

FOREWORD TO “INCENTIVES AND INCOMPLETE INFORMATION”

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It is a unique pleasure to contribute to this volume in honor of Bob Wilson, my esteemed advisor, on the occasion of his 65th birthday. This contribution is meant to be a token of admiration, gratitude and friendship. My good fortune started in 1967, when Bob Wilson visited CORE for a year and left there a warm and durable impression. As a student in Louvain University, I didn’t meet him at the time, but had some knowledge of his articles since they were already much quoted; however, when later I declared to my friends at CORE that I wanted to do a Ph.D. in the States, I was strongly advised (especially by my master-thesis director, Jean Gabszewicz) to go to Stanford and work with Bob Wilson. This is what I did, for my greatest happiness.

That the article I have chosen to put in this volume owes much to what I, personally, have learned from Bob Wilson is clear. When I arrived in Stanford in 1970, I knew little about game theory, and nothing about games with incomplete information. I had then the privilege to learn these topics (and others, such as social choice) in Bob Wilson’s wonderful classes and in the regular interviews he was kind enough to give me. Most fascinating to me was his fundamental concern to explain, using a game-theoretic approach, how economic institutions, and in particular markets, do work in practice. My co-author Louis-André Gérard-Varet also had the privilege to receive Bob Wilson’s advice, as a post-doctoral fellow at the IMSSS in 1974–1975. The benefit for us both of this enlightened learning came soon, when back to CORE I was hired, together with Louis-André Gérard-Varet, on a Belgian State research contract devoted to the economic analysis of transboundary pollution.¹ It is in this context that we read a report by an OECD scientist (Smets, 1973) who was proposing to replace the often advocated Polluters-Pay-Principle by a ‘Principle of Reciprocal Compensation’ (equivalent to what was to be called later a Vickrey-Clarke-Groves mechanism, and thus implying outcome efficiency and incentive compatibility in dominant strategies), and who formulated very clearly the problem of balancing the agency budget. It was only natural, after receiving from Bob Wilson a perfect training on “modeling incomplete information” (the title of notes prepared

¹Bob Rosenthal, who was Bob Wilson’s first doctoral student, also contributed to this contract, while a visitor at CORE.

by him for his class), to recognize that the strategic revelation of damage and abatement costs generates a game with incomplete information. This triggered our first result for this contract (Theorem 6 in the present article): if utility is fully transferable and beliefs are “independent” (or “free”), one can construct a mechanism which is Bayesian incentive compatible (BIC), outcome efficient, and budget balancing.²

But, the present article means much more to me than the publicity given to this result. It is the first visible output of a long-term project that I worked on in close collaboration with my friend Louis-André Gérard-Varet until his death in January 2001. In this long-term perspective, the main contribution of the article is the formulation, for the transferable utility case, of the direct revelation problem as a system of linear inequalities, to be solved in the transfers while taking some efficient decision rule as given, and the application to this system of the “theorem of the alternative.” This is how the “compatibility condition” is derived as another sufficient condition on the beliefs (Theorem 7), much weaker than the independence condition. As we show in a recent paper,³ jointly written with Jacques Crémer, the compatibility condition as defined here is close to be the weakest condition.⁴ This weakest condition is now known to hold generically, and to be necessary and sufficient to ensure that any BIC mechanism can be transformed into a BIC mechanism that balances the budget.

However large the efforts represented by this whole set of results are, they have to be complemented by others in the direction so well developed in the work of Bob Wilson, since here only direct revelation mechanisms are investigated. Indeed, as he tells us,⁵ “it suffices in principle to study direct revelation games in order to find efficient trading rules.” However: “There often remains a motive, of course, to translate an efficient direct revelation game back into a form of the

²This result was first presented at the Belgian-Israeli Conference in Jerusalem in 1975 (see CORE DP 7519). However, the references at the end of the present article are incomplete. The same result is derived in Arrow K. J. (1979) “The Property Rights Doctrine and Demand Revelation Under Incomplete Information” in *Economics and Human Welfare: Essays in Honor of Tibor Scitovsky*, ed. by M. J. Boskin, New-York: Academic Press, and in d'Aspremont, C. and L.-A. Gérard-Varet (1975) “Incentive Games with Incomplete Information: An Application to a Public Input Model” in *Colloque International de Théorie des Jeux*, Institut des Hautes Etudes de Belgique.

³d'Aspremont, C., J. Crémer, and L.-A. Gérard-Varet, 2002, “Balanced Bayesian Mechanisms,” mimeo, CORE.

⁴In the notation of the present article this condition becomes: there is no $\lambda \in \Lambda$ such that, $\forall i \in N, \forall \alpha \in A, p_i(\alpha_{-i}|\alpha_i) \sum_{a_i \neq \alpha_i} \lambda_i(a_i, \alpha_i) = \kappa(\alpha) + \sum_{a_i \neq \alpha_i} \lambda_i(\alpha_i, a_i)p_i(\alpha_{-i}|a_i)$, whenever $\kappa \neq 0$. It is identical to the compatibility condition except for a permutation of α_i with a_i in the left-hand-side λ_i 's, and can be immediately derived from Theorem 7. This variant of the compatibility condition was introduced (without mentioning the difference) in d'Aspremont C. and L.-A. Gérard-Varet (1982): “Bayesian Incentive Compatible Beliefs,” *Journal of Mathematical Economics*, **10**, 83–103. An example showing the difference was constructed by Johnson S., J. W. Pratt, and R. J. Zeckhauser (1990): “Efficiency despite mutually payoff-relevant private information: the finite case,” *Econometrica*, **58**, 873–900, who define an equivalent condition (LINK).

⁵Wilson, R. B. (1985) “Efficient Trading” in *Issues of Contemporary Microeconomics and Welfare*, ed. by G. Feiwel, Albany: State University of New York Press, p. 183.

sort more usually found in practice.” As he explained further, although direct revelation mechanisms use the simplest kind of equilibrium (truth-telling), they are complicated, since their rules integrate specific features of the economic environment such as the participants information structure; in practice, trading rules are more simple (e.g. independent of the number of participants and of their beliefs), and the complexity is shifted to the equilibrium strategies, to be computed by the participants themselves on the basis of their knowledge of the economic environment.