in everyday life. So one has to consider numeracy as something broader, more integrated than functional mathematics, which is where I believe we are today.

In dealing with students, I find that they are trained—and I use that word advisedly—in Cremin's inert literacy, notably routine procedures. My students come to class and say, "Just tell me how to do this: give me the formula, and I'll go home and do it." I have to fight them. I have to say, "No, no. The questions are vague (I don't use that word) and the answers are long and it's a process." It's not too different from the scientific process, from a data analysis investigation where you have to read, you have to understand, you have to critique, you have to glean information. Then you have to formulate a mathematical or statistical problem, which is usually pretty simple to solve once you get it formulated. Students usually can do the solving part. What they can't do is the first three parts, and then they don't reflect back because they've never been asked to do so.

Students can do the middle part that isn't explicit in Cremin's definition, solving some mathematical situation that has been modeled for them. That's what we've done for them in traditional courses. We've given them the model and asked them to do something with the model. So they are strongest at something—performing procedures—that in Cremin's language is inert rather than liberating. Getting them to critique, reflect, and apply is more difficult, particularly getting them to critique and reflect because they don't see that as part of a course with a math prefix. That is one reason not to have a math prefix in this because the students come with a certain set of expectations when you do that.

**LS:** One of the issues that those who teach QL share with mathematicians who teach calculus is a frustration with the mathematical proficiency of their students. A related difficulty that encumbers quantitative literacy courses (and most courses in the category of “math for poets”) is a tendency to devolve into skills at a pretty elementary level.

To pursue that issue, I picked a typical question from the middle of your recent QL exam and compared it (Table 1) with a similar released item from the Arkansas exam that is given to all students at the end of eighth grade. Both questions are on probability. Granted, one question cannot represent an entire test, but it can give a realistic sense of the general level at which students are expected to perform. The first<sup>2</sup> is from your QL assessment; the second is from the 8<sup>th</sup> grade Arkansas exam.

**Table 1. Comparing Two Questions**

<p>A sample of University of Arkansas students consists of 30 freshman, 25 sophomores, 43 juniors, and 22 seniors. If one of the students in this sample was chosen at random, what is the probability that the student would not be a junior?</p> <p>A. 43/120    B. 43/77    C. 77/43    D. 77/120    E. 77/100</p>																							
<p>Deb and Antoinette used a polyhedral die with 10 sides to play a game. They threw the die 100 times and recorded their results in the table below:</p> <p><i>100 Throws of Polyhedral Die:</i></p> <table> <tr> <td>Numbers on Die</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td>No. of Times Rolled</td> <td>14</td> <td>17</td> <td>12</td> <td>7</td> <td>10</td> <td>8</td> <td>12</td> <td>5</td> <td>11</td> <td>4</td> </tr> </table> <p>Based on the data, what is the experimental probability that Deb will throw a 7 on her next roll?</p> <p>A. 3 out of 25    B. 1 out of 20    C. 1 out of 10    D. 4 out of 5.</p>		Numbers on Die	1	2	3	4	5	6	7	8	9	10	No. of Times Rolled	14	17	12	7	10	8	12	5	11	4
Numbers on Die	1	2	3	4	5	6	7	8	9	10													
No. of Times Rolled	14	17	12	7	10	8	12	5	11	4													

Granted, the readings and discussion questions in your course casebook are much more subtle than the questions on your pre-and post-test. But anyone looking only at the assessment—as many politicians and members of the public often do—could be excused for wondering if QL is really much different from middle school mathematics. How do you explain to your faculty colleagues that what you’re doing is worth college credit if the assessment measures mostly topics that are in the Arkansas eighth grade curriculum?

<sup>2</sup> This is item #6 on the pre- and post-test for the course Mathematical Reasoning in a Quantitative World at the University of Arkansas. The pre- and post-testing is part of a research project to evaluate different approaches to teaching quantitative reasoning.

**BM:** Well, it's a tough sell. These are similar questions. The first is an item on a pre- and post-test to see what kind of questions students can answer in the beginning and what kind of questions they can answer at the end of our course. The questions most students can answer at the beginning are elusive; it's not difficult to find questions they cannot answer. In fact it's somewhat difficult to find appropriate questions. We try not to give questions in these kinds of tests that will be covered directly in the course, but that's what we've been doing in many traditional situations. We believe that you have to be able to adapt the reasoning from the course to answer a question that was not covered in the course. It's adaptive reasoning that we're trying to assess rather than just recall or procedural fluency. I believe that is not the attitude of the Arkansas eighth grade test.

My colleagues stood back when I decided to develop this course, and I've had questions about the level of this course from audiences like this. At the American Statistical Association meeting I remember somebody asking, when I described this course, "Is this a college-level course?" And my answer was "Yes, any course is a college-level course if the students don't know the material in the course." Even though Arkansas is testing with similar items in the eighth grade, the performance of my students on these "eighth grade items" is—well, discouraging at best, or maybe abysmal is the right word. I mean the performances on these tests are really, really very low. The polyhedral die problem is probably one of the tougher eighth grade questions, and I suspect the performance on that question is pretty low too. One should also note that many of my students were eighth grade students in Arkansas not very long ago.

I would be very happy if my students could answer such questions. They're liberal art students, mostly humanities, social science, music, and journalism majors. They're not geology majors or physics majors, although my experience with geology and physics majors, and math majors, is that they may not be able to answer these questions either. What you have to sell these courses on is the entire reasoning process not the calculation in the middle.

My colleagues in mathematics judge everything, the sophistication of any course, based on the sophistication of the mathematics that's included. This course is not a methods course or a mathematics content course. It is called mathematical reasoning, and my colleagues have accepted that, mainly because the administration loves it, other faculty love it, and most students love it because they think they learn something. Some of my colleagues have said this is really good stuff because it is process rather than calculation. It's a tough sell but it's a sell we must make because if students can't answer this question we should keep asking it and try to get them to learn to answer not only this question but lots of questions like this question.

**LS:** Another issue that frequently arises in conversations about QL—especially among mathematicians—concerns the pros and cons of trying to teach mathematics in context. In 2001 Bernie and I helped organize a national colloquium on numeracy with a lot of speakers on a large variety of topics. One of the underlying themes that emerged from almost everybody at the colloquium was that to achieve QL, mathematics has to be taught contextually. If it's not contextual, then while it may be mathematics, it's not QL.

We had selected as one of our closing speakers Hyman Bass, one of the most respected research mathematicians in the United States and one of the very few who has actually thought deeply about education issues. Here's what he said after having sat through two days of discussion on the importance of context for quantitative literacy (Bass 2003):

Contextualization ... is a laudable goal but it is often treated naïvely, in ways that violate its own purpose. Serious modeling must treat both the context and the mathematics with respect and integrity. Yet much contextualized curricular mathematics presents artificial caricatures of contexts that beg credibility. Either many of their particular features, their ambiguities, and the need for interpretation are ignored in setting up the intended mathematics, which defeats the point of the context, or else many of these features are attended to and they obscure the mathematical objectives of the lesson. Good contextualizing of mathematics is a high skill well beyond that of many of its current practitioners.

So my question, Bernie, is this: How many members of your department could teach the QL course you are teaching? What about the high school teachers in Arkansas?

**BM:** Right now three of us—three faculty members—are teaching the course. We tried it with graduate assistants, and even with close mentoring they had limited success. It's difficult. As for Arkansas teachers, we have some plans for professional development. Fortunately we just received an NSF MSP grant, and one of the things we're going to do is professional development with some of the teachers in our partner districts to help them with contextual mathematics or with quantitative reasoning or quantitative literacy. MSP stands for Math and Science Partnerships. It's an NSF program where you work with school districts. There are big bucks involved if you're interested. We got \$7-million; we're really tickled. But it's really hard work because you work in partnership with school districts, and you learn from them and they learn from you. You do a lot of professional development with teachers, or at least that's what we proposed to do, and I hope we can be successful with that.

Actually Hy's right about the difficulty with contextualization. We have been using the wrong pedagogy in mathematics for a few centuries. What we do is present finished mathematics, and we then give some phonied up applications as exercises.

But the way we do mathematics is not that at all. The way we do mathematics, as researchers in mathematics, is we look at examples from the situation we are trying to study and then we develop the mathematics. We teach mathematics exactly opposite from the way we create it. And that's part of the problem; the students don't have much experience with dealing with contextual mathematics. That's why the contexts in the course that I teach are the most important thing there. The mathematics is pretty simple.

We couldn't do the kind of mathematics that Hy is talking about in context. We couldn't solve the huge systems of partial differential equations that Hy has in mind, or I think he has in mind. (I'm putting words in his mouth.) But what we can do is get students to deal with contexts that mean something to them, from their lives around them such as the cost of tuition.

You have to work at finding something that students are interested in. This week and last week have been eye openers for me because I was trying to get them to understand how much \$2.2-trillion is. They just look at newspaper headlines reporting that the loss in the values of stocks for a single week in September was \$2.2-trillion and say, "I don't know." Is a \$12-trillion national debt a serious thing? Again, they say, "I don't know." In a think-aloud session four students were trying to figure out how much \$1.2-trillion dollars is. That \$1.2-trillion is one estimate of the cost of the Iraq War (Leonhardt 2007). They decided they were going to measure \$1.2-trillion in how many houses it would buy. First, they decided the cost of a house was \$30,000. (Loud laughter from audience.)

You must find contexts that the students can work with and finding those contexts is tough. But if you can find contexts where the student can practice, practice, practice, then you can develop quantitative literacy—it's a habit of mind and habits require practice, practice, practice. You should send students out of your classroom with a venue for them to continue to learn. Unless we can give them a venue for continuing to learn and a context where they can do this, like the newspaper, then we are going to be less successful than we would be otherwise.

**LS (to the audience):** I'd like to shift gears a little bit now. We've been talking about mathematics and quantitative literacy because that's the subject of this meeting. But many of you may not know that 20 years ago Bernie came to the math community and said to people, "You guys really ought to wake up. You're going to have to start learning to do assessment."

This was way before the accrediting agencies and Congress and Margaret Spellings and all these other people were telling higher education that assessment

and accountability is important. Bernie had the foresight to see this “really big thing” coming down the road and urged the math community to wake up and start working on it. As a result—no surprise here—Bernie became chair of the Mathematical Association of America’s assessment committee for about a decade. Way before anybody else was thinking about assessment, his committee produced a report emphasizing that assessment wasn’t a one-time thing where you just give a test and see how well students do. Instead, it is a cycle of setting goals, assessing progress, then revisiting goals and making adjustments.

Jump to the present. In the introduction to Bernie’s QL casebook<sup>3</sup> (Madison and Dingman 2008), he describes what he calls “the typical QL encounter”—what a person should be doing when they see a headline about a \$12-trillion national debt. So I took his recipe for a QL encounter and put it side by side with his earlier recipe for the assessment cycle (CUPM, 1995) (Table 2):

**Table 2. Comparing Two Recipes**

<b>Quantitative Literacy</b>	<b>Assessment Cycle</b>
Glean relevant information	Set goals
Take up the challenge	Select methods
Estimate to see if assertions are reasonable	Gather evidence
Do the mathematics	Draw inferences
Generalize the situation	Take action
Reflect on the results.	Re-examine goals and methods

They seem to line up pretty well. There really isn’t a whole lot of difference other than the context—the subject matter you’re dealing with. Hence my question to Bernie—I guess it’s not a question, but an observation that Bernie can refute if he wishes: Would you say that quantitative literacy is the Assessment Cycle of Democracy?

**BM:** Lynn is so good at developing such segues, hitting both themes of this workshop in one question: “Is quantitative literacy the assessment cycle of democracy?”

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<sup>3</sup> This casebook was first published in a preliminary custom version in August 2007. Published and sold by Pearson Custom Publishing, a limited numbers of copies are available during the 2008–2009 academic year (contact Carie Jones at carie.jones@pearson.com). A second, revised and improved version will be published in summer 2009 with Stuart Boersma and Caren Diefenderfer as co-authors.

That's not a bad line to pursue. When Lynn and Bob Orrill got me to work with them in 2000, or whenever it was, I was wondering, "What kind of title is the title to that book—*Mathematics and Democracy*?" Then I began to understand. More recently people are puzzling over the quantitative reasoning needed to support democratic processes. We need to solve the puzzle and improve QL education—get students, particularly college graduates, to understand quantitative arguments that are posed to them in these debates between conservatives and liberals, between Republicans and Democrats—and now with all of this financial stuff that's going on, understand even the words being used.

So, yes, I'm surprised at the similarity of these multi-step processes. The steps on the left-hand side are the ones that I tried to articulate a minute ago and I don't think that I got them all out. There are six here—sometimes I use five, sometimes I use six. But that's what I think the typical quantitative literacy encounter is. On the right is an assessment cycle, which all of you know.

And then, of course, you repeat the assessment cycle. We know that. We repeat the QL, but probably in a different context. So there is one difference here, Lynn. This one on the right never ends, and the one on the left can end in a particular context.

**LS (to the audience):** I've got two more issues to bring up. One is pretty complicated and then I'll end on another that leads, I hope, directly into your working sessions. The complicated one might benefit from some background. I mentioned a minute ago Bernie's uncanny foresight in urging the math community to think about assessment way before it became a political necessity. I also recall his saying around 1990 that the relatively unknown governor of his state would be the next president of the United States. So I have learned to listen to his predictions.

In preparing for this interview I found another. In a talk Bernie gave in 2004 he listed a few "characteristics of U.S. democracy" that he said increased the need for quantitative reasoning among our citizens. Two of the eight were these:

- Free market system with minimally regulated labor markets
- Recent deregulation of markets and services

If the Federal Reserve had listened to Bernie three or four years ago, maybe we would be in a different position today.

I'm not going to ask Bernie directly about those assertions. Instead, I want to use them to set up a question that is more directly germane to the purpose of this workshop: What is the purpose of QL as a goal in higher education? When you

look at instruments such as the Collegiate Learning Assessment<sup>4</sup> and other broad assessments of what we expect graduates to be able to understand and do based on a bachelors degree qualification, it is quite a bit more sophisticated than those two probability questions we were looking at. These assessments seem to suggest that QL should enable graduates to understand today's financial crises.

To put this to the test, I selected a variety of potential explanations for the financial meltdown from the *New York Times* and other places. What I want to do is to let Bernie (and the rest of you) focus on the question of how can we get our students to be ready to deal intelligently with these kinds of issues when they graduate from college. I'll introduce this with examples of simple but very different hypotheses. [Note: During the actual interview, the following hypotheses were merely summarized rather than presented in detail.]

One is that the traders on Wall Street were under pressure to produce simple and quick results (Hansell 2008). As a consequence they put oversimplified data into otherwise good models. According to this theory, financial firms chose to program their risk-management software with overly optimistic assumptions and oversimplified data, thereby radically underestimating the risk of complex mortgage securities. For example, to simplify the rating process, some trading desks took the most arcane security made of slices of mortgages and entered it into the computer as if it were a simple bond with a set interest rate and duration—thereby hiding details of risk from bond raters. In other cases, to keep capital needs as stable as possible many computer models looked at several years of trading history instead of just the last few months. “It was like a weather forecaster in Houston talking about the onset of Hurricane Ike by giving the average wind speed for the previous month,” reported Hansell.

Physicist Mark Buchanan looked at the economy as a dynamical system and asked, as we once did of the solar system, is it stable? He argued that economic markets have internal dynamics unrelated to actual facts or balance sheets and that these dynamics are not taken into account by traditional equilibrium models (Buchanan 2008):

Simulations show how [internal dynamics] can push the market toward instability. ... The instability doesn't grow in the market gradually, but arrives suddenly. Beyond a certain threshold this virtual market abruptly loses its stability in a 'phase transition'

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<sup>4</sup> The Collegiate Learning Assessment, developed by the Council for Aid to Education, measures the institutional contributions to the learning gains made by students. See <http://www.cae.org/content/pdf/CLABrochure2008.pdf> (accessed Dec 31 2008).

akin to the way ice abruptly melts into liquid water. Beyond this point, collective financial meltdown becomes effectively certain. This is the kind of possibility that equilibrium thinking cannot even entertain.

Then you have people who blame the financial mess on the way economists calculate risk. These days they generally use the Black-Scholes formula, named for two economists who won the Nobel Prize in 1997 for its discovery. From one perspective, this formula is a method for calculating risk, but from another perspective it is just a mathematical theorem that has a bunch of hypotheses. And as is typical when people apply mathematics, they tend to ignore the hypotheses. For example, some hypotheses in the Black-Scholes theory assume that certain variables are normally distributed. But if a variable does not fit that hypothesis, you may well under-represent extreme risks (Rozeff 2008):

Finance professors got all happy when they discovered a means of understanding risk ... but they failed to heed warnings ... that the distributions of returns had infinite variance, which makes very unlikely events occur much more often than a normal distribution suggests. Black even wrote a note called "The Holes in Black-Scholes," pointing out problems in his own option pricing model .... But teachers and students went their merry way, happy to have *any* kind of model.

Finally I want to mention an insightful article written well before the current crisis not by an economist or a mathematician but by Mary Poovey, a humanities professor at NYU. In a paper entitled "Can Numbers Ensure Honesty?" she analyzes why people believe that numbers embody objectivity even when they don't understand where they came from or what they really mean. But she goes on to warn that widespread belief in abstract models has a specially insidious effect on modern accounting (Poovey 2003):

The belief that makes it possible for mathematics to generate value is not simply that numbers are objective but that the market actually obeys mathematical rules. The instruments that embody this belief are futures options or, in their most arcane form, derivatives....Futures and derivatives trading depends on the belief that the stock market behaves in a statistically predictable way... this belief inspires derivatives trading to escalate in volume every year."

One appeal of applying mathematical equations to equities trading is...equations like Black-Scholes that enable the financial community to disaggregate components of commerce and reassemble from these parts new financial products that combine

different risk profiles....Risk used to be viewed as uncertainty about the future, an irrational factor that one sought to protect against. Now that risk has been objectified, divided, and reassembled so that it can be traded, it becomes mathematically predictable—that is, rational, abstract, and subject to management [through statistical devices]....These financial instruments mobilize both the belief that numbers are objective and true and the belief that the market conforms to mathematically produced statistical probabilities.

Very few people inside or outside the global financial community question whether the foundational assumptions implicit in financialization are true....[Recently] investors have begun to suspect that numbers do not always embody objectivity, but few have stopped to question the assumptions that make the largely unseen world of derivatives work: that the market obeys the logic of statistical probability and that the estimates that mathematical equations silently make do not matter.

But what if markets are too complex for mathematical models? What if unprecedented events...affect markets in ways that no mathematical model can predict? What if the mathematical models traders use to price futures actually influence the future?

I've enumerated these contrasting views in some detail to illustrate the variety and complexity of what we as a nation—and as voters—need to talk about. Congress needs to debate hypotheses just like these because if they don't know what caused the problem, they're never going to get the solution right. Bernie, you warned in 2004 that deregulation requires increased quantitative literacy. Do you really think that QL, even in its most optimistic form, would be a noticeable defense against the hazards of the modern economy? Can you help us out of this mess?

**BM:** Well, if they had just listened to me!

Let me talk about the context in which I mentioned the quantitative burden imposed by deregulation. The point I was making when I wrote this was that the quantitative reasoning burden on U.S. citizens is higher than in any country that has ever existed. And these two bullets are two of the reasons. You can just go right on down the list of examples that Lynn cited to see why we need our citizens to be quantitatively literate.

At lunch today [Project Kaleidoscope director] Jeanne Narum showed me an op-ed piece in the *New York Times*. The writer was saying that what the culprits in the recent financial implosions acted on was equivalent to believing that if you flip a coin three times and it comes up heads that it's more likely to come up tails

the next time. Of course, a lot of my students would believe that. And when I test them, that's what actually comes out.

The thing about using mathematics to model an economy—well, I don't need to say anything about that because the editorial writers Lynn cited did it for me. Mathematical theorems are so precise with such critical hypotheses that you had better be careful when you're using them to model a complicated situation like the weather or the economy.

Let me just give one example. In the last few days, the headlines in all the papers across the U.S. have focused on one number: the Dow Jones Industrial Average. Now, the Dow Jones Industrial Average is, in the words of Dan Okrent who was public editor of the *New York Times*, “mathematically preposterous” (Okrent 2005). If you examine how it's computed, it really is mathematically preposterous. It's a very weird average. It concerns only 30 stocks out of something like 7000 stocks traded on these exchanges. And a \$1 change in a \$100 stock is treated the same as a \$1 change in a \$3 stock. It changes the average in the same way. So what is it we're looking at to decide if our economy is healthy? It's largely this weird average. In 1928 when 30 stocks became the Dow Jones base, they divided the sum of 30 numbers by 30. But now, because of stock splits through the years, they divide the sum of 30 stock prices by about 0.11. That's why you get up to maybe 8000 today. (It was at 14,000 not so long ago.)

I think that shows you the complexity of how our economy operates. The psychologists can probably tell you the answer to this question much better than I can. Neil might want to take a shot at it. This is really a psychological thing. The Dow Jones Industrial Average has an enormous influence. But it is undeserved influence. Sorry. It's a bad number. But it has an enormous effect on our economy.

We have to worry about deregulation. Think about the complications in people's lives when we deregulated the banking industry. Think about the complexity in people's lives if we change the Social Security System and move it to partly-private-partly-public. Think about the complexity of airline fares—just the deregulation of airlines. Every one of these deregulations has poured tons of demands on our population. And, I'm sorry, but our people aren't prepared to deal with that. Black-Scholes and many much simpler things are not among the capabilities of the average person.

**LS:** Well, thank you Bernie. I've one more question for Bernie and then we'll open it to questions. I went through the reports that all of you wrote in preparation for this workshop and looked at what you listed as the deficiencies and challenges required to bring about transformation on your campuses. Here's a summary (Table 3).

**Table 3. Deficiencies and Challenges**

<p><b>Deficiencies:</b></p> <ul style="list-style-type: none"> <li>Faculty time and energy</li> <li>Resources for faculty development</li> <li>Faculty conviction of the value of QR/QL</li> <li>Faculty experience with QR</li> <li>Reflective and informed discussion</li> <li>Faculty training in educational theory</li> <li>Faculty-wide consensus about QR/QL</li> <li>Consensus about the value of assessment</li> <li>Convincing holistic models of assessment</li> <li>Clarity in what is being assessed</li> <li>Enthusiasm for assessment</li> <li>Resources for administration and assessment</li> <li>Grant funding</li> <li>Timely feedback of assessment results</li> </ul>	<p><b>Surpluses:</b></p> <ul style="list-style-type: none"> <li>Faculty fatigue (with curricular initiatives)</li> <li>Adjunct faculty teaching GE courses</li> <li>Faculty and departmental inertia</li> <li>Silo thinking that sees QR as responsibility of STEM depts</li> <li>Commitment to academic freedom and departmental autonomy</li> <li>Temptation to lower standards</li> <li>Skepticism about assessment</li> <li>Large classes</li> </ul>
<p><b>Challenges:</b></p> <ul style="list-style-type: none"> <li>Maintaining consistency and coherence across departments</li> <li>Avoiding a check-list approach to QL/QR requirements</li> <li>Fairly assessing QL courses taken at different stages in a 4-year curriculum</li> <li>Student (and faculty) habit of compartmentalizing knowledge</li> <li>Difficulty of creating high quality assessments</li> <li>The increasing diversity of student backgrounds</li> <li>Making good use of assessment results</li> </ul>	

The left column lists things that are in short supply like faculty time, resources, and faculty experience with QR. The column on the right, what I called “surpluses,” include such things as faculty fatigue, too many adjuncts teaching GE courses, too much skepticism about assessment. Down at the bottom are more-general challenges.

You have been both a department chair and a dean, so I wonder if you want to offer any suggestions for how best to overcome some of these hurdles.

**BM:** Let me say something about that. I've dealt with most of the things on the right. In earlier parts of my career, I was a department chair for ten years and then for ten years I was Dean of Arts and Sciences with twenty departments. It was really fun work. But it was really tiring. One reason I can get away with doing things at my institution that most faculty can't get away with or won't try is my long tenure at different positions.

First of all, I have the time. I have the luxury to do things. I stopped being Dean a few years ago and I just focus my attention on these kinds of things.

The other thing is that I have a lot of latitude. I don't worry about merit raises. (One reason is, there aren't any.) I don't worry about promotion or tenure. I don't worry about doing mathematical research. I've done that, been there. So I can get away with a lot of things.

But I do understand all of these pressures on faculty members. And they are mostly disciplinary pressures because the disciplines decide what it means to be successful for disciplinary faculty. I saw that as Dean in every discipline that I dealt with. It wasn't just mathematics.

Right now I have a talented young assistant professor working with me. His name, Shannon Dingman, is on the cover of that book that we distributed here. I was very pleased to get him vested in the project. Developing credentials and documenting accomplishments are more critical to Shannon than they are to me, and Shannon's going to be around longer than I am. I put Shannon on as co-PI on the grant that I got from NSF-CCLI last year because NSF asked me to add some people to help. I got Stuart Boersma to help me, and Caren Diefenderfer, in addition to Shannon. Shannon is also a co-PI on our new NSF-MSP grant. One of the very nice things about being a senior professor is that you can help talented younger faculty members, and, in turn, they can help you with fresh ideas and renewed energy.

You have to protect young faculty members if you're going to engage them in this kind of work because developing a new course is not something they are going to get promoted for. And doing assessment is probably not something you're going to get promoted for. So you have to worry about the kinds of things in this chart, and I do worry about them. But I'll only worry about them for other people. My point is, I don't have to worry too much about me.

**LS:** Let me add one comment. There's a big difference between numbers like \$1.2-trillion as the cost of the war because as we all know there are Democratic numbers, there are Republican numbers, there are all sorts of different people who calculate that differently. If you ask about the cost of the bill that Congress just passed with regard to the bailout, there you can look in the law itself and see what the number is. So there's a difference in how difficult it is to find the source of the number. I think we need to get our students to be aware of that as well.

**Question:** I'm Stacey Lowery Bretz, a chemist at Miami University in Ohio. I just want to offer an observation. Our university recently opened a center for writing excellence, and it's a big deal, a big, big, big deal, endowed by lots of money, and all disciplines are expected to incorporate writing—even in my department where people have the perception that we don't write. Actually, most days I feel that all I do is write.

But if we were to say we were going to have a center for quantitative literacy on our campus, one of the barriers we would run into is the faculty themselves, not because they think it should be the statistics department's responsibility but because many admit that they themselves are quantitatively illiterate. I was shocked to find members saying this in the faculty learning community I joined at the start of this school year. Somehow it's socially acceptable for faculty to say, "I don't need to know those things."

So I'm curious to know how we're going to make the case that it's not math being the leaders but it's math being partners with all of these departments when there are faculty themselves not capable of quantitative reasoning. It's going to be, in my opinion, threatening to have that exposed, that somehow we're going to create a campus culture of quantitative literacy while there are people who should be on board with that who themselves are going to be challenged to model quantitative literacy.

**BM:** I agree. I mean it's interesting because one of the things that got my course to be more accepted among my math colleagues was they couldn't answer some questions. Further, students would go to the learning resource center and other tutoring centers where they couldn't find anybody to help them. They were told, "we don't know how to do that." It's because they don't think that way. It's clear you're going to have to do some faculty development. I have colleagues who freely admit that they're quantitatively illiterate. I have had history faculty come to me and say "I don't know what you're doing here but it sounds good." I have had journalism faculty ask me to come over and talk to journalism faculty because they say "we don't know how to do this." The journalism faculty asked for multiple copies of this new casebook so that faculty could use examples in the courses that they teach. I was just happy that they wanted to learn about the cases.

I think you're absolutely right. But I think we're making progress where people are understanding that even though we don't understand what QL is, we recognize it when we see it. And we recognize that it's important.

**LS:** Let me just add one bit to that and that is to put in a plug for Carleton's QuIRK program. If you read about it—or talk to the Carleton folks about it—you'll see that it is embedded in an existing curricular writing structure, which I

think is a very good way to begin dealing with the similarities between literacy as writing and quantitative literacy.

**Question:** Dale Ciphers, physics department, Bowdoin College. The answer to your prior question touched on something I have a question about. Students come to us with prior experiences and they're taught mathematical skills often separate from other things; it's what I call "disembodied math." They have a good background in disembodied math, and when you put in quantitative reasoning, or quantitative literacy, how do you start to address the clash of expectations when these students come into some course and think you might be doing things wrong because they've never seen it being done that way before.

**BM:** Once somebody told me that when you take a job the most important thing is to choose your predecessor well. Now I say that the reason I haven't had problems with students in my QL course is that they didn't at all like the math course they were taking before, and it wasn't good for them. It was a traditional finite mathematics course and it was delivered via computers; there was all sorts of howling about that. So I haven't had the problem you describe, but I can certainly understand the possibilities.

Students do say to me, "I've never had a math course like this before," and I say, "Of course not, because this is different." But students soon realize that it's in their world. One student wrote on an evaluation, "This course takes off the table once and for all the question of 'Where will I ever use this?'" I keep repeating to them: "I don't make these vague questions. I don't ask these questions that don't have clear algorithmic solutions. That's part of your world. It's all around you." So that's the way I defend it: it's part of their world, and they should understand the world around them.

**Question:** Hi. I'm Ed Rothman, University of Michigan; I'm a professor of statistics. One of the things a buddy of mine talked about was that we can't learn from examples alone. Are there principles that underlie the sort of case studies that you've talked about?

**BM:** What do you mean by principles?

**Rothman:** Well, if we were to say—I'll pick a simple example—that everyone doing their best may not be best. Let's think about that. If we have a series system where you have one machine capable of 500 per hour followed by another machine in the process that's capable of only 200 per hour, and your objective is to get as much throughput as possible with as little work in process, you know, there's an underlying calculus principle here, but in simple terms you wouldn't want to run the first machine at 100% efficiency. And then there are metaphors of that elsewhere that students would have to come up with. So teaching the

example could be a useful pedagogical tool to lead to a more general one, rather than thinking about maximizing a function of several variables, one variable at a time.

**BM:** We're working on that, but it's difficult to structure something in terms of principles and underlying truisms when you are actually trying to see if there are such truisms. What I do know is that we do have certain rules in the class. For example, you don't make an argument without evidence, you don't make an assertion without support. You give evidence to support anything that you state. A student said to me yesterday, "But you didn't say to do that" and I said to him "It's a rule. It's a rule. You never make a statement without giving evidence to support your answer."

And we talk about what constitutes good evidence. I know that here at Carleton I've heard them talk about the same thing. We will never make this quite as codified as some of our more traditional courses because it's not possible. It's much too amorphous. It's much too integrated into other disciplines.

One difficulty that I have when I'm teaching the course is that I wander into disciplines where I don't know what I'm talking about. I'm learning, but I have to say, "I don't know that." So I don't think we can codify it yet, but I'm hoping that we can.

Learning what conceptual difficulties students have in dealing with these problems is part of that. What I'm finding is they're reluctant to use algebra. That's true. They don't like to compute with unknowns. I have lots of evidence that they don't like to compute with unknowns. So we have to work on getting them to compute with unknowns, because they don't recognize needing them. They don't think you can multiply by something you don't know, or divide by something you don't know. So we're beginning to develop some things that I think you would agree are principles, but we are a long way from having it codified in any reasonable sense.

**Question:** Hi. I'm Tom Ellman; I'm a computer scientist from Vassar College. I'm here with a few colleagues from Vassar, so I want to state that I'm speaking for myself. I'd like to follow up on something that was mentioned earlier having to do with faculty who themselves may have some imperfect knowledge, or imperfect quantitative literacy. A related issue has to do with what I imagine might be some problems I'd encounter in my college if I tried to teach the kind of course that Bernie is talking about. I fear that a course like that would have a kind of feel or association of being remedial education, and that would trouble people a lot. What that really says to me is that we have a problem admitting to ourselves and maybe to our public audiences (people who might be applying to our schools, for example), just how bad things are.

Of course, there's a huge range of students that we see, some are very, very bright, some very well prepared, and others not. Some who are not so quantitatively literate have other tremendous strengths. But in any case, certainly, it's been my experience that quantitative strengths among many students are quite bad, and we need to find a way to own up to this. So I'm just wondering if either of you could talk about how we could do that in a way that helps us confront these issues.

**LS:** Let me take a crack at that. I did some careful thinking about this awhile ago when I kept being confronted with that same kind of challenge: just how do you distinguish quantitative literacy from remedial courses? It's a different version of what I just asked Bernie: How do you justify for college credit a course whose mathematical techniques are strictly those that are part of high school or even middle school curricula?

If you look, for example, at the case studies in Bernie's volume (and other people's projects where they are underway), the contexts in the situations that are presented are not, for the most part, the kinds of contexts and situations that you'd expect middle school and high school students to deal with. Just because a theoretical physicist sometimes uses algebra doesn't mean that what you're doing with it is necessarily high school level work, even though the particular mathematical techniques may be the same. So I think we have to focus on providing sophisticated college-level challenges and then not worry about what level mathematical tools lie beneath them. In fact, even if they've taken a fair amount of mathematics, most people who leave college and go off into the world of work are not going to remember (or use) much of anything beyond algebra I.

**Question:** Michael Tucker, Fairfield University, Department of Finance. We use cases in business schools that are quite effective in bringing the context to problem solving. I teach a course in financial modeling that I would say is more process than content. My colleagues would assert that they're teaching derivatives, etc. I am teaching applications. I use Excel extensively. I believe it's the thinking process that really counts, but it's very difficult for the students to grasp that. They may know some of the context, but they don't know how to use it in answering the questions. They can't even get to the point of asking a question, which is what you were emphasizing earlier. However, there is resistance from financial colleagues about a course like this because you can't point to the content you're teaching, because it is about process. I think that is really what we're up against. I don't even know what you're calling quantitative reasoning. I guess that's what I'm doing, but process doesn't have the cachet that content has.

**BM:** I couldn't agree more. The content does carry the disciplinary cachet. That's why mathematicians look at such courses as remedial—because the mathematics therein is mathematics that students should already know before they come to college. That's missing the point of the whole course, because it's about process. The content in mathematics is fairly simple, but if students don't know how to use it, it's not valuable to them.

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