

ON THE THEORY OF THE FIRM IN AN ECONOMY WITH INCOMPLETE MARKETS

Steinar Ekern

Robert Wilson*

Abstract

This article establishes conditions sufficient to ensure that a decision of the firm is judged to be desirable by any one shareholder (*e.g.*, the firm's manager) if and only if every shareholder judges it to be desirable. One such condition is that the decision would not alter the set of distributions of returns available in the whole economy. Another is that shareholders are interested only in the mean and variance of the returns from their portfolios. The analysis allows for the possibility of incomplete markets.

1 Introduction

One approach to the problem of selecting decision criteria for a firm is to identify those circumstances in which a manager, if delegated the task, would have an incentive to choose the “preferred” alternatives. Supposing that the manager is also a shareholder, such a circumstance would be one in which all shareholders are necessarily in unanimous agreement as to which alternatives are preferable. Then, acting in his self-interest, the manager would of his own volition make decisions which would be endorsed unanimously by the other shareholders. This circumstance is a special case of the general problem of constructing managerial incentives analysed by Wilson.¹

It is well-known² that the problem is fully solved when there is a “complete” set of markets for state-contingent claims. In this case it is in the interest of each shareholder to maximize the value of the firm. The firm's state-contingent returns are evaluated at the market prices for state-contingent claims, which for

*The authors are indebted to Alan Kraus, Hayne Leland, Robert Litzenberger and Niels C. Nielsen for discussions on this topic.

¹In [12].

²See, for example, Debreu [1].

each shareholder are equal to his marginal rates of substitution for contingent income.

The problem has substance, therefore, only when there are incomplete markets for state-contingent claims, as in the formulation of Radner.³ In the case of incomplete markets, shareholders are unable to insure against every contingency, and therefore their marginal rates of substitution for contingent income may differ. Consequently, proposed changes in the firm's state-distribution of returns may be met with a divided response among the shareholders.

The main purpose of this note is to demonstrate, nevertheless, that if the alternative decisions available to the firm would not alter the set of state-distributions of returns available in the whole economy, then the shareholders would be unanimous in their preferences. In fact, each shareholder would use the current market prices for existing securities to evaluate proposed changes in the firm's distribution of returns. This result shows that the effects of inoperative markets are limited to proposals which would change the set of state-distributions of returns available in the whole economy. Proposals which would not change this set can be evaluated in terms of the prevailing prices for existing securities. In particular, failure to obtain unanimity on a proposed project is a signal that separate incorporation of the project as a new firm would enlarge the feasible set of state-distribution of returns available to the shareholders.

An interesting example of this result occurs for the model of Diamond⁴ and its generalization by Leland⁵, which is analyzed in detail in a companion paper in this volume. In this special case, the technology of the firm is such that *every* proposal would leave unchanged the set of available state-distributions of returns. Consequently, unanimity is always assured in Diamond's and Leland's models.

A further special case of some interest is a demonstration that unanimity always obtains when shareholders value only the mean and variance of their portfolios. In particular, the unanimous preference of the shareholders is not necessarily such as to maximize the market price of the firm. This phenomena has also been demonstrated recently by Stiglitz and analyzed in successive papers by Jensen and Long and Fama.⁶ Here we show that it can also be explained by an arbitrage process, following an earlier line of argument by Wilson.⁷ In a companion paper in this volume, Merton and Subrahmanyam⁸ demonstrate that, if firms maximize their market price, then free entry of firms will ensure productive efficiency, satisfying the unanimous preferences of shareholders. Their demonstration is essentially equivalent to ours since in the mean-variance

³In [8].

⁴In [2].

⁵In [6].

⁶In [10], [5], and [4], respectively.

⁷In [13].

⁸In [7].

framework every shareholder holds equal proportions of every risky firm.

It is worth noting that security markets are only one means of risk sharing. General conditions for unanimity have been presented by Wilson.⁹

We turn now to the formulation of our model and the derivation of our results. Our model is highly simplified, and our results in this limited context are at most indicative. However, in Ekern¹⁰ the results are demonstrated to hold in a somewhat modified form when many of the simplifying assumptions to be made below are relaxed. Also, in his note in this issue, Radner provides an alternative (and, we believe, superior) formulation in terms of the Arrow-Debreu model.¹¹

2 Formulation

Our basic model is very simple. We first state its major premises [formulae (1) and (2) below] and then show how these can be derived from more fundamental assumptions.

Assume that there are several individuals indexed by $i \in I$ and several firms indexed by $j \in J$. There are just two time periods, now and then, all decisions being made now and all returns from firms occurring then, depending upon which state $k \in K$ obtains then. It will suffice to suppose that there is only one commodity, which serves as money. The return of firm j in state k is a known function $r_{jk}(x_j)$ of a decision variable x_j , which for the sake of simplicity we assume to be differentiable. Individual i selects a portfolio $s^i = (s_j^i)$ in which s_j^i is the fraction of firm j which he owns (short sales, which are allowed, correspond to negative s_j^i 's). Let p_j denote the price of firm j (i.e., the price of a unit fraction).

Our first premise is that at a market equilibrium there exists for each individual i a set of weights $\omega^i = (\omega_k^i)$ for the states such that for each firm j ,

$$\sum_k \omega_k^i r_{jk}(x_j) = p_j \quad (1)$$

For most models one derives a version of (1) as a portfolio optimality condition for each individual. The weight ω_k^i is then individual i 's marginal rate of substitution between present income and future income in state k . Of course the equilibrium price p_j is determined to ensure that $\sum_i s_j^i = 1$, the equality of demand and supply, and necessarily $p_j \geq 0$ or the firm dissolves. It is useful to regard the vector $p = (p_j)$ of firms' prices as a function $p(x) = [p_j(x)]$ of the vector $x = (x_j)$ of firms' decisions.

⁹In [13].

¹⁰In [3].

¹¹In [9].

Our second premise is that at a market equilibrium an individual i prefers to increase the decision variable x_j of firm j if and only if

$$s_j^i \sum_k \omega_k^i r'_{jk}(x_j) > 0 \quad (2)$$

This criterion expresses the requirement that individual i prefers an increment in the decision variable x_j if it would increase his “demand-price” for shares of the firm, provided he is not presently a short-seller. It agrees with the market-value criterion [i.e., $p'_j(x_j) > 0$] only under certain restrictive assumptions which are amplified below.

For those readers who find the above formulation of our basic model to be unfamiliar we provide in the next paragraphs the simplest one of its several possible derivations from more familiar premises.

Consider a model of the type employed by Irving Fisher. There is only a single commodity, which is available now either for consumption now or for investment by firms to yield returns for consumption then. Individual i is endowed with e_i units of this commodity now, and also with a fraction \bar{s}_j^i of each firm j . Given the firms' investment levels $x = (x_j)$ and prices $p(x) = [p_j(x)]$, individual i chooses now consumption $c_i(x)$ and a portfolio $s^i(x) = [s_j^i(x)]$ to maximize his expected utility

$$\sum_k u_{ik} \left[c_i, \sum_j s_j^i r_{jk}(x_j) \right] f_{ik} \quad (3)$$

subject to the budget constraint

$$c_i + \sum_j s_j^i p_j(x) \leq e_i + \sum_j \bar{s}_j^i p_j(x) \quad (4)$$

Here, u_{ik} is individual i 's utility function for consumption now and then if state k obtains, and f_{ik} is his assessed probability that state k will obtain. As noted by Radner, (4) omits consideration of firms' inputs now. Let u'_{ik} denote individual i 's marginal utility of consumption then if state k obtains. Assuming sufficient mathematical regularity properties, and allowing short sales, a necessary condition for optimality is that there exists a Lagrangian multiplier $\lambda_i = \lambda_i(x) \geq 0$ for the budget constraint such that for each firm j ,

$$\sum_k u'_{ik} f_{ik} r_{jk}(x_j) = \lambda_i p_j(x) \quad (5)$$

Ordinarily $\lambda_i > 0$ and therefore if we let $\omega_k^i = \lambda_i^{-1} u'_{ik} f_{ik}$ then (5) implies our first premise (1).

It is worth noting that if there exists a firm 0 whose return $r_{0k}(x_0) = r_0(x_0)$ is the same in every state, then $\left(\sum_k u'_{ik} f_{ik} \right) r_0(x_0) = \lambda_i p_0(x)$ and therefore $\sum_k \omega_k^i = p_0(x) / r_0(x_0)$; of course, by construction $\omega_k^i \geq 0$. One can think of ω_k^i

as the price that individual i would be willing to pay now for a claim to one unit of the commodity then in state k , which in the special case of complete markets must be the same for every individual. It should be noted also that the formulation of our first premise (1) requires one to take account of the means of financing. If the firm purchases the commodity amount x_j now for investment by issuing shares, then (1) stands as it is, but if (say) it issues a riskless bond at a price $b = p_0/r_0(x_0)$ then $r_{jk}(x_j) = R_{jk}(x_j) - x_j/b$, where $R_{jk}(x_j)$ measures the gross return before bond payments in state k (a similar formulation holds for risky bonds).

Now consider a proposal to increment the investment level x_j of firm j . Let $U_i(x)$ denote individual i 's maximum expected utility, given the investment vector x ; i.e., $U_i(x)$ is (3) evaluated at $c_i = c_i(x)$ and $s^i = s^i(x)$. One can show directly by means of the calculus that

$$\frac{\partial U_i(x)}{\partial x_j} \Big|_{\bar{x}} = s_j^i(\bar{x}) \sum_k u'_{ik} f_{ik} r'_{jk}(\bar{x}_j)$$

provided that $e_i = c_i(\bar{x})$ and $\bar{s}^i = s^i(\bar{x})$; that is, provided that the system is appraised *at* equilibrium, in which all individuals are content with their current holdings. Consequently,

$$\lambda_i(\bar{x})^{-1} \partial U_i(\bar{x}) / \partial x_j = s_j^i(\bar{x}) \sum_k \omega_k^i r'_{jk}(\bar{x}_j)$$

which implies our second premise for $x = \bar{x}$.

Thus, once the economy is in equilibrium, given an investment choice x by firms, each individual i will use (2) as the criterion by which to evaluate proposals to change the investment levels. This criterion agrees with the market-value maximization criterion only if one takes the weights ω_k^i as fixed, which is well-known to be valid only for firms which are price-takers in an economy with complete markets, in which case the weights are themselves prices common to every individual.

A variety of other formulations lead repeatedly to our basic premises (1) and (2), and therefore we adopt them as the central features of our basic model. In Section 4 we will, however, offer an alternative formulation which is more appropriate for the mean-variance framework commonly employed in the theory of capital markets.

3 Unanimity

The claim that we made in the Introduction was that if a proposal would not alter the state-distributions of returns available in the economy, then the firm's shareholders (those individuals who own *positive* fractions) will either unanimously approve it or unanimously disapprove it. In terms of our basic model a project for firm j is simply an opportunity to increase (or decrease) the decision

variable x_j . [Radner provides a more rigorous formulation of a “project” as a “feasible (local) direction of change.”¹²]

Proposition. *The shareholders of a firm will approve or disapprove unanimously a project which would not alter the set of state-distributions of returns available to individuals in the whole economy.*

The argument runs as follows. For each firm j let $r_j(x_j) = [r_{jk}(x_j)]$, the vector of state-dependent returns; thus, $r_j(x_j)$ is the state-distribution of returns available by purchasing firm j . In the whole economy each individual can obtain any state-distribution of returns in the subspace S spanned by the set of return vectors $[r_j(x_j)]$ for all firms, subject only to his budget constraint. The project to change the decision variable x_j of firm j does not alter the feasible set of state-distributions of returns if and only if the vector $r'_j(x_j) = [r'_{jk}(x_j)]$ of marginal state-dependent returns lies in S ; that is,

$$r'_j(x_j) = \sum_{h \in J} \alpha_h^j r_h(x_h) \quad (6)$$

for some numbers α_h^j , one for each firm $h \in J$. Consequently, employing first (6) and then (1) in the criterion (2), individual i prefers (say) to increase x_j if and only if

$$\begin{aligned} 0 &< s_j^i \sum \omega_k^i r'_{jk}(x_j) = s_j^i \sum_k \omega_k^i \sum_{h \in J} \alpha_h^j r_{hk}(x_h) \\ &= s_j^i \sum_{h \in J} \alpha_h^j \sum_k \omega_k^i r_{hk}(x_h) \\ &= s_j^i \sum_{h \in J} \alpha_h^j p_h \end{aligned} \quad (7)$$

Since (7) has the same sign for every shareholder, it follows that the project will be approved or disapproved unanimously by the shareholders.

An evident special case occurs when S is in fact the whole space of state-distributions of returns, for which the usual mode of proof relies upon showing that the individuals' weights are identical and equal to the prices of state-contingent claims.

A further illustration is provided by a generalization of Diamond's¹³ model, also analyzed recently by Leland and Ekern.¹⁴ Suppose that firm j 's returns have the special form $r_{jk}(x_j) = g_j(x_j) + h_j(x_j)l_{jk}$ (Diamond supposes that $g_j \equiv 0$) and that there exists a riskless firm 0 for which $r_{0k}(x_0) = r_0(x_0)$. Short

¹²In [9].

¹³In [2].

¹⁴In [6] and [3], respectively.

sales are allowed. Then

$$\begin{aligned} r'_{jk}(x_j) &= g'_j(x_j) + h'_j(x_j) l_{jk} \\ &= \alpha_0^j r_0(x_0) + \alpha_j^j r_{jk}(x_j) \end{aligned}$$

where

$$\alpha_j^j = \frac{h'_j(x_j)}{h_j(x_j)} \quad (8a)$$

and

$$\alpha_0^j = \frac{[g'_j(x_j) - \alpha_j^j g_j(x_j)]}{r_0(x_0)} \quad (8b)$$

and, therefore, the proposition implies that unanimity obtains among the shareholders.

A corollary of the criterion (7) is that, for a range of choices of the decision variable x_j by firm j which do not alter the available set of state-distributions of returns, the choice unanimously preferred by the shareholders is the one for which $\sum_{h \in J} \alpha_h^j p_h = 0$. (Of course the components α_h^j and the price p_h normally depend upon x , as in Leland's model above.) It is in this sense that the firm is required to be a "price taker" in order to achieve productive efficiency. Note that the manager of firm j actually needs only to know the components (α_h^j) and the market prices (p_h) for all firms $h \in J$; no further information about shareholders' preferences is required, provided unanimity obtains. For example, in the special case of Leland's model the optimality condition takes the simple form $\alpha_0^j p_0 + \alpha_j^j p_j = 0$, using the formulas (8) above for α_0^j and α_j^j , which in his companion paper in this volume Leland interprets as "marginal cost equals average cost" in the context of his model. As Leland observes, an important ramification of this result is the consequence that firms' investment, production, and choice-of-technique decisions are affected by the market price of the firm, and indeed, by the market prices of all firms.

4 Mean-variance model

A similar analysis is applicable when the individuals are interested only in certain parameters of their portfolios. We shall show here that unanimity obtains whenever the shareholders value only the mean and variance of their portfolios. Here we suppose that individuals' probability assessments agree.

Let $R(x) = [R_j(x_j)]$ be the vector of the firms' mean returns and let $V(x) = [V_{jh}(x_j, x_h)]$ be their covariance matrix. If individual i selects the portfolio s^i , then his return will have mean $s^i R(x)$ and variance $s^i V(x) s^i$. We assume that each individual's expected utility (3) takes the special form $u_i [s^i R(x), s^i V(x) s^i]$, where we have deleted consumption now for simplicity. Let $V_j(x)$ be the j th row of $V(x)$ and assume there exists a firm 0 with zero

variance to its return. It is then easily seen that optimality of the portfolios requires that, for each individual i and each firm j ,

$$R_j(x_j) - 2\omega_i V_j(x) s^i = \left[\frac{R_0(x_0)}{p_0} \right] p_j \quad (9)$$

where $\omega_i > 0$ is individual i 's marginal rate of substitution between mean return and variance. The quantity $P = 2(\sum \omega_i^{-1})^{-1}$ is often called the "price of risk" because (9) implies that

$$p_j = \frac{R_j(x_j) - P \sum_{h \in J} V_{jh}(x_j, x_h)}{\frac{R_0(x_0)}{p_0}} \quad (10)$$

A further consequence of (9) and (10) is that each individual acquires the same fraction $s_j^i = P/(2\omega_i)$ of each firm j .

The portfolio optimality condition (9) corresponds to our earlier premise (1). Corresponding to the criterion (2), there is now the criterion that individual i prefers to increase the decision variable x_j of firm j if and only if

$$s_j^i \left[R_j'(x_j) - 2\omega_i \sum_{h \in J} V_{jh}'(x_j, x_h) s_h^i \right] > 0 \quad (11)$$

where $V_{jh}'(x_j, x_h) = \partial V_{jh}(x_j, x_h) / \partial x_j$. N.B.: If $v_j(x_j) = V_{jj}(x_j, x_j)$, then $V_{jj}' = (1/2)v_j'(x_j)$. One obtains (11) by differentiating individual i 's maximum expected utility partially with respect to x_j , as was done in Section 2 for the state model. Stiglitz¹⁵ notes that (11) differs slightly from the criterion implied by maximization of the market value (10) when P is taken to be fixed.

We shall suppose that the submatrix of $V(x)$ corresponding to the risky firm is nonsingular. It then follows that there exists a solution α^j to the equation

$$V_j'(x) = \alpha^j V(x)$$

where $V_j'(x) = [V_{jh}'(x_j, x_h)]$ is the vector of marginal covariances. Hence the criterion (11) reduces to

$$\begin{aligned} 0 < s_j^i [R_j'(x_j) - 2\omega_i V_j' s^i] &= s_j^i [R_j'(x_j) - 2\omega_i \alpha^j V(x) s^i] \\ &= s_j^i \left\{ R_j'(x_j) - \alpha^j \left[R(x) - \left(\frac{R_0(x_0)}{p_0} \right) p \right] \right\} \end{aligned} \quad (12)$$

using (9). Thus, the shareholders of each firm j will be unanimous in their response to a proposal to change the decision variable x_j .

An alternative derivation of the unanimity property is provided by substituting the share formula $s_h^i = P/(2\omega_i)$ into (11), which shows that (11) and

¹⁵In [10].

(12) can be reduced to

$$s_j^i \left[R_j'(x_j) - P \sum_{h \in J} V_{jh}'(x_j, x_h) \right] > 0 \quad (13)$$

Again it is evident that the manager of a firm needs only to know the market price of risk, P , to satisfy the unanimous preferences of the firms' shareholders. (He must, however, avoid the temptation to replace V_{jj}' by v_j' in (13), as he would were he to seek to maximize the firm's market value.)

Another derivation of the criterion (13) is obtained by considering the market opportunities of shareholders. For the sake of simplicity, consider a risky firm j whose return is uncorrelated with all other firms. Let $\sigma_j(x_j)$ be the standard deviation of returns as a function of the decision variable x_j ; i.e., $\sigma_j(x_j)^2 = v_j(x_j) = V_{jj}(x_j, x_j)$. An individual i will obtain from his shares s_0^i and s_j^i in the riskless firm and firm j a mean return of $s_0^i R_0 + s_j^i R_j(x_j)$ and a standard deviation of $s_j^i \sigma_j(x_j)$. Consider a proposal to change the decision variable x_j to $x_j + dx_j$, which would change his mean return to $s_0^i R_0 + s_j^i R_j(x_j + dx_j)$ and his standard deviation to $s_j^i \sigma_j(x_j + dx_j)$. He could obtain the same mean and standard deviation with shares $s_0^i - ds_0^i$ and $s_j^i + ds_j^i$ satisfying the two equalities $(s_0^i - ds_0^i) R_0 + (s_j^i + ds_j^i) R_j(x_j) = s_0^i R_0 + s_j^i R_j(x_j + dx_j)$ and $(s_j^i + ds_j^i) \sigma_j(x_j) = s_j^i \sigma_j(x_j + dx_j)$. Solving these equations for ds_0^i/ds_j^i and letting dx_j go to zero, one obtains

$$\frac{ds_0^i}{ds_j^i} = \frac{R_j(x_j) - \sigma_j(x_j) R_j'(x_j) / \sigma_j'(x_j)}{R_0} \quad (14)$$

Now if $ds_0^i/ds_j^i > p_j/p_0$, then individual i would prefer to reorganize his portfolio rather than to increment the decision variable x_j . Hence, his criterion for preferring to increment the decision variable x_j is that

$$\frac{ds_0^i}{ds_j^i} < \frac{p_j}{p_0} \quad (15)$$

However, employing (10) and (14), one sees that this is just the criterion

$$R_j'(x_j) - P \sigma_j(x_j) \sigma_j'(x_j) > 0 \quad (16)$$

which is precisely the same as the criterion (13) in this case.

Still another derivation of the criterion (13) has been given by Wilson¹⁶ based on the requirement that the individuals share risk efficiently. For example, if each individual i has a constant Arrow-Pratt measure of risk aversion $r_i = 2\omega_i$, and the firms' returns are jointly normally distributed, then efficient risk sharing requires each firm j to act as though it has a constant measure of risk aversion

¹⁶In [11].

$r_j = \left(\sum_i r_i^{-1} \right)^{-1} = P$. Thus, each firm takes the market price of risk as its measure of risk aversion. This in turn is readily shown to imply (13).

One consequence of the preceding arguments is the conclusion that the shareholders of a firm unanimously prefer that the market price of the firm *not* be maximized, which conflicts with one's intuition. Jensen and Long, Fama, and most recently Merton and Subrahmanyam¹⁷ have argued that various modifications of the notion of perfectly competitive markets are desirable to expiate the paradox. Here, we conclude with an example which is designed to show that the fault may lie instead with the mean-variance model itself. Consider as before a firm j whose returns are uncorrelated with all other firms. Further, suppose that $R_j(x_j) = a_j x_j$ and $\sigma_j(x_j) = b_j x_j$; viz., the firm's returns have stochastic constant-returns-to scale. Then the criterion (16) implies that the efficient choice of the decision variable is $x_j = a_j / (P b_j^2)$. But then according to (10) the market price of the firm is $p_j = 0$! The scheme advocated by Merton and Subrahmanyam escapes this phenomenon only by supposing a sequence of successively smaller firms, each with a smaller market price converging to zero in the limit.

5 Summary

The substance of our results is the demonstration, admittedly for an overly simplified model, that even with incomplete markets a unanimous response from shareholders can be expected in many cases to firms' proposals. The simplest models, such as Diamond's, Leland's, and the mean-variance model, imply unanimity in every case. The more general model asserts that unanimity will fail to obtain only if the proposal would alter the state-distribution of returns available in the economy. In the latter case, formation of a new firm, offering new securities, provides an evident market test.

References

- [1] Debreu, G. *The Theory of Value*. New York: John Wiley, 1959.
- [2] Diamond, P. "The Role of the Stock Market in a General Equilibrium Model with Technological Uncertainty." *The American Economic Review*, Vol. 57, No. 4 (September 1967), pp. 759-776.
- [3] Ekern, S. *On the Theory of the Firm in Incomplete Markets*. Ph.D. dissertation, Stanford University, 1973.

¹⁷In [5], [4], and [7], respectively.

-
- [4] Fama, E. "Perfect Competition and Optimal Production Decisions under Uncertainty." *The Bell Journal of Economics and Management Science*, Vol. 3, No. 2 (Autumn 1972), pp. 509-530.
- [5] Jensen, M. and Long, J. "Corporate Investment under Uncertainty and Pareto Optimality in the Capital Markets." *The Bell Journal of Economics and Management Science*, Vol. 3, No. 1 (Spring 1972), pp. 151-174.
- [6] Leland, H. "Production Theory and the Stock Market." *The Bell Journal of Economics and Management Science*, Vol. 5, No. 1 (Spring 1974), pp. 125-144.
- [7] Merton, R. and Subramanyam, M. "The Optimality of a Competitive Stock Market." *The Bell Journal of Economics and Management Science*, Vol. 5, No. 1 (Spring 1974), pp. 145-170.
- [8] Radner, R. "Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets." *Econometrica*, Vol. 40, No. 2 (March 1972), pp. 289-303.
- [9] Radner, R. "A Note on Unanimity of Stockholders' Preferences among Alternative Production Plans: A Reformulation of the Ekern-Wilson Model." *The Bell Journal of Economics and Management Science*, Vol. 5, No. 1 (Spring 1974), pp. 181-184.
- [10] Stiglitz, J. "On the Optimality of the Stock Market Allocation of Investment." *Quarterly Journal of Economics*, Vol. 86, No. 342 (February 1972), pp. 25-60.
- [11] Wilson, R. "Comment" (On Stiglitz, reference [10] above). Working Paper No. 8, Institute for Mathematical Studies in the Social Sciences, Stanford University, April 1972.
- [12] Wilson, R. "Construction of Incentives for Decentralization under Uncertainty." *La Decision*, G. Guilbaud, ed., Paris: Centre National de la Recherche Scientifique, 1970.
- [13] Wilson, R. "The Theory of Syndicates." *Econometrica*, Vol 36, No. 1 (January 1968), pp. 119-132.