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Roberta Distante
Fondazione Eni Enrico Mattei, roberta.distante@feem.it

Ivan Petrella
Department of Economics, Mathematics and Statistics, Birkbeck, University of London

Emiliano Santoro
Department of Economics and Finance, Catholic University of Milan and Department of Economics, University of Copenhagen

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Asymmetry Reversals and the Business Cycle*

Roberta Distante† Ivan Petrella‡ Emiliano Santoro§

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Abstract

The cross-sectional dynamics of the U.S. business cycle is examined through the lens of quantile regression models. Conditioning the quantiles of firm-level growth to different measures of technological change highlights a deep connection between counter-cyclical skewness and the transmission of aggregate disturbances. Asymmetry reversals emerge as the dominant source of cyclical variation in the probability density, generating a powerful amplification of aggregate shocks to firm technology. Designing and validating heterogeneous firm business cycle models should necessarily account for this empirical finding.

JEL classification: C21; E32.

Keywords: Corporate Growth, Conditional Quantiles, Business Cycles, Asymmetry Reversals.

1 Introduction

Over the last two decades increasing interest has been devoted to understanding how microeconomic decisions affect the macroeconomy. Caballero (1992) has argued that probability theory imposes strong restrictions on the joint behavior of a large number of units that are less than fully synchronized. In line with this insight, a number of authors have recognized the importance of tracking the business cycle behavior of firm-level dispersion over several dimensions, such as investment, output growth, productivity and price-setting. Complementing the study of major macroeconomic aggregates with the analysis of the business cycle from the cross section has proven to be an important disciplining device for heterogeneous firm models (see, e.g., Bachmann and Bayer, 2011). This paper examines time-variation in the distribution of firm growth and its implications for business cycle dynamics. We estimate the quantiles of U.S. quoted companies' growth rates of real sales, conditioning them on both firm-specific characteristics and alternative measures of technological change. Unlike the approach followed so far – which focuses on a restricted subset of empirical moments – we characterize the cyclical behavior of the entire density of firm growth, as well as its response to aggregate shocks that are commonly

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†Fondazione Eni Enrico Mattei, Milan. *Address*: Corso Magenta 63, 20123 Milan, Italy. *E-mail*: roberta.distante@feem.it.

‡Department of Economics, Mathematics and Statistics, Birkbeck, University of London. *Address*: Malet St, London WC1E 7HX, UK. *E-mail*: i.petrella@bbk.ac.uk.

§Department of Economics and Finance, Catholic University of Milan and Department of Economics, University of Copenhagen. *Address*: Via Necchi 5, 20123, Milan, Italy. *E-mail*: emiliano.santoro@unicatt.it.

regarded as important drivers of the business cycle. A key result is that skewness in the density of firm growth should be carefully accounted for when imposing empirical restrictions on heterogeneous firm models. In fact, counter-cyclical asymmetry emerges as a powerful amplifier of technology shocks, implying substantial reallocation of probability mass from one side of the distribution to the other. As a result, shifts and contortions in the density of firm growth play a crucial role in shaping macroeconomic fluctuations, making a strong case for business cycle models that emphasize the importance of microeconomic adjustment for aggregate dynamics (e.g., Caballero *et al.*, 1995, Caballero and Engel, 1999, Bachmann *et al.*, 2013).

Historically, a great deal of attention in the literature on industrial demography has been devoted to exploring the static properties of the distributions of firm size and growth. A number of theoretical and empirical contributions have focused on the assessment of the theoretical proposition known as Gibrat's law (Gibrat, 1931), which predicts randomness of firm growth rates. Up to early 90s the general consensus seemed to be in line with this prediction, along with indicating that the distribution of firm size was a member of the log-normal family whose variance becomes asymptotically infinite (Gibrat's law in strong form). More recent evidence has put this conventional wisdom into question, as the tendency towards larger firms in the economy has been reversed and several studies find evidence of a negative relationship between growth rates and firm size.[1] Altogether, these results have spurred a renewed interest in the study of firm size distribution (see, e.g., Machado and Mata, 2000 and Cabral and Mata, 2003), while little attention has been received by firm growth and its drivers. The present study represents an important attempt to bridge the industrial dynamics tradition with the business cycle literature. In this respect, the importance of allowing for asymmetric time-variation in the density of company growth rates is warranted by recent findings of Holly, Petrella, and Santoro (2013). According to this study systematic changes in the density display leading properties with respect to the business cycle, so that shifts in the probability mass may propagate and amplify macroeconomic fluctuations, as originally hinted by Caballero (1992) and Caballero and Engel (1992, 1993). We build on these findings and add some important insights to this line of inquiry.

Conditional quantiles show that changes in the asymmetry of the distribution are the dominant source of business cycle variation in the cross-section of firm growth, while changes in the degree of dispersion are of second order importance for propagating and amplifying macroeconomic fluctuations. The business cycle induces an inversion in the asymmetry of the distribution of firm growth, generating counter-cyclical skewness. In addition, lower quantiles display stronger co-movement with respect to aggregate fluctuations, as compared with the right tail of the density, thus signalling that dispersion tends to increase during contractionary episodes. Both features have been widely documented by Higson *et al.* (2002, 2004) and Holly, Petrella, and Santoro (2013). However, these facts do not help us interpret how aggregate disturbances transmit throughout the entire spectrum of firm growth and how this reflects into fluctuations at the macroeconomic level. To dig deeper into the cross-sectional dynamics underlying the business cycle we condition the distribution quantiles to different measures of technological change. We highlight a deep connection between systematic asymmetry reversals and the amplification

[1] See Hall (1987), Evans(1987a,b), Dunne, Roberts, and Samuelson (1989). Consistent with the assumption of decreasing returns to scale, these works show that small firms tend to grow faster than large ones. This implies a mean reversion effect on firm size, which introduces an overall limit on the variance of the size distribution, as firm size converges in the long run towards an optimal level.

of shocks to the aggregate economy. An aggregate technology disturbance induces a substantial reallocation of probability mass over the domain of firm growth, reflecting into marked changes in the skewness of the density. Along with this effect, asymmetries in the response of the tails of the density reflect location and scale shifts that have little or no role in amplifying the response of real GDP to aggregate disturbances.

A large body of theoretical and empirical literature is expanding around the analysis of the business cycle from the cross section.[2] In this respect, the importance of the results in this study traces back to their core implications for designing and validating heterogeneous firm business cycle models. From an empirical viewpoint, we show that resorting to methods that go "beyond the mean" may unveil relevant information when firm-level heterogeneity is pervasive and matters for the transmission of aggregate disturbances. We demonstrate that neither spread preserving shifts nor scale shifts in the density of firm growth play a major role in the transmission of technology shocks. By contrast, asymmetry reversals are the key to uncover the cross-sectional dimension of macroeconomic fluctuations. Formulating models that emphasize the role of heterogeneity in amplifying and propagating aggregate technology shocks should necessarily start from these considerations.

The remainder of the paper is laid out as follows: Section 2 describes the data and presents a preliminary exploration; Section 3 introduces the quantile regression framework and presents some evidence on the dynamics of the density of firm growth at both business cycle and secular frequencies; Section 4 explores the transmission of alternative measures of technological change on the cross-section of firm growth and the associated aggregate dynamics; Section 5 discusses the implications of our results for the theoretical literature on heterogeneous firm business cycle models; Section 6 concludes.

2 Data and Preliminary Evidence

We employ annual accounting COMPUSTAT data over the 1950-2010 period. Nominal sales are deflated by the GDP deflator. The resulting measure of real sales is taken as a proxy for firm size, which is denoted by s_{it}.[3] We then compute annual growth rates as $g_{it} = \log s_{it} - \log s_{it-1}$.[4]

Holly, Petrella, and Santoro (2013) have extensively shown that the empirical distribution of growth rates in the US displays shifts and contortions that are correlated with the business cycle.[5] To account for cross-sectional cyclical variations, Table 1 investigates the co-movement between the sample moments computed from the quantiles of the distribution of firm growth and the rate of growth of real GDP.[6] In the first two columns we report measures of static and dynamic correlation (Croux *et al.*, 2001) between each of the moments and real GDP growth.

[2] See Bachmann and Bayer (2011), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann, Caballero, and Engel (2013), among others.

[3] Various measures – including the value of assets of a firm, employment and sales – have been traditionally used to proxy firm size. Where data have been available for the various measures the results have generally been invariant to the measure of size (see Evans, 1987a and Hall, 1987).

[4] We remove firms growing (declining) beyond a 100% rate. Appendix A reports some descriptive statistics of the resulting sample. Replicating the analysis under alternative cut-off intervals confirms that our results are not qualitatively affected by extreme observations.

[5] The p-values of the Kolmogorov-Smirnov, Cramer-von Mises and Kuiper tests all agree on indicating that the distribution is not time invariant.

[6] Quantile-based statistics are typically seen as more robust than sample moments in the presence of outliers (see, e.g., Pearson, 1895). The estimation of conditional quantiles is detailed in the next section.

The third column reports pairwise measures of business cycle concordance that capture the proportion of time the cycles of two given series spend in the same phase (Harding and Pagan, 1999, 2002).[7] All measures of co-movement show that standard deviation and skewness behave counter-cyclically, while kurtosis follows a marked pro-cyclical pattern. These features have been originally documented by Higson *et al.* (2002, 2004)[8] and may be usefully summarized in Figure 1, where we sketch the typical shape of the density during contractions and expansions in economic activity. As shown by Holly, Petrella, and Santoro (2013), an economic slowdown generally translates into a density that shifts to the left and a relative increase in the probability mass on the left-hand side of the mode (LHS henceforth). From a visual viewpoint the right-hand side of the resulting density (RHS henceforth) assumes a characteristic tent shape – which is typical of Laplace benchmark – while the LHS is more bell-shaped – thus resembling a Gaussian. This picture reverses during expansions, though we appreciate lower dispersion about the modal rate of growth, as compared with contractions.

Insert Table 1 here
Insert Figure 1 here

Figure 2 graphs the time series of distribution quantiles. These display different degrees of co-movement with the business cycle, with lower quantiles showing stronger correlation than higher ones. This is at odds with the view that the business cycle reflects into a spread preserving shift in the mean of the distribution, which would instead imply that all quantiles display analogous reactiveness to sources of exogenous perturbation. Overall, the density has become more sparse over time, as formerly documented by Comin and Philippon (2006) and Comin and Mulani (2006), among others. However, the conditional quantiles allow us to appreciate a key aspect: increasing dispersion emerges as a phenomenon that primarily hinges on the evolution of firms in the tails of the distribution, while the interquantile range displays very moderate trending behavior. This emphasizes the importance of employing quantile-based techniques to deal with the cross-sectional dynamics of firm growth, so as disentangle the heterogeneous behavior of different parts of the density.

Insert Figure 2 here

3 Quantile Regression Analysis

The ultimate scope of our study is to understand whether changes in the density may propagate and amplify macroeconomic shocks. To address this task, estimation methods that "go beyond the mean" have to be used. In fact, there is no reason to anticipate that the marginal effects of the covariates on the shape of the density are invariant over the spectrum of growth. To this end, conditional quantile regressions have become increasingly popular and may usefully serve our purpose (Koenker and Bassett, 1978 and Koenker, 2005). Quantile regressions are especially useful whenever the heterogeneity of conditional distributions is not just captured by location

[7] For each series different cyclical phases are identified by applying the Bry and Boschan (1971) algorithm. McDermott and Scott (2000) show that concordance is symmetric around 0.5. Therefore, a concordance of one (zero) between two given series indicates they systematically experience the same (opposite) cyclical phase.

[8] DeVeirman and Levin (2011) report analogous evidence on the relationship between firm growth volatility and the business cycle.

shifts, but also by scale shifts and/or asymmetry reversals. In technical terms this may be stated as saying the distribution of firm growth conditional on certain covariates does not belong to a location family. In this setting, one should expect to observe significant discrepancies in the estimated 'slopes' at different quantiles (Machado and Mata, 2000).

Let $\tau \in (0,1)$. The τth quantile of the distribution of a generic variable y, given a vector of covariates $\mathbf{x}$, is:

$$Q_\tau\left(y|\mathbf{x}\right) = \inf\left\{y|\, F\left(y|\,\mathbf{x}\right) \geq \tau\right\}, \tag{1}$$

where $F\left(y|\,\mathbf{x}\right)$ denotes the conditional distribution function. A least squares estimator of the mean regression model would be concerned with the dependence of the conditional mean of y on the covariates. The quantile regression estimator tackles this issue at each quantile of the conditional distribution. In other words, instead of assuming that covariates shift only the location or the scale of the conditional distribution, quantile regression looks at the potential effects on the whole shape of the distribution. The statistical model we opt for specifies the τth conditional quantile of firms' growth rate as a linear function of the vector of covariates, $\mathbf{x}_{it}$, as well as time effects, $\gamma_{t,\tau}$:[9]

$$Q_\tau\left(g_{it}|\gamma_{t,\tau}, \mathbf{x}_{it}\right) = \gamma_{t,\tau} + \mathbf{x}_{it}'\boldsymbol{\beta}_\tau,\ \ \tau \in (0,1)\,. \tag{2}$$

As discussed by Koenker (2005), the marginal change in the τth quantile due to the marginal change in the jth element of $\mathbf{x}$ does not imply that a subject in the τth quantile of one conditional distribution would still be there, had the corresponding value of its x_j changed. Moreover, quantile estimation is influenced only by the local behavior of the conditional distribution of the response near the specified quantile. Therefore, no parametric form of the error distribution is assumed. Estimates depend on the signs of the residuals: outliers in the values of the response variables influence the model's fit to the extent that they are above or below the fitted hyperplane.

3.1 Size, Age and Business Cycle Co-movement

We consider alternative specifications of the quantile regression framework. The first model includes firm-level (lagged) size and age in the vector of covariates. We also consider time effects, which aim at controlling for the evolution of the distribution over time.[10] The resulting framework generalizes the first order Galton–Markov model that has often been used to explore the relationship between firm size and growth:

$$g_{it} = \beta s_{it-1} + u_{it}, \tag{3}$$

where u_{it} is an error term, which is assumed to be *i.i.d.* across firms and over time. Note that $\beta < 0$ implies that small firms grow faster than bigger ones, while for $\beta > 0$ the opposite holds true. Gibrat's Law holds instead if the estimated parameter $\hat{\beta}$ is not significantly different from

[9] Ideally, one would prefer to implement quantile panel regressions, allowing for both firm-specific and time effects (see, e.g., Powell, 2010). However, this is computationally demanding, even in the presence of a limited number of covariates. Appendix B shows that our estimates are robust to the exclusion of firm-specific effects, comparing Powell's panel estimates with those from a quantile regression with pooled data. We conclude that alternative assumptions about the error structure are of second order importance in the present context.

[10] In the pooled data setting time effects are estimated by including time dummies.

zero, so that growth turns out to be stochastic and independent of size. As remarked in the introduction, linear frameworks have delivered mixed evidence on Gibrat's Law.[11] Explicit tests started in the 1950s and have generally found that Gibrat's law serves as a good approximation of the relationship between firm size and growth (see, e.g., Hart and Oulton, 1996). But earlier studies (Samuels, 1965; Singh and Whittington, 1975) found a tendency for large firms to grow faster than small ones, while later studies (Hall, 1987; Evans, 1987a,b; Dunne *et al.*, 1989) found a tendency for small firms to grow faster. Our estimates add important insights to this large body of evidence. Figure 3 plots the quantile treatment effects (QTE) associated with each quantile. The QTE of firm size is an affine transformation of the control distribution and crosses the zero axis at zero. In other words, size acts as a scale shifter that exerts positive (negative) effects on LHS (RHS) of the median rate of growth. This is consistent with a pattern of competitive convergence, as reported by Fama and French (2000) with respect to firm profitability. Firms that grow below the median growth rate tend to a have a comparatively better performance the larger they are, whereas size represents an obstacle to fast growing firms. This result emphasizes the potential dangers of neglecting heterogeneity in the influence of firm size on firm growth for companies that grow at different speeds. Analogous observations apply when considering the role of firm age. Throughout the entire spectrum of firm growth the QTE is always negative. This is in line with Evans (1987a), who finds that firm age is also important for the variability of firm growth and the probability of dissolution. Therefore, age is never advantageous, and more so for quantiles above the median rate of growth. Notably, this relationship is broadly consistent with the predictions of Jovanovic (1982), whose theory of firm growth is based on entrepreneurs learning about their abilities over time.

Insert Figure 3 here

We now move to a second specification that accounts for two distinctive features we have detailed in the preliminary analysis, namely strong cyclicality of some moments of the distribution and increasing dispersion over time. To this end we include, along with firm-specific (lagged) size and age, a business cycle indicator (Δy_t) and a time trend (t). This amounts to set $\gamma_{t,\tau} = \alpha_\tau \Delta y_t + \delta_\tau t$, thus parameterizing the time effects in (2) so as to disentangle business cycle variation from secular patterns in the distribution. The resulting estimates are graphed in Figure 4. The cross-sectional impact of firm size and age is robust to alternative specifications, even those that allow for sector-specific determinants of growth.[12] Therefore, in the remainder of this section we will focus on the role of time effects.

Insert Figure 4 here

The QTE associated with the time trend is symmetric, though it is not centered at zero. This pattern is typical of a location and scale shift of the distribution. According to this, not only dispersion increases over time, but also the median growth rate does, though at a very small pace. This finding may be seen as providing indirect support to Davis and Kahn (2008) and Davis, Faberman, Haltiwanger, Jarmin, and Miranda (2010), according to whom

[11] See Sutton (1997) for a comprehensive review of the literature.

[12] Distante, Petrella, and Santoro (2013) focus on the impact of size and age on firm growth, reporting additional evidence at the sectoral level. In line with the estimates presented here, firm size generally acts as a scale shifter of the distribution, while age emerges as a location and scale shifter.

upward trending dispersion in the distribution of public companies might be driven by a marked shift in the selection of publicly traded firms occurred in the early 1980s. In fact, the secular pattern of the median growth rate is compatible with including in the sample relatively small but rapidly-growing companies.

Some important aspects emerge from inspecting the QTE of GDP growth. This function attains its minimum at about one – above the median rate of growth – implying that GDP growth displays near perfect co-movement with firms that grow above the median rate, a feature that is compatible with a location shift of the distribution. Most importantly, the QTE displays a marked U-shaped pattern, indicating that skewness is negatively correlated with the cycle. In addition, lower quantiles denote stronger co-movement with respect to aggregate fluctuations, as compared with the right tail of the density, thus signalling that dispersion tends to increase during contractionary episodes. Both features have been widely documented by Higson et al. (2002, 2004) and Holly, Petrella, and Santoro (2013), as exemplified in Figure 1. The novel element we retrieve from this picture is that the cyclical behavior of the firm growth density primarily reflects into asymmetry reversals. In turn, counter-cyclical scale shifts are the manifestation of changes in the skewness being themselves asymmetric between contractions and expansions. These aspects certainly deserve closer attention. In fact, the literature on heterogeneous firm models has fundamentally underestimated the role of higher moments in the transmission of aggregate disturbances, while focusing on the cyclical behavior of dispersion over several dimensions of firm-level activity. To dig deeper on these aspects, the next section explores the reaction of different growth quantiles to aggregate shocks that have been classically considered as potential drivers of macroeconomic fluctuations.

4 Transmission of Technology Shocks

The analysis so far has revealed varying degrees of co-movement between different parts of the distribution of firm growth and the business cycle. As it stands this evidence does not tell us much as to whether changes in the density may influence aggregate dynamics, or whether such movements are to be seen as simple cross-sectional projections of the business cycle. The next step addresses these issues by exploring the transmission of structural shocks onto the cross-section of firm growth and, in turn, aggregate dynamics. To this end, we make use of local projections along the lines of Jorda (2005). This represents a very convenient methodology in our setting, as it does not require specifying a model and extrapolating responses from increasingly distant horizons. The main idea behind this approach is that impulse response functions can be generally thought as the difference between two conditional forecasts:

$$IRF_\tau(t,h,d) = \mathbf{E}\left[Q_\tau\left(g_{it+h}|\mathbf{x}_{it}, v_t = d\right)\right] - \mathbf{E}\left[Q_\tau\left(g_{it+h}|\mathbf{x}_{it}, v_t = 0\right)\right], \ \tau \in (0,1), \tag{4}$$

where $Q_\tau\left(g_{it+h}|\mathbf{x}_{it}, v_t\right)$ denotes time $t+h$ quantile estimate, conditional on a generic set of covariates, $\mathbf{x}_{it}$,[13] as well as the shock of interest, v_t. Moreover, d denotes a generic one-standard deviation shock. For each horizon h we compute a direct forecast by means of the following quantile regression:

$$Q_\tau\left(g_{it+h}|\mathbf{x}_t, v_t\right) = \mathbf{x}_{it}'\boldsymbol{\beta}_\tau^h + \phi_\tau^h v_t, \ \tau \in (0,1), \quad h = 0,1,...,H, \tag{5}$$

[13] This includes firm size, age and a time trend.

so that for each h we can compute the impulse response function to the shock as $IRF_\tau(t,h,d) = \phi_\tau^h d$.

We consider alternative measures of technological change as computed by Fernald (2012).[14] Technology shocks are retrieved from adjusting the Total Factor Productivity (TFP henceforth) for factor utilization, as indicated by Basu, Fernald, and Kimball (2006). The series are then decomposed into utilization-adjusted TFP series for equipment investment, denoted by TFP_t^I, and consumption (intended as everything other than equipment investment and consumer durables), which is denoted by TFP_t^C. Figure 5 reports the QTE at each period after the shock has occurred. The upper panel graphs the responses to a TFP_t^C shock. Overall, we detect strong cross-sectional heterogeneity. A first striking finding is that, on impact (i.e., $h = 0$), the QTE is negative for the first few quantiles, while the others only display moderately positive responses, with the 40th quantile denoting the strongest reaction. The reaction of lower quantiles gradually increases after the initial shock, implying that the response at the lower end of the distribution of firm growth takes some time to build up. Therefore, on impact good performers benefit from the positive technology shock, while bad ones are left further behind. As time goes by, lower quantiles are the ones benefiting the most from the technological advance. These developments reflect into an initial scale shift of the distribution, while at $h = 2$ the QTE displays an asymmetry reversal. It is in this period that the shock exerts the strongest effects, with the tails denoting much stronger reactiveness, as compared with the median growth rate, which responds on a one-to-one basis. Over the last two periods the lower end is still the most reactive, but the shock gradually absorbs, and so the dispersion does, as signalled by the fact that the QTE mostly lies in the negative quadrant.

The responses to TFP_t^I, which are reported in the bottom panel of Figure 5, share some similarities with those to TFP_t^C. We still detect an initial (asymmetric) scale shift, which tends to absorb in the last two periods. However, in this case "good performers" are the ones that over time benefit the most from the technological impulse, reflecting the existence of implementation lags and costs entailed by capital goods investment. To see this, note that at both $h = 2$ and $h = 3$ the QTE implies an asymmetry reversal. However, while in the first case the maximum response is attained at the lower end of the density, in the next period the same quantiles display much stronger responsiveness. This signals that relatively worse performers take much longer to pick up an investment-specific technological advance, while they are more responsive to shocks that do not entail major adjustments in the rate of capital utilization.

Insert Figure 5 here

4.1 Asymmetry Reversals and Aggregate Dynamics

Section 3 has shown that the business cycle acts as a treatment capable of inverting the asymmetry of the distribution of firm growth. Also technology shocks induce asymmetry reversals and, importantly, they do so when the overall cross-sectional response reaches its peak after the initial impulse has occurred. In light of this, it seems relevant to pose the following question: do asymmetry reversals have any implication for aggregate dynamics? To address this point, we need to compute the average response to a generic aggregate shock $v_t = d$:

[14] The time series of these shocks span over the same time-window under investigation.

$$\overline{IRF}\left(t,h,d\right)=\int IRF\left(t,h,d\right)f\left(g_{t+h}\right)dg_{t+h}. \tag{6}$$

The average response can be conveniently obtained from the QTE graphed in Figure 5:

$$N^{-1}\sum_{\tau=1}^{N}\phi_{\tau}^{h}d, \tag{7}$$

where N denotes the number of bins between the 5th and the 95th quantile. In turn, if we denote with ϕ_{50}^{h} the treatment effect associated with the median quantile, we can decompose (7) into $N^{-1}d\left(\sum_{\tau=1,\tau\neq 50}^{N}\phi_{\tau}^{h}+\phi_{50}^{h}\right)$. Thus, it is immediate to derive the following condition:

$$IRF_{50}\left(t,h,d\right)\gtreqless\overline{IRF}\left(t,h,d\right)\Leftrightarrow\phi_{50}^{h}\gtreqless\bar{\phi}_{-50}^{h}, \tag{8}$$

where $IRF_{50}\left(t,h,d\right)$ is the impulse response function associated with the median, while $\bar{\phi}_{-50}^{h}\equiv\left(N-1\right)^{-1}\sum_{\tau=1,\tau\neq 50}^{N}\phi_{\tau}^{h}$. According to (8) the mean response is greater than the median one whenever ϕ_{50}^{h} is smaller than the average of all other QTE.

Both $\bar{\phi}_{-50}^{h}$ and ϕ_{50}^{h} are reported in each panel of Figure 5: importantly, in the first few periods after the shock has occurred the inequality $\bar{\phi}_{-50}^{h}>\phi_{50}^{h}$ consistently holds true. In particular, the median response tends to lie well below $\bar{\phi}_{-50}^{h}$ when the QTE implies an asymmetry reversal, due to the tails of the density displaying much greater responsiveness. Figure 6 confirms that treatment effects that are capable of altering the asymmetry of the distribution imply a powerful amplification of the mean response, as compared with the median one. It must be stressed that scale shifts are not crucial to this result. In fact, amplification of the mean response could also be observed with a symmetric QTE, as long as (8) is met. Note also that greater swings of the mean growth rate in the presence of asymmetry reversals are necessarily compatible with the mean lying at the right (left) of the median during contractions (expansions). In fact, the rule of thumb according to which positive (negative) skewness implies a mean lying at the right (left) of the median is often violated in the case under scrutiny. This is due to the skewed part of the density being highly leptokurtic, as compared with its counterpart on the other side of the mode.[15] These features are compatible with the analysis of Holly, Petrella, and Santoro (2013) and the representation of Figure 1.

Insert Figure 6 here

A word is due on the connection between the average growth rate and the rate of growth of real GDP, so as to develop some intuition on how asymmetry reversals may play a role in amplifying the response of aggregate dynamics to aggregate disturbances. To this end, assume there are N firms in the economy. Real GDP at $t-1$ can be defined as $Y_{t-1}=\sum S_{it-1}$, so that its growth rate equals:

$$g_{t}^{GDP}=\bar{g}_{t}+g_{t}^{R}, \tag{9}$$

where $g_{t}^{GDP}\equiv\Delta Y_{t}/Y_{t-1}$, $\bar{g}_{t}\equiv N^{-1}\sum_{i}g_{it}$ and $g_{t}^{R}\equiv\sum_{i}\left(S_{it-1}/Y_{t-1}-N^{-1}\right)g_{it}$.

In a world of small firms with initial size N^{-1}, g_{t}^{R} would be zero, and so the GDP growth rate would equal the average growth in the economy. In this case $\bar{\phi}_{-50}^{h}>\phi_{50}^{h}$ would ensure that

[15] von Hippel (2005) reviews cases in which skewness and unimodality need not imply a particular ordering of measures of location, presenting situations that are compatible with the evidence we report.

the aggregate response is also greater than the median one. By contrast, in a world where the size distribution of firms is sufficiently fat tailed – as in Gabaix (2011) – $g_t^R \neq 0$ whenever the effects of a given economy-wide shock on some relatively bigger firms do not wash out in the aggregate. In this case the position of large firms in the domain of firm growth may have a role in the transmission of exogenous shocks. On *a priori* grounds there is no reason to expect that $\partial g_{t+h}^R / \partial v_t$ is necessarily positive, as this depends on the aggregation of large firms' responses. However, Figure 3 has shown that size acts as centripetal force over the growth domain, exerting a near symmetric impact on either side of the median rate of growth. Thus, aggregating the growth rates of large firms should have limited impact on g_t^R, so that the response of real GDP growth should be predominantly driven by composition effects stemming from asymmetry reversals, as captured by $\bar{g}_t$.

5 Discussion

Over the last two decades the business cycle literature has been seeking for alternative forms of non-linear micro adjustment that, combined with micro-level heterogeneity, may be relevant to aggregate outcomes. The basic premise of these contributions is that firm-level heterogeneity in terms of output, employment and investment implies a large, continuous pace of reallocation of real activity across production sites. In turn, such an adjustment process may involve substantial frictions, so that the ultimate impact of an aggregate shock depends on the location of individual firms with respect to their adjustment thresholds, which determines time-varying elasticities of macroeconomic aggregates to aggregate shocks (King and Thomas, 2006). Under such circumstances representative agent frameworks necessarily suffer from a "fallacy of composition", as they do not distinguish between statements that are valid at the individual level and those that only apply to the aggregate (Caballero, 1992). Heterogeneous firm models have emerged to address these issues. Nevertheless, a clear consensus on the relevance of microeconomic decisions for the aggregate economy is far from being reached. To give a quick account of how the debate has evolved around these issues, we find instructive to focus on firm-level investment. After a first generation of partial equilibrium models that have supported the importance of lumpy investment for the macroeconomy (Caballero *et al.*, 1995, Caballero and Engel, 1999), Thomas (2002), Veracierto (2002) and Kahn and Thomas (2003, 2008) have shown that, in a general equilibrium setting, investment lumpiness is irrelevant to the cyclical properties of aggregate dynamics. More recently, this view has been questioned by Bachmann, Caballero, and Engel (2013) upon methodological grounds that mark the distinction between partial and general equilibrium components of the impact of aggregate shocks on aggregate endogenous variables (investment, in the specific case under scrutiny).

Regardless of the specific structure of the model economy, our study makes a strong case for business cycle frameworks that emphasize the importance of microeconomic adjustment for aggregate dynamics. We go even further, indicating that non-convexities and lumpy adjustment at different margins of firm-level decisions should be tailored on some specific cross-sectional criteria. In fact, our evidence suggests that technological change should not simply induce a spread preserving shift in the mean of the distribution, nor do scale shifts play a major role in propagating and amplifying technology shocks. By contrast, mechanisms that are capable of inducing asymmetry reversals are to be seen as promising avenues to impose sound empirical restrictions on heterogeneous firm models. So far plant-level dispersion over several domains

of firm activity has represented a key disciplining device. However, replicating the cyclical behavior of firm growth volatility – mostly in the form of counter-cyclical scale shifts – does not ensure *per se* a powerful propagation and amplification of technology shocks. Furthermore, we should stress that asymmetry reversals are likely to enhance the capacity of heterogeneous firm models embedding non-convexities along different margins of plant-level activity to generate counter-cyclical volatility of gross production (see, e.g., Šustek, 2011). This should be seen as a promising avenue to reproduce non-trivial business cycle asymmetries (see, among others, Neftci, 1984; Hamilton, 1989; Sichel, 1993; Morley and Piger, 2012).

A final word is due on the interaction of aggregate shocks with the moments of the cross-sectional distribution. In this respect, Caballero and Engel (1993) have formulated increasing-hazard models where larger variance leads to larger responses of aggregate employment to aggregate shocks, due to direct interaction. The intuition behind this result is that more weight on the tails of the distribution reflects higher average hazard, so that the fraction of firms that hire workers is proportionally larger (and so the one that fire workers) when the shock is large. There is a close connection between this property of partial adjustment frameworks and the behavior of conditional quantiles. Asymmetry reversals imply higher responsiveness of the tails, regardless of the size of the shock. Therefore, more weight on the tails of the density means greater reallocation of probability mass following an aggregate technology shock, due to a non-zero net flow of production units from one hand of the distribution to the other.

6 Concluding Remarks

Recent years have borne witness to the development of various heterogeneous agent frameworks whose main goal is to understand whether the dynamics of major macroeconomic aggregates is non-trivially affected by the decisions of different microeconomic actors. At the firm-level, a number of researchers have regarded higher moments of company growth rates as important elements to discipline and validate business cycle models. This paper has shown by means of quantile regression techniques that shifts and contortions in the density of firm growth of real sales matter for the transmission of aggregate disturbances. In fact, changes in the asymmetry of the distribution are the dominant source of cross-sectional dynamics at the business cycle frequency. Projection methods allow us to extrapolate the responses of each quantile to different sources of technological change, so as to characterize the behavior of the entire distribution of firm growth. The analysis highlights a deep connection between systematic asymmetry reversals and the amplification of aggregate disturbances. The formulation of heterogeneous firm models that aim at describing business cycle dynamics should account for these facts, identifying mechanisms that are capable of inducing asymmetry reversals over the domain of firm growth, so as to generate non-trivial propagation and amplification of aggregate technology shocks.

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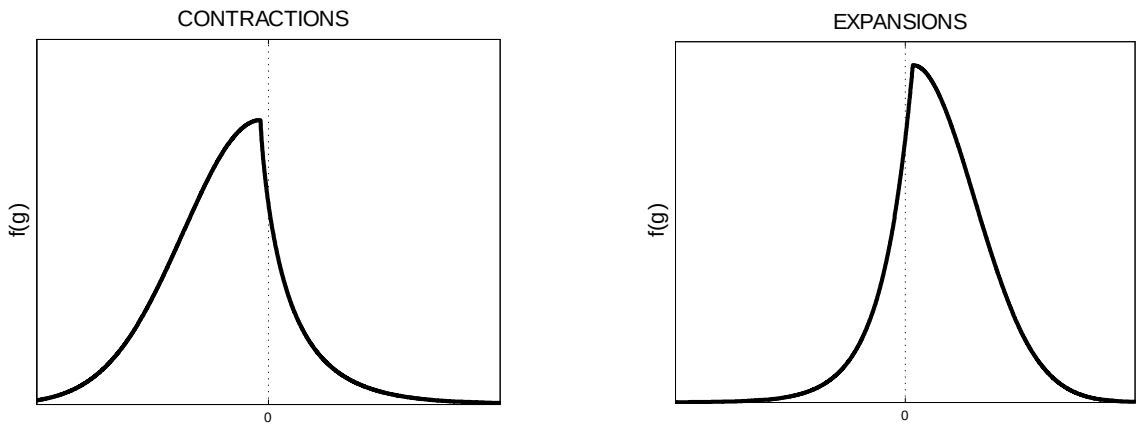
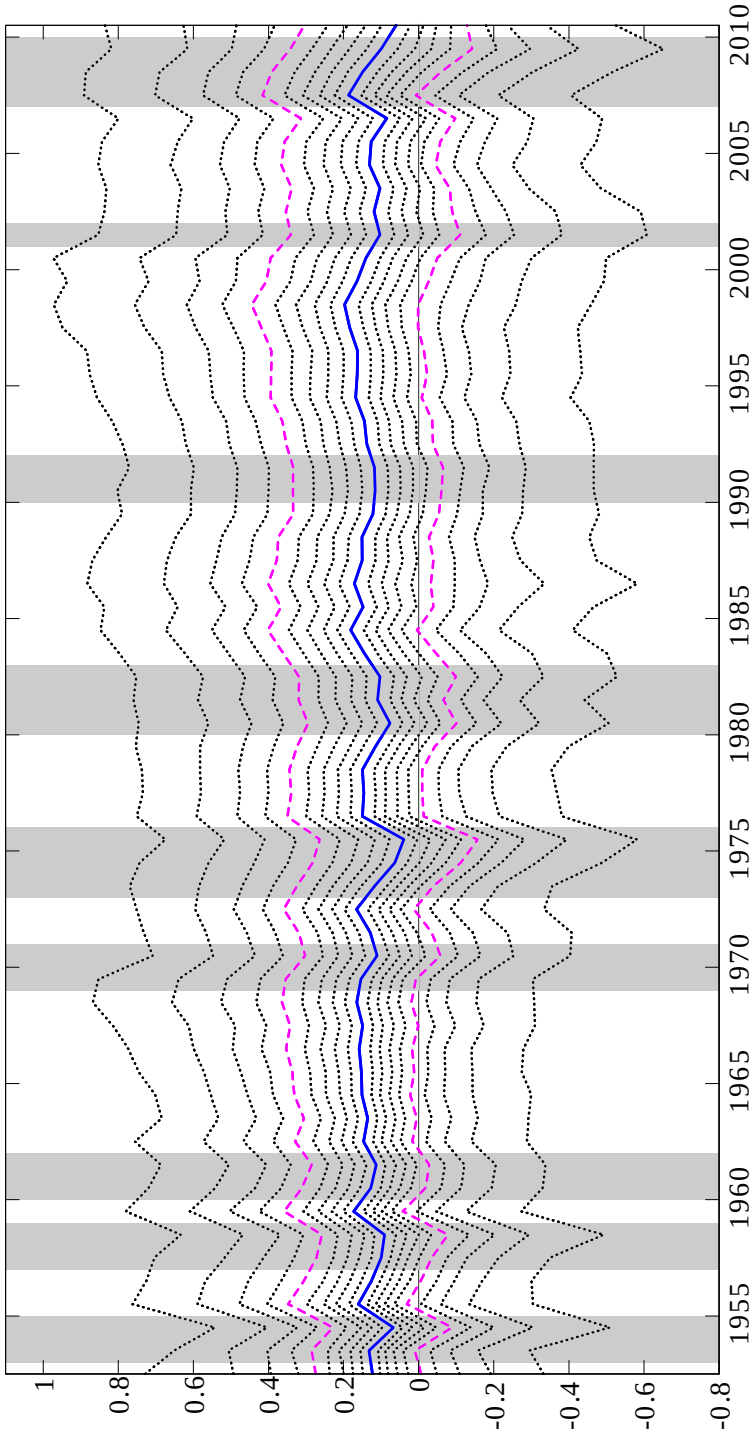
Figures and Tables

FIGURE 1. FIRM GROWTH DENSITIES



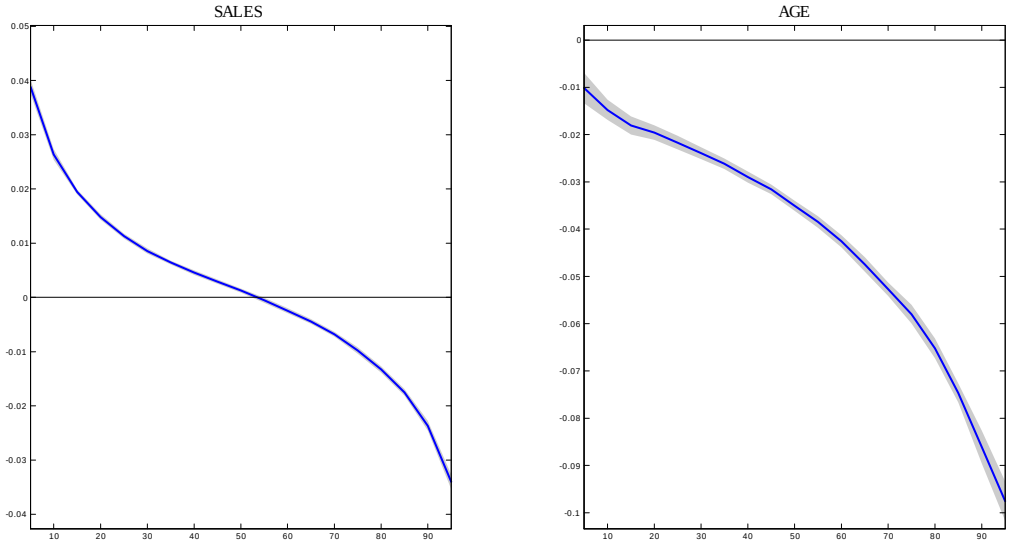
Notes: Figure 1 sketches the density of firm growth during contractions (LHS panel) and expansions (RHS panel). Contractions are generally characterized by positive skewness and higher dispersion about the modal rate of growth, while expansions tend to translate into positive skewness and lower cross-sectional dispersion.

FIGURE 2. QUANTILES OF FIRM GROWTH



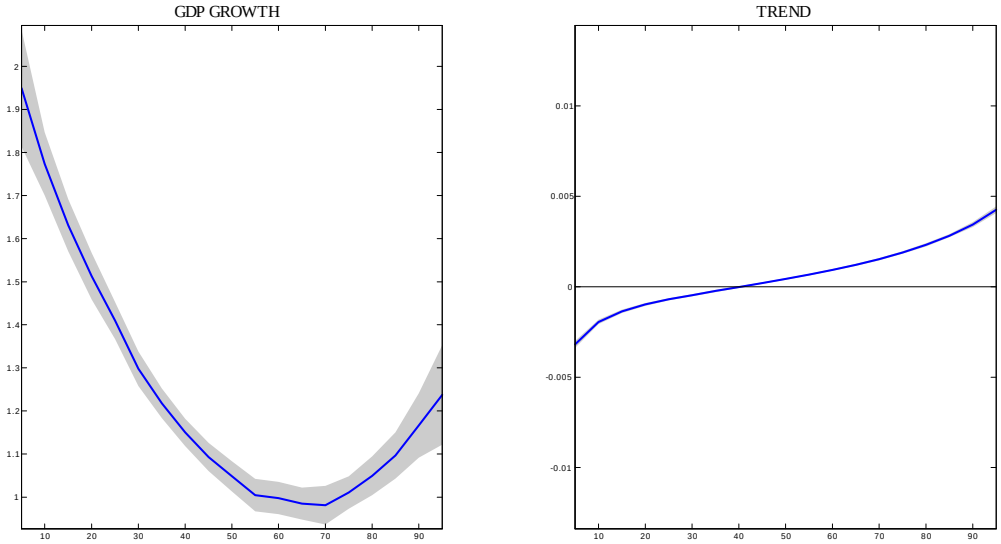
Notes: Figure 2 graphs the quantiles of firm-level growth of real sales over the 1950-2010 time window. The continuous line denotes the median, while the dashed lines denote the 25th and 75th quantiles. Recessionary episodes as identified by the NBER are denoted by the vertical bands.

FIGURE 3. QUANTILE TREATMENT EFFECTS (Size and Age)



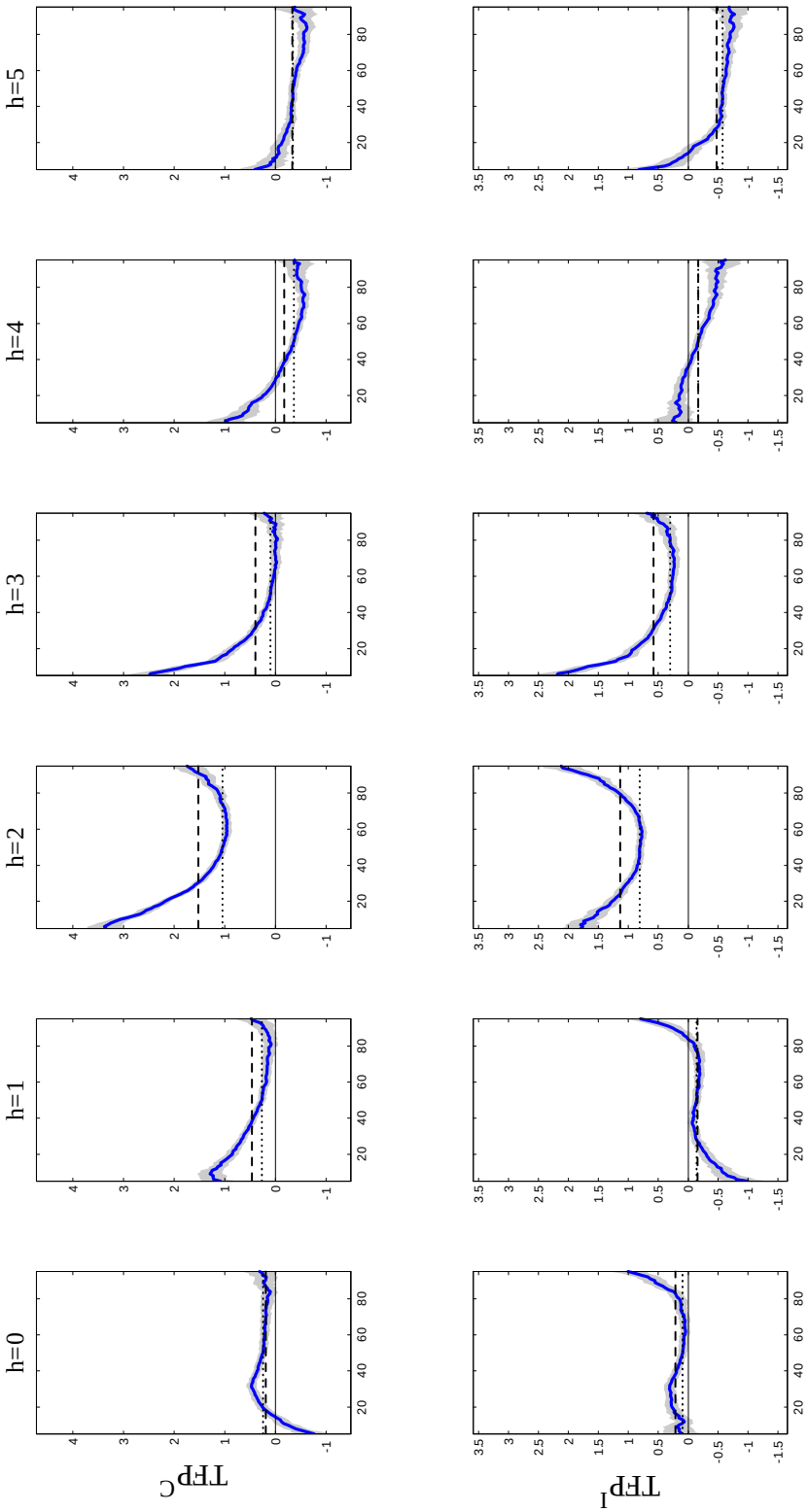
Notes: Figure 3 graphs the estimated QTE associated with firm-specific lagged real sales (left panel) and age (right panel), together with the 95% confidence interval.

FIGURE 4. QUANTILE TREATMENT EFFECTS (GDP Growth and Time Trend)



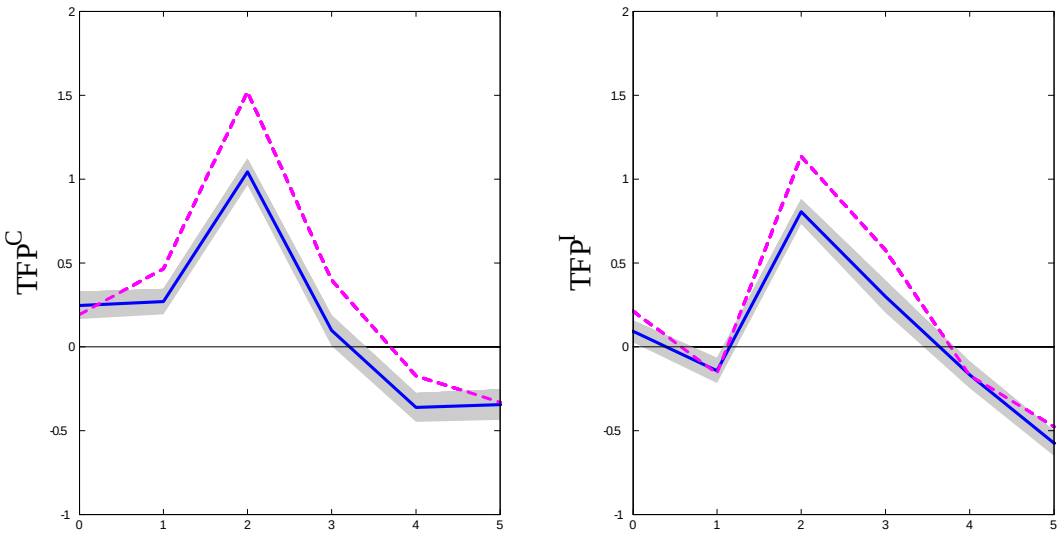
Notes: Figure 4 graphs the estimated QTE associated with real GDP growth (left panel) and a time trend (right panel), together with the 95% confidence interval.

FIGURE 5. QUANTILE TREATMENT EFFECTS (Impulse Response Functions)



Notes: Figure 5 graphs the estimated QTE associated with the responses to a TFP shock, together with the 95% confidence interval. The upper panel considers a TFP series for consumption, while the bottom one considers a TFP series for equipment investment. The dashed (dotted) line denotes the mean (median) response.

FIGURE 6. MEDIAN AND MEAN RESPONSE TO A TECHNOLOGY SHOCK



Notes: Figure 6 graphs the median QTE associated with a TFP shock (continuous line) and the mean response to the same disturbance (dashed line). The left hand panel graphs the responses to a TFP consumption shock, while the right hand panel considers a TFP series for equipment investment.

TABLE 1. SUMMARY STATISTICS

		Corr.	Dyn. Corr.	Conc.
Location	$Q_{0.50}$	0.6508	0.7023	0.5423
Scale	$(Q_{0.75}-Q_{0.25})/(Q_{0.75}+Q_{0.25})$	-0.6954	-0.7138	0.3898
Skewness	$(Q_{0.75}+Q_{0.25}-2Q_{0.50})/(Q_{0.25}-Q_{0.75})$	-0.7019	-0.6885	0.3220
Kurtosis	$(Q_{0.90}-Q_{0.10})/(Q_{0.75}-Q_{0.25})$	0.7824	0.8131	0.6271

Notes: *Corr.* is the correlation of the moment with the real GDP growth rate. *Dyn. Corr.* is a measure of dynamic correlation (Croux *et al.*, 2001), which accounts for correlation at a specific frequency band: in the present case we choose the business cycle frequency in the range $[\pi/4, 3\pi/4]$, which corresponds to a cycle of $6-32$ quarters. *Conc.* stands for the business cycle concordance indicator of Harding and Pagan (1999): this is bounded between 0 and 1 and indicates independence between two given series whenever it equals 0.5.

Appendix

Appendix A: Statistical Evidence

TABLE A1. COMPUSTAT DESCRIPTIVES

Variable	Mean	SD	Min	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	Max
Real sales s (mln \$)	1,298.66	5,814.29	0.001	31.75	141.61	602.64	267,265.90
Growth rate g	0.059	0.229	-1.00	-0.040	0.053	0.155	1.00
Age	15.70	11.66	2	7	12	21	61

Note: 216,282 observations for 10,478 firms over 60 years between 1951 and 2010. We kept only observations for which the growth rate of real sales was included in the interval $(-1, 1)$, dropping about 5,500 observations.

TABLE A2. SECTORAL REPRESENTATION

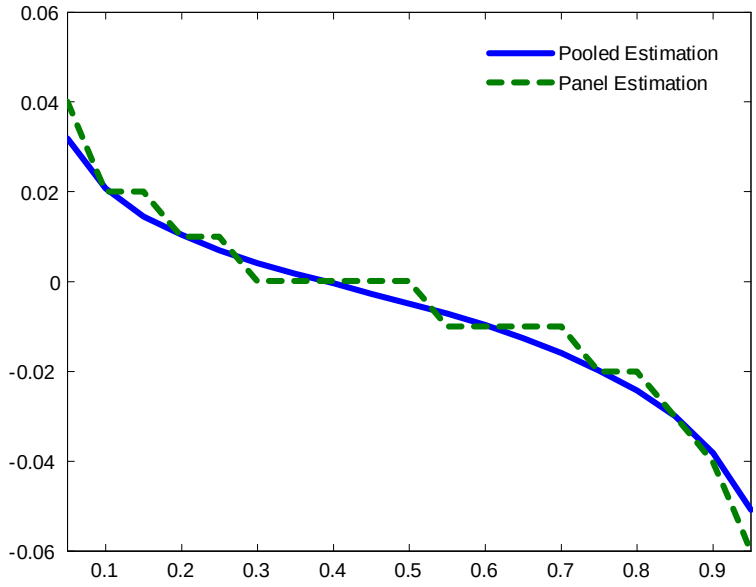
Sector	Frequency	Percent
Agriculture	718	0.33
Mining	8,294	3.83
Construction	2,303	1.07
Manufacturing (durables)	36,284	16.78
Manufacturing (nondurables)	62,370	28.84
Transportation	29,032	13.42
Trade	22,857	10.57
Financial and Other Services	54,424	25.16
TOTAL	216,282	100.00

TABLE A3. COMPUSTAT DESCRIPTIVES BY SECTOR

Agriculture					Mining				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	342.54	787.21	1.67	7,313.79	Real Sales	698.61	2241.36	0.003	32,044.04
Growth	0.051	0.239	-0.882	0.991	Growth	0.063	0.310	-0.997	1.00
Age	14.15	0.4002	2	50	Age	14.64	10.55	2	61
Construction					Manufacturing (durables)				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	733.25	1372.09	0.002	13,718.21	Real Sales	2,032.68	8,542.48	0.001	267,265.90
Growth	0.043	0.289	-0.984	0.994	Growth	0.053	0.208	-0.998	0.998
Age	15.54	11.09	2	61	Age	17.01	12.60	2	61
Manufacturing (nondurables)					Transportation				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	1,050.65	5,445.49	0.001	149,939.4	Real Sales	1,555.49	4,432.01	0.003	78,068.14
Growth	0.053	0.242	-1.00	1.00	Growth	0.059	0.175	-0.997	0.998
Age	15.97	11.69	2	61	Age	19.40	14.07	2	61
Trade					Financial and Other Services				
Variable	Mean	SD	Min	Max	Variable	Mean	SD	Min	Max
Real Sales	1,648.62	7,104.13	0.003	253,339.4	Real Sales	937.54	4296.52	0.001	111,911.2
Growth	0.062	0.200	-0.995	0.996	Growth	0.068	0.248	-1.00	1.00
Age	14.71	10.87	2	61	Age	13.17	9.14	2	61

Appendix B: Pooling vs. Panel Estimates

FIGURE B1. QUANTILE TREATMENT EFFECTS (Size): PANEL VS. POOLED ESTIMATES



Notes: Figure B1 graphs the estimated QTE associated with lagged firm-level real sales.