INTRODUCTION TO A PAPER INSPIRED BY BOB WILSON, WELL-KNOWN OPTIMIZATION THEORIST

L. Peter Jennergren

It is a matter of some interest that Bob Wilson, the famous game theorist, actually started out in a somewhat different field, optimization theory. That is, Bob's early work was in the area of operations research (or management science) rather than in economics. His doctoral dissertation *A Simplicial Algorithm for Concave Programming* (Harvard Business School, 1963) is a noteworthy contribution to mathematical programming. A recent quick SSCI/SCI search for references to this dissertation resulted in no less than 75 hits (roughly). One of those hits contains "Wilson" in the title and refers in the abstract to Bob's algorithm as "the classical Wilson method."[1] In the years after the dissertation, Bob published some papers about different aspects of mathematical programming. From seminars and courses, I remember him as tremendously knowledgeable about optimization (*e.g.,* integer programming).

When I came to the Stanford Business School Ph.D. program in 1968, I was particularly interested in operations research (I had developed that interest during a previous year as an assistant at a research outfit associated with the Swedish defense). At the Business School, I was immediately assigned to Bob as my adviser. This was only natural, since Bob was still considered at least partly as an operations researcher. At the time, operations research had a somewhat more dignified name at the Stanford Business School: Operations and Systems Analysis (OSA). In a letter to me dated April 16, 1970, Bob signs off with the title OSA Doctoral Liaison.

When the time came to start writing a dissertation, I suggested some ideas about decomposition methods in mathematical programming and relationships between such methods and planning processes in divisionalized firms. Bob agreed to serve as my main dissertation adviser. That may not have been a foregone conclusion on his part, given that his own interests had already started to shift more in the direction of decision analysis and game theory. However, he did have the advantage of having recently served as main adviser for another dissertation with a very similar perspective, Ed Zschau's *A Primal Decomposition Algorithm for Linear Programming Problems* (Graduate School of Business,

[1] A. Fischer, "Modified Wilson's Method for Nonlinear Programming with Nonunique Multipliers", *Mathematics of Operations Research*, Vol. 24, 1999, pp. 699-727.

Stanford University, 1967). He fetched a copy for me of that dissertation from Ed Zschau's office (Zschau was an Assistant Professor at the Stanford Business School at the time). As he handed it to me, he said: "We are very proud of this dissertation." It was comforting to me that the general dissertation area that I was considering could potentially lead to a dissertation that would make Bob proud.

My own dissertation, *Studies in the Mathematical Theory of Decentralized Resource Allocation*, finished in 1971, is largely about dual (price-directive) decomposition methods. However, there is also a brief section where I propose a revision of the Zschau algorithm, resulting in a sequential trading scheme among subproblems that will in the limit provide an over-all optimal allocation of coupling constraint right hand sides. In particular, if sequential subsets of $m + 1$ subproblems are jointly optimized, where the number of coupling constraints is m, then this process will converge to a solution that is optimal for the original decomposable mathematical programming problem. When I discussed this trading procedure with Bob, he gave me to read a preliminary version of a fascinating paper by Ross M. Starr "The Structure of Exchange in Barter and Monetary Economies" (subsequently published in *Quarterly Journal of Economics*, Vol. 86, 1972, pp. 290-302). In effect, Bob suggested to me that my revision of the Zschau algorithm had some similarity to certain trading schemes for bringing exchange economies to Pareto-optimal allocations. The characteristic features of the trading schemes in question were that commodities should be exchanged in subsets of traders, and that no trader should get worse off at any stage of the process.

Over the 10 years or so after finishing my dissertation, I returned off and on to this idea. A few other researchers were on to similar ideas, and a modest number of papers were written about this class of trading schemes. My contributed paper to this Festschrift summarizes some of those papers, by myself and others.