Uranium and Nuclear Power:
The Role of Exploration Information in Framing Public Policy

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Abstract

As addressing climate change becomes a high priority it seems likely that there will be a surge in interest in deploying nuclear power. Other fuel bases are too dirty (coal), too expensive (oil, natural gas) or too speculative (solar, wind) to completely supply the energy needs of the global economy. To the extent that the global society does in fact choose to expand nuclear power there will be a need for additional production. That increase in demand for nuclear power will inevitably lead to an increase in demand for uranium. While some of the increased demand for uranium will be satisfied by expanding production from existing deposits, there will undoubtedly be pressure to find and develop new deposits, perhaps quite rapidly. Looking forward, it is important that policies be put in place that encourage an optimal allocation of future resources towards exploration. In particular, I argue there is a valid concern that privately optimal levels of industrial activity will fail to fully capture all potential social gains; these sub-optimal exploration levels are linked to an departure between the private and social values of exploration information.

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1 Introdution

When the Fukushima Daiichi nuclear reactor melted down on March 11, 2011 many thought the nuclear industry would collapse along with it. Indeed, following the disaster the Japanese government temporarily suspended operation of all nuclear power plants; shortly thereafter and the German and Swiss governments also decided to close all of their nuclear power plants. The future of nuclear power was far from clear.

Twentyone months later, the nuclear industry is alive and well. The Japanese Prime Minister recently acknowledged the need to restart some of the country's nuclear plants if they are to maintain their standard of living, and German Chancellor Angela Merkel has argued that the German plan to walk away from nuclear power is "unworkable" (International Herald Tribune, 2012). The British government has publically acknowledged the need to expand their nuclear portfolio (Pfeifer, 2011), and in the United States (U.S.), plans are underway to build and deploy the first new nuclear power plant in over three decades. Orders for future power plants in China are robust. These factors, among others, have lead some to forecast a renaissance for nuclear power in the coming years.[2]

This potential rebirth of nuclear energy has a historical antecedent. Nuclear power burst onto the scene in the U.S. starting in the late 1960s (and later in Europe and Asia). With this emerging interest in nuclear energy, nuclear power production and installed nuclear power capacity rose sharply in the 1970s in the U.S. (IAEA, 2011), as Figure 1 illustrates. With this sharp increase in the use of nuclear energy, demand for the key input—uranium concentrate, also known as U_3O_8—also increased sharply. The result was a dramatic run-up in the price of U_3O_8, as shown in Figure 2.[3] The key point here is the strong growth in prices during the 1970s (which parallels

[2] See Joskow and Parsons (2012) for a general discussion of the putative renaissance, with particular emphasis on the post-Fukushima environment. Davis (2012) offers a much less optimistic view of the future, arguing that construction costs are so high, and alternative fuel costs so low, as to render nuclear power irrelevant.

[3] For much of the history of the industry, data were not publicly available on contracted prices, so I use the values published by NUEXCO (the Nuclear Exchange Corporation), which are a broadly-accepted proxy for the short-term price of uranium oxide. For expositional simplicity, I will refer to the NUEXCO value as the 'price' of U_3O_8.

the growth in prices during the first decade of the 21^{st} century). Indeed, the future for the uranium market looked very rosy in the 1970s, until the reactor at Three Mile Island, near Harrisburg Pennsylvania, suffered a partial meltdown in March of 1979. In very short order, the interest in nuclear energy in the U.S. waned; plans to build new plants were put on hold, and the demand for uranium collapsed, along with the price of the resource. But only a few years later, the industry recovered; indeed, during the next 10-15 years U.S. nuclear power production rose steadily, from about 10% to about 20% of the U.S. portfolio. The prospect for nuclear power to survive the setback associated with the Fukushima accident is non-negligible.

The use of nuclear power raises a host of policy-related questions, including those based on concerns over the risk of a disaster like the one that afflicted the Fukushima power plant; disposal of spent fuel rods; and matter related to International security. All these issues are important, and worthy of discussion. But in this paper I want to pursue a different, largely overlooked concern: the possible inefficiencies related to the amount of exploration undertaken by private interests. Increases in the demand for nuclear power will lead to an associated increase in demand for uranium with which to create the fuel rods for power production. This increased demand is likely to manifest itself in expanded extraction of known uranium deposits, as well as increased pressure to find new deposits. With this increased pressure will come expanded exploratory efforts, in both the U.S. and other parts of the world.

When exploring for an exhaustible resource such as Uranium, a firm is typically motivated by a desire to either find deposits of the resource, or to more accurately define the scope and quality of a known deposit. Exploration also leads to the production of information that can increase the expected value of future ventures. But production of information carries with it the potential for externalities. Two forms of externality are possible. Because the information produced from exploration is a form of public good, there is a concern that agents will free-ride on the activities of others (Gilbert, 1979; Peterson, 1975; Stiglitz, 1975). Under this scenario, agents would be less inclined to explore, leading to smaller than socially optimal levels of exploration.[4] Alternatively, agents might be inclined to use the information obtained from other agents' exploration efforts to speculate on future prices of the resource (Gaffney, 1967; Gilbert, 1979). With this motivation, agents would be more inclined to explore, because of the potential speculative bonus associated with exploration information, leading to larger than socially optimal levels of exploration.[5]

The ability to more accurately predict future rates of discovery may be of interest to society as well. With such information, rates of consumption of the resource could be adjusted to more accurately align current and future generations' welfare. Since exploration information may be valued by both private interests and society, and for different motives, it is natural to ask if firms allocate the socially optimal level of resources towards the generation of that information; that is to say, if firms explore at socially optimal level. Equivalently, we might ask if firms produce the amount of exploration information that society desires. Were this information an economic good– that is, if it had a market determined price-it would seem reasonable to expect that the answer to either question would be yes. However, there is no direct compensation available for this information. Given this lack of direct compensation, previous authors have suggested that the answer to either question might well be no. The possible reasons for suboptimal allocation of resources are: (i) exploration information might benefit agents not required to pay for that information; or (ii) agents who produce the information might use it for speculative purposes (e.g. in futures market transactions) that may have little value to society.

At the end of the day, which of these two motives is more important is an empirical question. My goal in this paper is to determine whether exploration was conducted at socially efficient

[4] This could be because the land is not as valuable as agents believe, so that the first firm drills a "dry hole" and the second firm changes its mind, ceasing preparations for exploration. Obversely, a successful venture may signal to the second firm that it should drill, even though it had not planned to. For discussion see Gilbert (1979); Isaac (1987); Peterson (1975) and Stiglitz (1975). Analogous results have been obtained regarding information production in arbitrary markets by Grossman and Stiglitz (1976).

[5] See Gaffney (1967) and Peterson (1975). In Peterson's model, every time a deposit is found, the number of undiscovered deposits declines. As this makes it harder to find a remaining deposit, all agents' search costs are increased. As the individual agent has no reason to consider this effect, there is a tendency towards over-exploration.

levels in the U.S. Focusing on the era between the Three Mile Island and Fukushima incidents, I argue that socially excessive levels of exploration were the norm. This result takes on particular significance in light of the potential expansion of industry behavior in the near future, with the associated increases in incentives to explore.

The paper proceeds with a short description of the underlying theoretical resource model in Section 2. I then sketch the learning model that underpins the exploration story in Section 3. Using these constructs, I discuss the procedure whereby one can determine the social efficiency of exploration levels in Section 4; I then apply this methodology to the U.S. Uranium industry in the era after 1981. I conclude with a brief discussion of the possible future path of nuclear power, and the uranium industry, in Section 5.

2 Exploration and Production

The model I use in this paper has N identical risk-neutral price-taking firms, with identical production and exploration cost functions.[6] I assume production costs depend on its production level y_i and remaining (proven) reserves R_i: $c(y_i, R_i)$, with costs increasing and convex in production, and both total and marginal costs declining functions of proven reserves: $\partial c/\partial y > 0, \partial^2 c/\partial y^2 > 0, \partial c/\partial R < 0, \partial^2 c/\partial y \partial R < 0$. Exploration costs, $c^e(x_i)$, are increasing and convex in x_i.

Since it is risk neutral, the typical firm's goal is to maximize the discounted flow of expected profits. The solution to this problem induces the *optimal value function* $V_i(R_t)$ for firm i; Bellman's principle implies the optimal value function corresponds to maximized current payoffs, assuming optimal decisions will be rendered in the future:[7]

$$V_i(R_t) = \max_{y_{it},\, x_{it}} \left\{ p_t y_{it} - c(y_{it}, R_{it}) - c^e(x_{it}) + \delta \mathcal{E}_1 \left[V_i(R_{t+1}) \right] \right\}. \tag{1}$$

[6] For a more detailed analysis of incentives to explore, see Isaac (1987), Lasserre (1991), Pindyck (1978) and Swierzbinski and Mendelsohn (1989). A more formal variant of my model is available in Mason (1986).

[7] As the information available in period t reflects the combination of information available in period $t-1$ with the information gleaned from exploration in period $t-1$, one can denote the expectations operators over results in

As this relation applies in period $t+1$ as well, one can also write

$$V_i(R_t) = \max_{y_{it},\, x_{it}} \Big\{ p_t y_{it} - c(y_{it}, R_{it}) - c^e(x_{it}) + \\ \delta \mathcal{E}_1 \left[p_{t+1} y^*_{it+1} - c(y^*_{it+1}, R_{it+1}) - c^e(x^*_{it+1}) \right] + \delta^2 \mathcal{E}_2 \left[V_i(R_{t+2}) \right] \Big\} \quad (2)$$

where y^*_{it+1} denotes the optimal level of production and x^*_{it+1} denotes the optimal level of exploration, as selected in period $t+1$. This recursive relation leads directly to the fundamental equation of optimality:

$$0 = \max_{y_{it},\, x_{it}} \Big\{ p_t y_{it} - c(y_{it}, R_{it}) - c^e(x_{it}) + \delta \mathcal{E}_1 \left[V_i'(R_{it+1})[-y_{it} + x_{it+1}\theta_t] \right] \Big\}. \quad (3)$$

It is apparent that the optimal level of production satisfies

$$p_t - \frac{\partial c(y^*_{it}, R_{it})}{\partial y_{it}} = \delta V'(R_{it+1}); \quad (4)$$

interpreting $\delta V_i'(R_{it+1})$ as the present discounted value of a unit of the resource left in the ground (*i.e.*, the shadow price of the resource), eq. (4) says that the optimal level of production balances the marginal value of the resource extracted today with its expected marginal value next period, the familiar Hotelling's (1931) r-percent rule. Because a unit of ore extracted this period can not be extracted next period, the impact of current extraction upon next period's optimal extraction is -1, as is the effect of current extraction upon next period's expected reserves. From eq. (1) it then

periods t and $t+1$ as

$$\mathcal{E}_1(\zeta_1) = \int \zeta_1 f(\eta_t | I_t) d\eta_t;$$
$$\mathcal{E}_2(\zeta_2) = \iint \zeta_2 f(\eta_t | I_t) f(\eta_{t+1} | I_{t+1}) d\eta_t d\eta_{t+1},$$

where ζ_1 is a arbitrary function of period t finds, hence η_t, and ζ_2 is a function of period $t+1$ finds, hence η_{t+1}, I_s is the available information in period $s = t, t+1$ and $f(\eta_s | I_s)$ is the relevant probability density function in period s.

follows that the optimal level of production satisfies

$$p_t - \frac{\partial c(y^*_{it}, R_{it})}{\partial y_{it}} = \delta \mathcal{E}_1 \left[p_{t+1} - \frac{\partial c(y^*_{it+1}, R_{it+1})}{\partial y_{it+1}} - \frac{\partial c(y^*_{it+1}, R_{it+1})}{\partial R_{it+1}} \right]. \tag{5}$$

The interpretation of this equation is that the marginal value of an incremental unit saved for next period comprises two terms: the marginal gain in profit, *i.e.*, resource rent, plus the discounted flow of future cost savings associated with a slightly greater future resource stock.

To determine the optimal level of exploration, note that a small increase in x_{it} raises expected finds, and hence next period's reserves, by an amount equal to the find rate θ_t; this raises optimal extraction next period by a like amount. Accordingly, the optimal level of exploration can be found by partially differentiating eq. (2) with respect to x_{it}:[8]

$$c^{e\prime}(x^*_{it}) = \delta \mathcal{E}_1 \left[\left(p_{t+1} - \frac{\partial c(y^*_{it+1}, R_{it+1})}{\partial y_{it+1}} + \frac{\partial c(y^*_{it+1}, R_{it+1})}{\partial R_{it+1}} \right) \theta_{t+1} \right] + \delta^2 \mathcal{F}\left(V_i(R_{t+2})\right). \tag{6}$$

The interpretation of this condition is that optimal current exploration equates current marginal cost with future expected net benefits. These net benefits come from the expected additions to rents from additional exploration, the expected future cost savings that arise from new finds, and the expected benefits associated with the refinement of future information sets, which I have written as $\mathcal{F}\left(V_i(R_{t+2})\right)$.

Because firms are identical, the first order conditions governing the optimal levels of production and exploration for a typical firm can be readily converted into conditions for industry activity:

$$p_t - \frac{\partial C(y^*_t, R_t)}{\partial y_t} = \delta \mathcal{E}_1 \left[V'(R_{t+1}) \right], \tag{7}$$

[8] As I noted above, the typical firm ignores its impact upon industry-level variables; in particular, it does not recognize its impact upon cumulative industry drilling, and hence it neglects the effect of current exploration upon next period's lagged cumulative industry-level drilling.

or, equivalently,

$$p_t - \frac{\partial C(y_t^*, R_t)}{\partial y_t} = \delta \mathcal{E}_1 \left[p_{t+1} - \frac{\partial C(y_{t+1}^*, R_{t+1})}{\partial y_{t+1}} - \frac{\partial C(y_{t+1}^*, R_{t+1})}{\partial R_{t+1}} \right]. \tag{8}$$

Likewise, industry-level exploration satisfies

$$C^{e\prime}(x_t^*) = \delta \mathcal{E}_1 \left[\left(p_{t+1} - \frac{\partial C(y_{t+1}^*, R_{t+1})}{\partial y_{t+1}} + \frac{\partial C(y_{t+1}^*, R_{t+1})}{\partial R_{t+1}} \right) \theta_{t+1} \right] + \delta^2 \mathcal{F} \left(V(R_{t+2}) \right). \tag{9}$$

In eqs. (7) and (9), $C(y,R)$ represents industry production costs, $C^e(x)$ is industry exploration cost and $V(R)$ is the aggregation of individual firms' optimal value functions, which one can think of as the industry-level optimal value function.[9]

Now consider the decision problem facing a beneficent social planner, whose objective is to maximize expected net social surplus, the sum of expected consumer and expected producer surplus. In the discussion below I will often refer to this as (social) welfare, and denote it by $W(y,x,R)$. In any period t, this equals

$$W(y,x,R) = \int_0^y p(v)\mathrm{d}v - C(y,R) - C^e(x), \tag{10}$$

where $p(\cdot)$ can be thought of as the Hicksian-compensated inverse demand curve for the resource. Similar to the private optimization problem, one may define the optimal value $\Omega(R)$. As above, one may expressing the optimal value function as:

$$\Omega(R_t) = \max_{y_t,\, x_t} \left\{ W(y_t, x_t, R_t) + \delta \mathcal{E}_1 \left[\Omega(R_{t+1}) \right] \right\}, \tag{11}$$

[9] This interpretation is subject to the qualification that there can be externalities associated with firms' decisions, so $V(R_{t+2})$ is not the optimal industry value function were firm decisions to be made cooperatively.

or

$$\Omega(R_t) = \max_{y_t,\, x_t} \Big\{ W(y_t, x_t, R_t) + \delta \mathcal{E}_1 \big[W(y_{t+1}, x_{t+1}, R_{t+1})] \Omega(R_{t+1}) \big] + \delta^2 \mathcal{E}_2 \big[\Omega(R_{t+2}) \big\}. \tag{12}$$

The socially optimal combination of production and exploration maximizes the expected present discounted flow of these payoffs.

Since $\frac{\partial W(y,x,R)}{\partial y} = p(y) - \frac{\partial C(y,R)}{\partial y}$ in any period t, the socially optimal level of production, $\hat{y}_t$, solves

$$p(\hat{y}_t) - \frac{\partial C(\hat{y}_t, R_t)}{\partial y_t} = \delta \mathcal{E}_1 \Big[p(\hat{y}_{t+1} - \frac{\partial C(\hat{y}_{t+1}, R_{t+1})}{\partial \hat{y} t + 1} - \frac{\partial C(\hat{y}_{t+1}, R_{t+1})}{\partial R_{t+1}} \Big]. \tag{13}$$

Alternatively, letting $\Omega(R)$ denote the optimal value function for society, the analogue to the value function for industry, $V(R)$, then the socially optimal level of production in period t solves

$$p(\hat{y}_t) - \frac{\partial C(\hat{y}_t, R_t)}{\partial y_t} = \delta \mathcal{E}_1 \Big[\Omega'(R_{t+1}) \Big]. \tag{14}$$

These conditions parallel the first-order condition determining privately optimal production levels.

The socially optimal exploration level, $\hat{x}_t$, solves

$$C^{e\prime}(\hat{x}_t) = \delta \mathcal{E}_1 \Big[\Big(p(\hat{y}_{t+1} - \frac{\partial C(\hat{y}_{t+1}, R_{t+1})}{\partial \hat{y}_{t+1}} \frac{\partial C(\hat{y}_{t+1}, R_{t+1})}{\partial R_{t+1}} \Big) \theta_{t+1} \Big] + \delta^2 \mathcal{E}_2 \big[\Omega'(R_{t+2}) \frac{\partial R_{t+2}}{\partial x_t} \big] + \delta^2 \mathcal{F} \big(\Omega(R_{t+2}) \big). \tag{15}$$

The effect of current exploration upon reserves in two periods arises here because those reserves are influenced by next period's finds, which in turn in influenced by $\phi(X_t)$. Accordingly, eq. (15) can be rewritten as:

$$C^{e\prime}(\hat{x}_t) = \delta \mathcal{E}_1 \Big[\Big(p(\hat{y}_{t+1}) - \frac{\partial C(\hat{y}_{t+1}, R_{t+1})}{\partial \hat{y}_{t+1}} \frac{\partial C(\hat{y}_{t+1}, Rt+1)}{\partial R_{t+1}} \Big) \theta_{t+1} \Big] + \delta^2 \mathcal{E}_2 \big[\phi'(X_t) x_{t+1} \Omega'(R_{t+2}) \big] + \delta^2 \mathcal{F} \big(\Omega(R_{t+2}) \big). \tag{16}$$

Because market price is dictated by the inverse demand function, at the observed level of market production, in any period t it must be the case that $p_t = p(y_t)$. It is therefore apparent that the rules for determining private and socially optimal levels of production are quite similar (Hotelling, 1931). In particular, if privately selected exploration levels were socially optimal in every period the expected additions to reserves associated with exploration during period t would then correspond to the expected additions that would emerge from the socially optimal level of exploration; in turn this implies the socially optimal shadow price of reserves would govern production choices in every period. Moreover, the probability distribution governing future expectations will be the same for society as for private interests. Accordingly, the privately optimal level of production would be socially optimal in every period. Evidently, everything turns on the social optimality of privately chosen exploration levels.

Considering then a comparison of eqs. (9) and (16), two differences are apparent. The first relates to the impact of current exploration upon future find rates, as measured by the first term on the second line in (16); I denote this effect by L_1:

$$L_1 = \delta^2 x_{t+1} \phi'(X_t) \mathcal{E}_2 \left[\Omega'(R_{t+2}) \right]. \tag{17}$$

The second effect corresponds to the value of information as gauged by the social value function, the second term on the second line in (16). I denote this effect by L_2:

$$L_2 = \delta^2 \mathcal{F} \left(\Omega(R_{t+2}) \right). \tag{18}$$

Determination of the degree and direction of social inefficiency in exploration requires evaluating terms related to the future optimal value function for society. In principle, this function will be based on an infinite discounted flow of effects, which is plainly impractical to determine empirically. An alternative is to truncate the effects at some point in the future, so as to render the calculations feasible. Such a truncation must include at least the terms for the two periods after decisions are made, *i.e.*, periods $t+1$ and $t+2$. Taking the view that additional effects will add progressively smaller amounts of useful information to the discussion, while simultaneously raising the complexity of the analysis, I proceed using an approximation to the optimal value function that considers only those two future periods. Under this approach, the two effects of interest reduce to

$$L_1 = \delta^2 x_{t+1}\phi'(X_t)\mathcal{E}_2\Big[p(\hat{y}_{t+2}) - \frac{\partial C(\hat{y}_{t+2}, R_{t+2})}{\partial \hat{y}_{t+2}} - \frac{\partial C(\hat{y}_{t+2}, Rt+2)}{\partial R_{t+2}}\Big] \quad (19)$$

$$L_2 = \delta^2 \mathcal{F}\left(\hat{C}S_{t+2}\right). \quad (20)$$

The expectation in eq. (19) corresponds to period $t+1$ rents, and so is positive, while the contribution from ϕ will be positive in early periods and negative later on—in particular, during the epoch I analyze later in the paper. That noted, the contribution from ϕ is likely to be numerically small, pointing to L_2 as the more important potential source of inefficiencies.

Determining the sign and magnitude of L_2 depends on the nature of deviations between social and private value functions. In principle, the social value function differs from industry's optimal value function in that the former involves the present discounted flow of both profits and consumer surplus, while the latter includes profits only. The concern that private exploration choices will not be socially optimal then reflects the concern that neglecting the value of information, as gauged through consumer surplus, could lead to systematic departures from socially optimal choices. One view is that agents opt to explore at inefficiently low levels because they prefer to wait for others to do the work. This incentive reflects the potential public good nature of information: if one agent's efforts reveal that exploration is more likely to succeed than was previously thought, other firms can free ride on the new information, capitalizing on the good news without paying the cost of exploration. On the other hand, if results are pessimistic, the free-riders avoid paying the extra cost. An alternative motivation to under-explore is strategic in nature: if agents expect larger levels of exploration by their rivals they will explore less (*i.e.*, exploration is a *stategic complement*). As this effect reflects a tendency to under-capitalize on potential marketing opportunities, one might expect it to be amplified as demand expands. In the other direction, if agents believe they can successfully speculate based on their own results, there can be an over-investment in exploratory activity. The idea here is that information from exploratory efforts allows revised prediction of discovery rates and total stock of the resource. These new predictions will be made by the economy when the information becomes public, and will have a predictable effect on the resource's spot and future prices, and consequently on the value of any related assets. If the firm thinks it can make these predictions before the economy, it will believe it can profitably play the futures market.[10]

The key point here is that privately optimal exploration choices will only correspond to the socially optimal exploration level in each period under very demanding conditions. In particular, this will only occur if the two external effects, related to the public bad nature of current exploration upon future find rates, combined with the marginal expected value of exploration information based on consumer surplus, cancel out. In practical terms, this requires L_2 to be very small.

3 A Statistical Learning Model

In this section, I sketch out a learning model associated with the exploration process.[11] The *find rate* θ_t, the rate at which an exhaustible resource is discovered per unit of exploration in any period t, is determined by two effects, one deterministic and one stochastic.

The deterministic effect reflects two influences. On the one hand, as the resource is exhaustible the find rate must eventually decline over time (on average). On the other hand, early on

[10] On a related note, if mineral rights are assigned by an auction process, firms have an incentive to speculate on the expected value of the lease. This is true since privately held information tends to increase expected profits from participating in an auction (Milgrom and Weber, 1982). Gilbert (1979) concludes that "there may well be redundant investment in exploration, as each firm seeks information to improve its expected profit from the auction." The phenomenon of excessive allocation of resources towards information production has been discussed by Hirschleifer (1971) in the context of allocation towards research and development, and by Goldberg (1977) in a contracting context.

[11] A thorough description of the learning model is available in Mason (1985).

there is a potential for agents to learn where and how to explore. As both phenomena seem likely to be related to past experience, I assume the deterministic component is a function of previous exploration, X_{t-1}:

$$\phi(X) = \alpha X^{\beta} e^{-\gamma X}, \tag{21}$$

where α, β and γ are all positive parameters. These parameters can be estimated using data published in U.S. Energy Information Administration (1983). This government publication tabulates average finds per unit of exploratory drilling, along with cumulative number of exploratory holes drilled, for the period 1948-1975.[12] Results from a log-linear regression based on eq. (21) are reported in Table 1. I note that all point estimates are statistically significant, and estimates of β and γ are positive, as predicted.

I assume the stochastic effect can be expressed by e^{η}, where the essential role of uncertainty enters via η. Combining these two effects, the find rate in period t can be expressed as

$$\begin{aligned} \theta_t &= \phi(X_{t-1}) e^{\eta_t} \\ &= \alpha X_{t-1}^{\beta} e^{-\gamma X_{t-1}} e^{\eta_t}. \end{aligned} \tag{22}$$

Modeling the evolution of learning requires imposing some distributional assumptions on η; the simplest way to do this is to assume that η is Normally distributed, or equivalently that the stochastic component in the find rate is log-normally distributed.

The economic actors in this scenario — either firms or some government agency — are unsure about the exact specification of the distribution over η. I assume this ignorance arise because agents know neither the mean of η, μ, nor the precision (inverse of variance), ρ. Learning then obtains from developing more informed guesses regarding possible values of μ and ρ. These new guesses, or estimates, are formed by combining old beliefs with new information, which is contained in the results of the previous year's exploration. The refined estimate of μ is a weighted

[12] Unfortunately, reporting of find rates was discontinued after 1975.

average of the old estimate and a sample estimate based on the new information, where the weight attached to each part reflects how much faith agents have in it. One imagine the faith attached to the sample estimate will depend on the number of observations in the sample, while the faith attached to the old estimate will depend on how precise agents felt that estimate was. The refined estimate of ρ will depend on the probable amount of dispersion associated with agents' original beliefs about η and upon the sample variance. One expects the weight attached to each ingredient will be related to the degrees of freedom associated with it (as this gives a measure of the confidence associated with the ingredient). Based on these updated values, then, the distribution over η can be "guessed" by integrating out the uncertainty posed by not knowing μ and ρ. I show below that this procedure yields a variant of the student's t-distribution.

To formulate these intuitions, one needs to know how to describe the original, or *prior*, beliefs about possible values μ and ρ. The analytics are greatly facilitated by using a distribution whereby the refined, or *posterior* beliefs, take on the same distribution as the prior beliefs. This can be accomplished by assuming that the joint distribution over μ and ρ is given by the *Normal-gamma* distribution (DeGroot, 2004).[13] It turns out that beliefs can be summarized by the 4-tuple $(\mu, \tau, \alpha, \beta)$; letting $(\mu_0, \tau_0, \alpha_0, \beta_0)$ represent the prior beliefs and $(\mu_1, \tau_1, \alpha_1, \beta_1)$ the posterior beliefs, the updating process described as follows:[14]

$$\mu_1 = \frac{\mu_0\tau_0 + n\overline{\eta}}{\tau_0 + n}; \tag{23}$$

$$\tau_1 = \tau_0 + n; \tag{24}$$

$$\alpha_1 = \alpha_0 + n/2; \tag{25}$$

$$\beta_1 = \beta_0 + \frac{n}{2}\left(s_\eta^2 + \frac{\tau_0}{\tau_1}(\overline{\eta} - \mu_0)^2\right). \tag{26}$$

[13] Possible values that μ can take, given that agents believe $\rho = r$, are distributed according to a normal probability density function with some mean μ_0 and some precision $r\tau_0$. The marginal distribution of ρ is a gamma distribution, whose parameters are α_0 and β_0.

[14] This is essentially Theorem 1 in DeGroot (2004); see also Mason (1985) for further discussion.

From this specification, it is straightforward to combine the probabilistic information underlying the data, *i.e.*, the sequence of 'draws' from exploratory ventures. The data from a period's worth of exploration can be regarded as a sample from the true distribution governing e^{η}; equivalently, one can think of this data as comprising a sample on η. The key point here is that the posterior distribution is of the same functional form as the prior distribution, *i.e.*, it is a Normal-gamma distribution, with the key difference between the posterior distribution and the prior distribution being the corresponding parameter vectors.

Finally, the reduced form distribution over θ is then found by "integrating out" the uncertainty associated with not knowing the mean and precision of θ. It turns out in this setting that the resultant distribution is a generalized version of the student's t-distribution (Mason, 1985), which facilitates calculation of its associated moments.

4 The Social Efficiency of Exploration for Uranium in the U.S.

The results from section 2 indicate that exploration levels will only be socially efficient if the marginal expected value of information, using consumer surplus as the valuation metric, is very small. That observation points towards an empirical methodology for evaluating the efficiency of exploration: if one was able to determine the market demand curve, it would be straightforward to derive consumer surplus. As future production levels will be tied to reserve levels, future expected consumer surplus will be tied to expected finds. Accordingly, one can determine the impact upon expected future consumer surplus that manifests from a small increase in current exploration levels, via the impact upon next period's information set.

To investigate the potential inefficiency in exploration empirical problem, then, I first obtain estimates of the demand for uranium. There are two epochs of note: immediately after the end of the Second World War, through the early 1970s, uranium was largely used for defense purposes; from the middle 1970s forward, uranium was largely used as an input into electricity production, with nuclear power reactors. Because this latter epoch is the more relevant today and in the future, I focus here on the period after 1980.[15]

In this era there is a distinction between U.S. production and U.S. consumption, as a non-trivial fraction of uranium is purchased from international sources. Despite this distinction, there are reasons to suspect that total production is not exogenous, and so one needs to take steps to address this potential endogeneity. To this end, I conduct an instrumental variables estimation procedure. The model above implies that firm's production decisions should be tied to proven reserves at the time those decisions are made (*i.e.*, at the start of the period). Accordingly, an obvious instrument is proven reserves at the end of the preceding period. During the period in question, the measure of reserves most widely accepted was based on a forward cost of $50/lb. of U_3O_8, which I denote as R_{50}.[16] In addition to reserves, a measure of immediate past cumulative industry drilling and immediate past exploration levels seem likely to be correlated with production, without being influenced by current price. Accordingly, I employ three instruments in estimating demand. As I discuss in the next section, there is a strong correlation between nuclear power plant capacity and the (real) price of crude oil. This correlation suggests the potential for the oil price to serve as a demand-shifting factor, and so I include it as a regressor.[17]

The results of this IV procedure are presented in the third column of Table 2; standard errors are listed below point estimates. For purposes of comparison, I also list OLS estimates (in the second column). I note that production levels exert a statistically and economically significant negative effect, corresponding to a downward-sloping demand curve. I note as well that with (inverse) demand represented as a linear function of output, consumer surplus is quadratic in output.

I also provide estimates of the supply curve for uranium traded in the U.S. As price is almost surely endogenous, I pursue an instrumental variables approach here as well; based on

[15] For a discussion of the social inefficiency prior to 1979, see Mason (1989).

[16] The concept of forward cost can be loosely interpreted as long run average cost, in that it takes into account both short term variable expenses and set-up costs associated with opening a new well. The precise interpretation is not critical; what matters is that the variate in question is consistent across the entire sample, as is the case here.

[17] Data on crude prices are taken from British Petroleum (2012).

the interpretation of crude oil price as a demand-shifter it is an obvious choice for an instrument. The results of this IV procedure are presented in the third column of Table 3; standard errors are listed below point estimates. For purposes of comparison, I also list OLS estimates (in the second column). I note that price levels exerts a small but statistically insignificant positive effect, corresponding to an upward-sloping (but highly inelastic) supply curve. I note as well that OLS and IV estimates differ markedly, underscoring the likely endogeneity of uranium prices.

Combining demand and supply information, one can derive a characterization of equilibrium output. For the purpose of evaluating expected marginal value of information, the key point here is that equilibrium output is linked to reserves, which implies that next period's anticipated finds will influence the value of exploration information two period's hence. Moreover, since consumer surplus is quadratic in output, while output is linear in finds, it is easy to see that the expected value of exploration information is based on the prospective mean and variance of the reduced form distribution governing η (which, as I noted in section 3, is a generalize version of the student's t-distribution). As a result, determining the expected marginal value of exploration information is numerically straightforward:

1. calculate the mean and variance of e^{η} based on observed exploration levels;
2. calculate the mean and variance e^{η} based on a counterfactual with slightly larger exploration levels;
3. use these two constructs to numerically approximate the marginal effect of a small increase in exploration.

Figure 3 illustrates the resultant expected marginal values of exploration information, period by period, for the time frame between 1981 and 2010.[18] The take-away point here is that these values

[18] There is a slight caveat to this point: in 2001 and 2002 there were so few domestic firms operating in the U.S. uranium industry that the EIA withheld reserve data for those years. Accordingly, there is a brief period in the sample where the requisite data are simply not available, and so I do not construct predicted values for those years. These omissions are reflected by the lack of values for 2001, 2002 and 2003 in the Figure.

are consistently positive; moreover, they increased dramatically over the final 5 years of the sample. Since the contribution from the other potential source of inefficiency, L_1, is also positive (though small), the combination of these two effects is most certainly positive in every period. The upshot is that inefficiently low exploration levels appear to be the norm in the U.S. during the era where uranium is used as an input into nuclear power production; moreover, this effect appears to be have been exacerbated as demand for uranium grew recently.

5 Peering Into the Future

The finding that U.S. firms tended to explore at inefficiently low levels over the past few decades raises questions about global exploration levels. Further, as the degree of under-exploration is apparently positively related to the level of demand for nuclear power, one is led to an evaluation of the future prospects for nuclear power. With that in mind, I note that global nuclear capacity and the number of global nuclear plants both increased steadily over a similar time-frame to that considered in the empirical analysis of the previous section (IAEA, 2011). In that regard, it is interesting to consider the year-on-year planned and realized additions to global capacity, which are illustrated in Figure 4. The dashed line, termed "planned additions to Global capacity", shows the anticipated capacity additions related to plants for which construction has started, while the solid line shows actual changes in Global capacity. The latter would include all of the former were the additions originally planned to actually pan out. While it is apparent that not all of these planned additions are ultimately completed, there is a clear link between the two series, with actual additions being offset from planned additions by somewhere between 7 and 10 years. In light of the apparent lag between planned capacity additions and actual additions, it is plausible that the world will see a steady increase in actual capacity during the next 5 years or so, which will imply an ongoing increase in demand for U_3O_8.

One plausible explanation for this large impending demand for nuclear power plants is the realization that nuclear energy may provide the best alternative source of energy to coal in light of fears of climate change.[19]

The potential significance of future carbon policy noted, there are other signs supporting a potential ongoing increase in demand. While short-term prices fell from a high of nearly $66 per pound of uranium concentrate in 2008 to a low just below $45.50 per pound in 2009, they actually rebounded in 2010. One possible explanation for this recent surge in planned capacity expansions that also offers an explanation for the uptick in activity in the 1970s, along with the retreat in the years after, is that both periods of expansion correspond to periods where energy markets were generally tighter, while the period of retreat corresponds to a period where energy markets were generally softer. One way to quantify the notion of tightness in energy markets is via the spot price for crude oil. While crude is not itself an important input into electricity production, it has strong correlation with coal and gas prices, both of which are key energy inputs. With this in mind, I plot the time series of real annual crude oil prices (in 2010 US dollars), from 1954 to 2010 along with anticipated capacity expansion in Figure 5. While it seems apparent that oil prices do not explain all features of planned capacity, there is a clear correlation between the two time series. As they are set in a global market, it is implausible that oil prices could be significantly impacted by planned capacity changes in the U.S. nuclear energy sector. It is more conceivable that causation could run in the other direction. This point is important, because to the extent that oil markets exert an important influence on nuclear energy, and thereby on the uranium market, one can anticipate the future of the latter by thinking about the future of the former.

Looking ahead, I believe there are good reasons to anticipate growing demand for uranium. Increased capacity at nuclear power plants, planned creation of additional plants (particularly in China, India and Korea) and increasing oil prices signal the potential for important increases in

[19] This point has been made in the mainstream news media (Calas, 2011) and in academic circles (Hoffert et al., 2002, 2003). These latter authors argue that major new finds are required if nuclear fusion power is to play a significant role in combating climate change. Huber and Mills (2005, p. 168) state unequivocally: ”[t]he one demonstrably practical technology that could decisively shift U. S. carbon emissions in the near term would displace coal with uranium.”

demand. The results from the preceding section point towards a tendency towards inefficiently low levels of exploration; the results from Figure 3, particularly in the last decade, indicate that expansions in demand may well magnify this tendency. To the extent that one accepts the likely expansion of global demand for uranium going forward, the potential for low exploration levels seems substantial in the near future. In turn, this under-exploration seems likely to generate a smaller than optimal resource base, which will hinder uranium production, thereby impinging on nuclear power production. To the extent that nuclear power provides an attractive substitute to coal, then, one implication of inefficiently low exploration is larger than desirable carbon emissions from coal-fired power production.

These observations point to arguments in support of measures that promote the use of nuclear power, for example though the encouragement of R&D into next-generation nuclear technology (anonymous, 2012). But there are also reasons to be concerned about overly exuberant support, as has been argued to apply to the U.S. uranium industry in its early days (Mason, 1989). Perhaps a satisfactory balancing of these two concerns could emerge from the construction of a carbon market; indeed, to the extent that one believes the support of nuclear power is an attractive option an argument can be made for a somewhat larger carbon price.

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Table 1: Estimates of Deterministic Component in Find Rate

parameter	point estimate	standard error
β	2.535	.7000
γ	.01482	.00276
$\ln(\alpha)$	-14.944	7.286
R^2	.5812	

Table 2: Estimates of Demand for Uranium, 1981-2010

regressor	OLS	IV
Q_{all}	-.7926	-.8473
	(.2222)	(.4022)
P_{crude}	.4771	.4878
	(.0824)	(.1047)
constant	32.278	34.168
	(6.469)	(10.848)
R^2	.5570	.5513

Table 3: Estimates of Supply of Uranium, 1981-2010

regressor	OLS	IV
P_{dom}	.0837	.2999
	(.1579)	(.2040)
R_{50}	.0476	.0696
	(. 0251)	(.0290)
lagged drilling	1.0387	1.0247
	(.2408)	(.2531)
lagged cumulative drilling	.00042	.00054
	(.00013)	(.00016)
constant	-822.96	-1091.8
	(278.27)	(327.55)
R^2	.5912	.5487

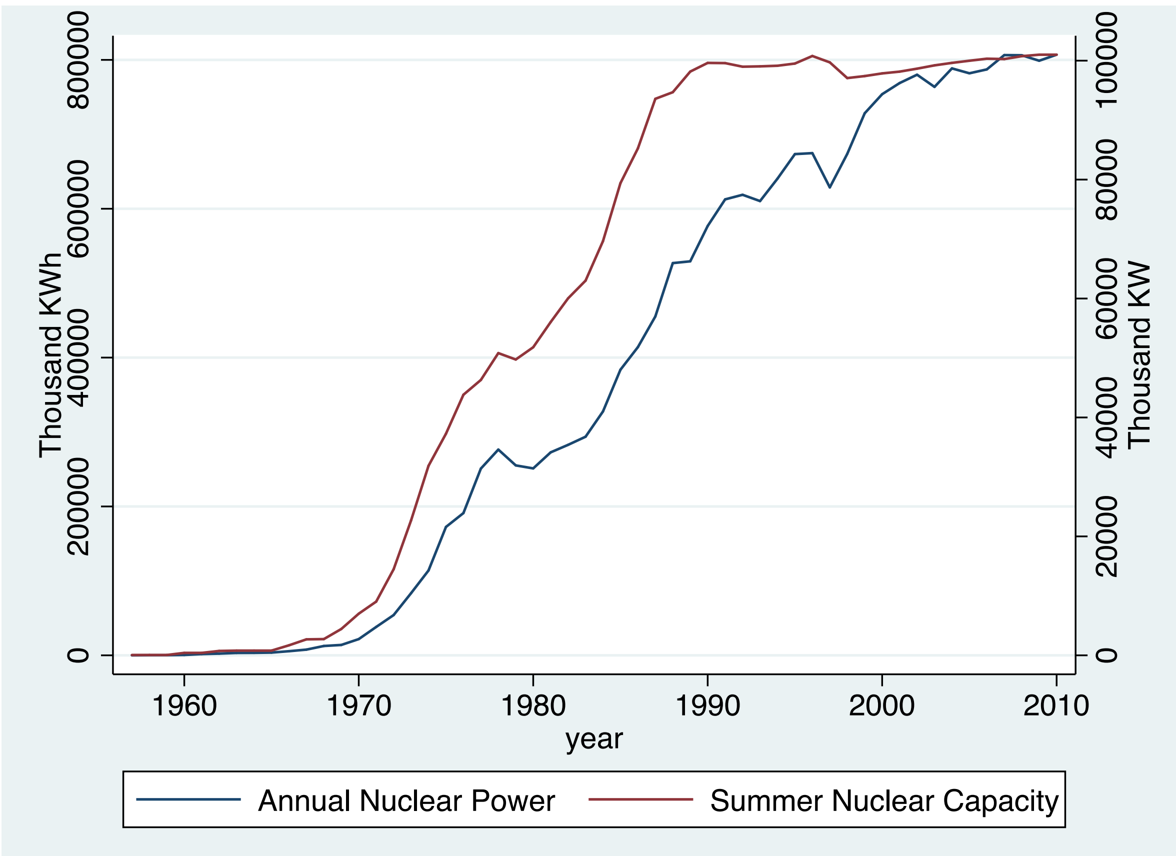

Figure 1: U.S. Nuclear Power Capacity, 1954-2010

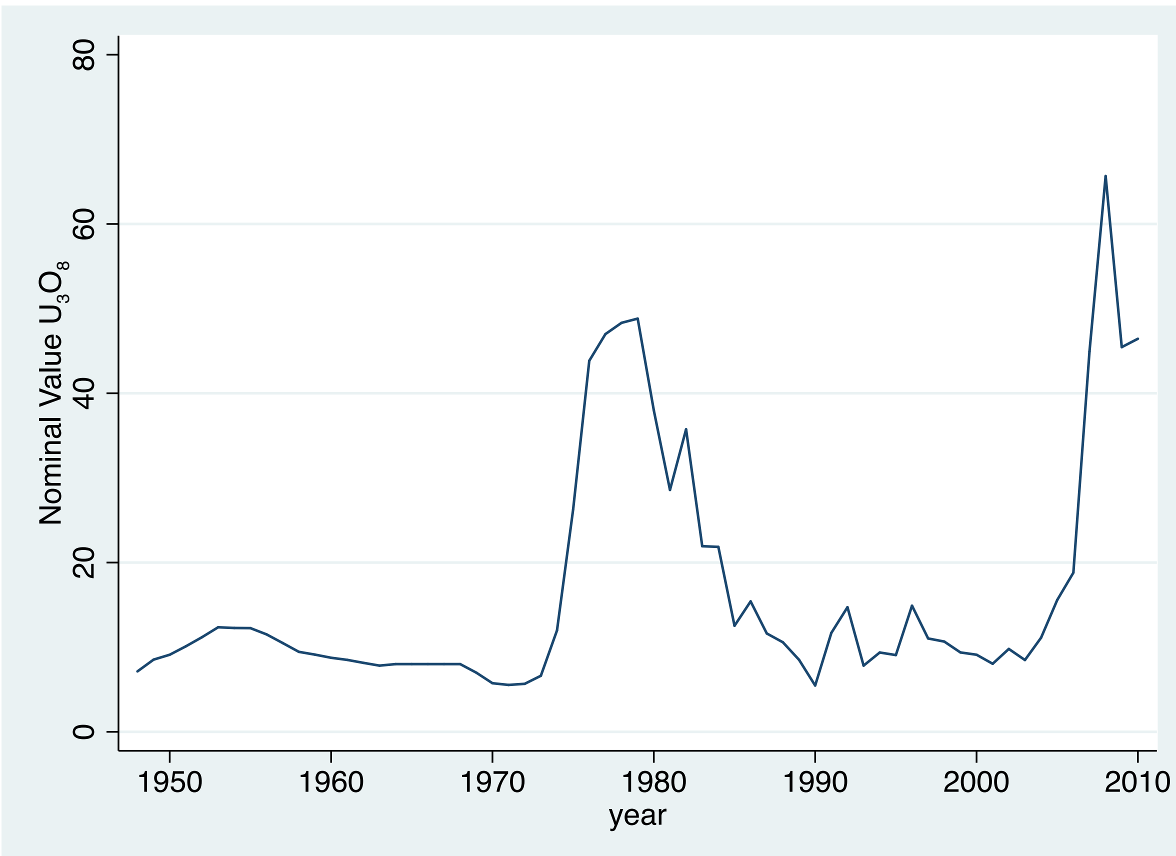

Figure 2: Uranium Prices: 1948-2010

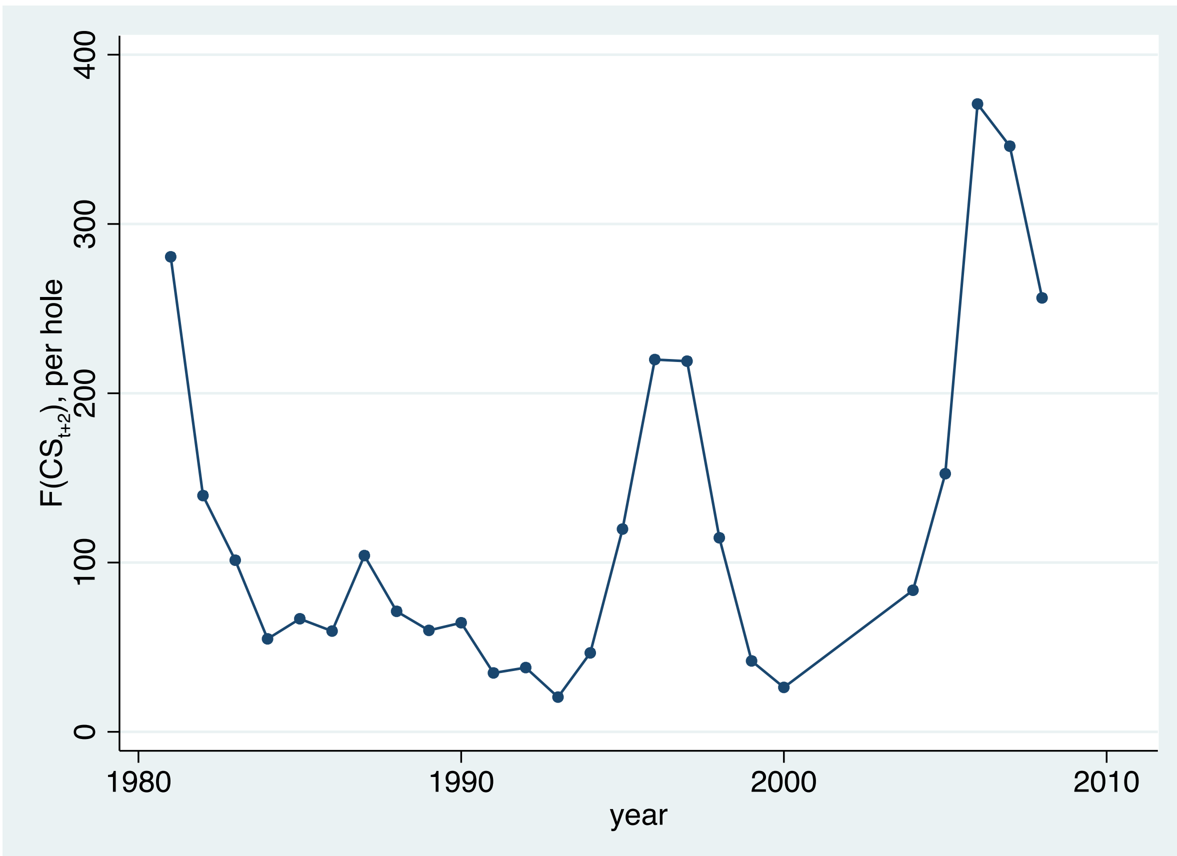

Figure 3: Predicted Marginal Value of Information Based on Consumer Surplus, 1981-2010

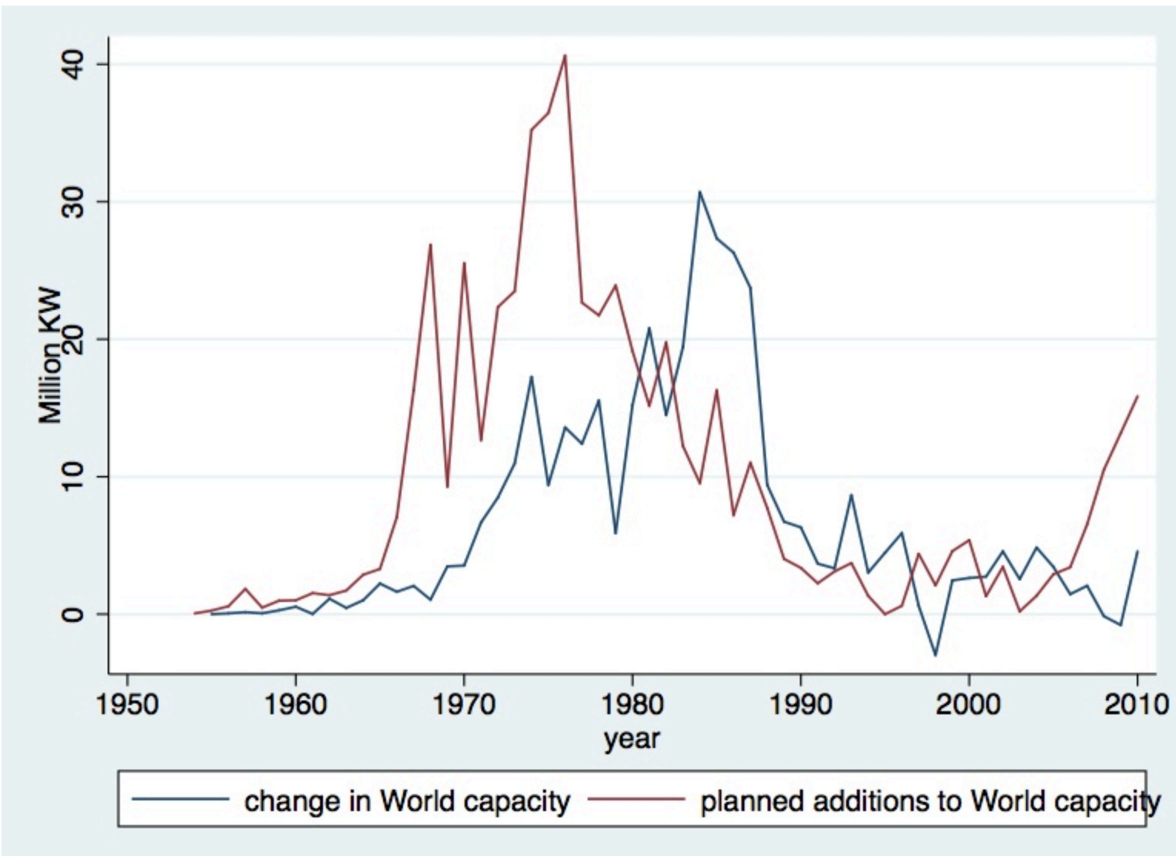

Figure 4: Planned and Realized Changes in Global Nuclear Power Capacity, 1954-2010

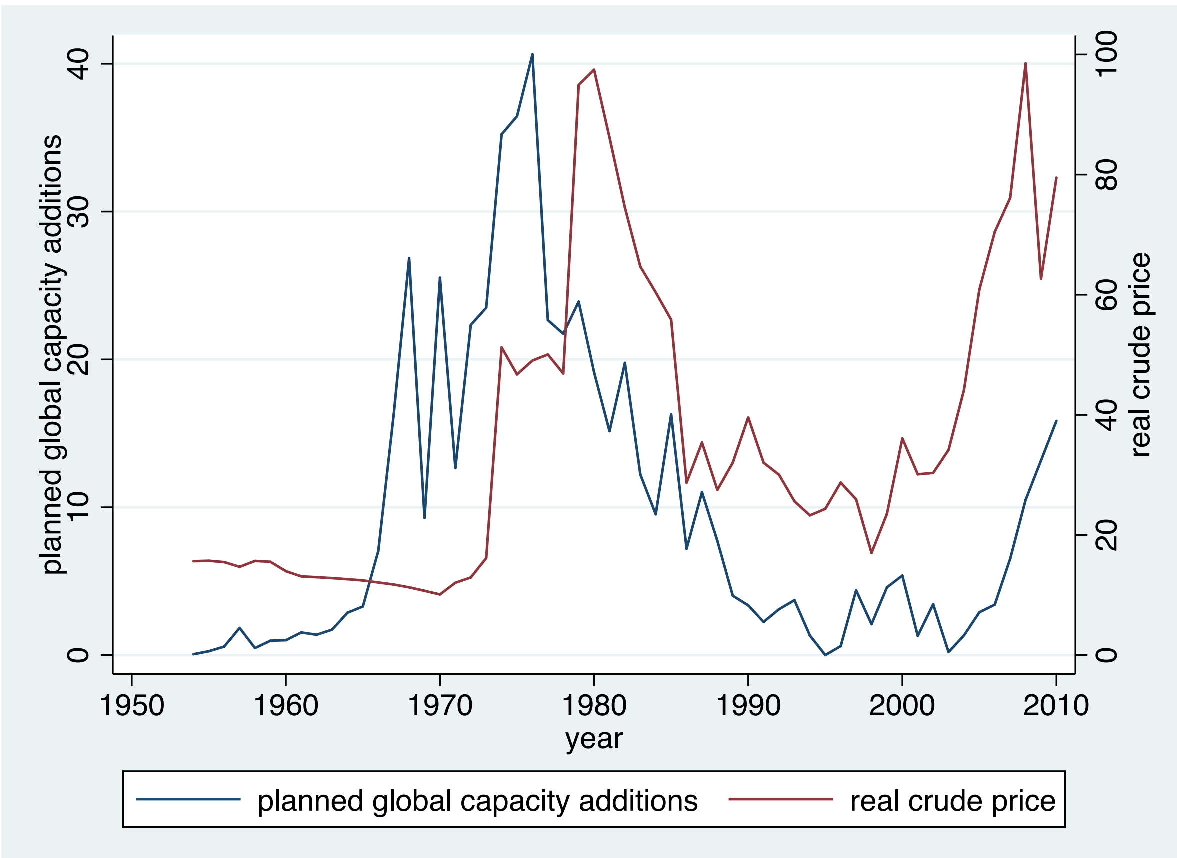

Figure 5: Real Crude Prices and Planned Changes in Global Nuclear Power Capacity, 1954-2010