A NONLINEAR MODEL OF THE U.S. HOG-CORN CYCLE

by Matthew T. Holt and Lee A. Craig*

IN RECENT YEARS THERE HAS BEEN renewed interest in empirical business cycle research. While the motives for this resurgence are varied, there is little doubt that two fundamentally related reasons underlie much of this recent renaissance. On the one hand economists have long observed that contractionary and expansionary phases of the business cycle are qualitatively different. Keynes (1936), for example, provides observations on properties of business cycles that are consistent with notions of asymmetry, suggesting that contractions are shorter and more turbulent than expansions. An immediate implication is that the underlying process governing business cycle behavior possesses certain features that are not apt to be captured by linear models. But not until recent times have economists developed econometric tools capable of depicting fundamental characteristics of asymmetric behavior in business cycles (Neftci 1984, and Falk 1986), the second reason for renewed interest in this line of research. The basic approach that has emerged is some form of regime-switching model wherein linear models are typically nested in a general framework that allows for only one or perhaps some combination of the elemental linear models to be active at any given time, depending on the cycle’s phase.

Broadly speaking, regime-switching models are categorized as belonging to one of two basic types. First, there is Hamilton’s (1989) Markov-switching model wherein regimes are determined by an unobserved and exogenous state variable. Alternatively there is a class of models for which it is explicitly assumed that the regime switch is endogenously determined by an observed state variable. Models belonging to this latter category include Tsay’s (1989) self-exciting threshold autoregression (SETAR) and Teräsvirta’s (1994) smooth transition autoregression (STAR) models. The STAR model has several advantages over the SETAR model in that, as its title suggests, regime change is potentially smooth. Moreover, unlike other specifications, the STAR setup admits the possibility that the economy may reside between regimes – a transitional state. Finally, several standard STAR models nest a SETAR representation. In recent years STAR models have been used to model nonlinear features of unemployment and aggregate output series for developed countries (Öcal

* Matthew T. Holt is Professor and Wickersham Chair of Excellence in Agricultural Research in the Department of Agricultural Economics at Purdue University in West Lafayette. Lee A. Craig is Alumni Distinguished Undergraduate Professor in the Department of Economics at North Carolina State University in Raleigh.

Aside from potential nonlinearity of the business cycle, considerable research has also focused on structural change and time-varying parameters in time series models (see, e.g., Stock and Watson 1996; Becker, Enders, and Hurn 2004). The basic idea is that structural breaks and parameter time variation occur because of institutional change, an evolving policy environment, or technological innovation. Recently, nonlinear models have been combined with specifications that facilitate structural change and parameter time variation. Lundbergh, Teräsvirta, and van Dijk (2003), Skalin and Teräsvirta (2002), and van Dijk, Strikholm, and Teräsvirta (2003), for example, combine the time-varying autoregression (TVAR) model of Lin and Teräsvirta (1994) with STAR models to obtain the time-varying STAR, or TV-STAR, model for unemployment and industrial production series.

In spite of the emerging popularity of nonlinear models in general and STAR models in particular for depicting aggregate business cycles, comparatively little research has focused on modeling similar attributes for primary commodity prices, a surprising result in that many commodity prices exhibit identifiable cyclical behavior (Labys, Kouassi, and Terraza, 2000) and, as well, may be associated with nonlinearity (Davidson, Labys, and Lesourd, 1998). To date, only Persson and Teräsvirta (2003) have used a STAR framework to explore commodity price dynamics. Specifically, they used a STAR model to examine the behavior of an aggregate non-fuel commodity price index relative to the price of traded manufactures, with evidence in favor of the nonlinear specification. There is clearly a need to investigate the potential of STAR models for capturing essential features of primary commodity price dynamics.

Perhaps the most widely recognized example of cyclic behavior in primary commodity prices is the U.S. hog-corn cycle. Beginning with Haas and Ezekiel (1926), Coase and Fowler (1937), and Ezekiel (1938), numerous studies have attempted to characterize the hog-corn relationship, typically by using linear models (Harlow 1960, Jelavich 1973, Larson 1964, Shonkwiler and Spreen 1986, and Hayes and Schmitz 1987). Alternatively, Chavas and Holt (1991) use quarterly data, 1910–1984, to show that the U.S. hog-corn cycle might be associated with deterministic chaos. Miller and Hayenga (2001), using band spectrum regressions, find evidence of asymmetric responses among weekly farm-level, wholesale, and retail pork prices. Alternatively, Chavas (1999) estimated a structural model of the U.S. pork market that included multiple expectation regimes (i.e., rational, quasi-rational, and naïve), and which is therefore fully capable of giving rise to complex dynamics (Brock and Hommes 1997).

In this chapter we employ for the first time a STAR framework, and more specifically, a TV-STAR framework, to investigate fundamental aspects of the U.S. hog-corn relationship. Our working hypothesis is that the hog-corn cycle exhibits nonlinear features and time-varying parameters, and that these features may be
adequately characterized by using a smooth transition approach. There are several reasons to believe \textit{a priori} that a TV-STAR framework might prove fruitful. First, and as already noted, prior research has found evidence of nonlinearity in the hog-corn cycle (e.g., Chavas and Holt 1991). Second, due to the inherent biological nature of hog production, it is far easier to sell breeding stock when expected profits are small or negative than it is to rebuild breeding herds when expected profits are large. Third, even if all agents in the pork market are fully rational, it is still possible to observe behavior that generates and responds to cycles (Rosen 1987; Rosen, Murphy, and Scheinkman 1994). Fourth, and aside from any price expectation issues, natural animal population dynamics are capable of giving rise to complex (i.e., nonlinear) behavior in livestock markets (Foster and Burt 1992, Chavas and Holt 1993). Finally, markets for corn and hogs change over time. For example, beginning with the postwar period there has been considerable technological innovation in hog production, including movement to total confinement operations, the advent of nearly continuous breeding-farrowing rotations, the now widespread use of antibiotics and growth hormones, and enhanced feed conversion and carcass quality through genetic improvements and dietary refinement. In the empirical analysis we employ a data set consisting of monthly observations for hog and corn prices, 1880–2002. Among other things, the large sample affords sufficient observations to isolate any potential nonlinear effects and, as well, to explore possibilities for structural change and parameter time variation.

The remainder of the chapter is organized as follows. First we provide a brief overview of the history of the U.S. hog-corn cycle. Then we describe the STAR testing-modeling-evaluation cycle, after which we discuss data. Then we present model estimates and discuss results, following which we evaluate the dynamics of the estimated nonlinear model by using a generalized impulse response function. The chapter ends with concluding remarks.

\textbf{Corn, Hogs, and Cobwebs}\footnote{This section is an abbreviated version of the history of the hog-corn cycle found in Craig and Holt (2003).}

In modern terms the expression “hog cycle” refers to the correlated—possibly lagged—component of the swings in the prices of corn and hogs over time. The combination of swine physiology, with its affinity for corn, and the inherent logic of supply and demand ensure that, as long as markets exist for both corn and hogs beyond the farm gate, there will inevitably be hog cycles, and indeed, such cycles were recognized in American agriculture, albeit initially at the local level, as early as 1818 (Buley 1950, vol. 1, pp. 520–521, 526). The subsequent history of American agriculture owes much to the hog-corn nexus, and can be summarized by
the observations of a nineteenth-century British journalist traveling in the United States: “The hog is regarded as the most compact form in which the Indian corn crop of the States can be transported to market” (quoted in Cronon 1991, pp. 208–209). By the end of the nineteenth century, the transportation revolution and this relationship between corn and hogs had generated something like a national cycle. In the twentieth century economists began to analyze the cycle, and two articles dedicated to the topic became classics (Coase and Fowler 1937, and Ezekiel 1938). Economists continue to analyze the cycle’s causes and consequences (see, most recently, the review in Chavas and Holt 1991). The existence and importance of the hog cycle in American economic history stems from at least three related factors: the capacity of swine to convert corn into meat, the importance of swine in American agriculture, and the sheer size of the U.S. market.

Despite its shortcomings as a staple in human diets – corn is low in glutenin and niacin (Collins 1993, Brinkley 1994) – corn is an ideal device for delivering carbohydrates to livestock, and hogs are particularly efficient in converting carbohydrates into meat. In addition to their ability to convert corn into meat, hogs possess several biological characteristics that contributed to their importance in the early farm economy relative to, say, cattle. These include early onset of breeding (within one year of birth), short gestation periods (four months), and large litter size.

With the rise of a national market, countless travelers and foreign observers noted that pork, served in various forms, was a popular dish in the U.S. “[T]he national taste certainly runs on pork…,” observed one traveler; pork was “the beau ideal of good cheer everywhere,” wrote another. When Americans were not eating pork directly, they were using its fat in the preparation of just about everything else (Gates 1960, pp. 215–216).

Once a sufficient combination of urbanization (town-building) and transportation development occurred, farmers began producing pork for the market as opposed to just on-farm consumption. Originally valued for its ability to forage, the hog’s subsequent lofty economic status required an off-farm market and a transportation “revolution.” Urban consumers provided the ultimate off-farm market for farm-produced fat and protein, but these products could be supplied from the hinterland only as long as the cost of transporting them did not itself consume their value. As the frontier moved west and the country back east urbanized, with improvements in graded roads, followed by the emergence of canals and later railroads, farmers farther out in the hinterlands not only had relatively low-cost access to urban consumers and world markets, but they also increasingly specialized in a relatively few products and increased their scale of production in those lines (Craig and Weiss 1997). In turn, processors of those agricultural products, themselves located in the urban areas to which the raw materials were shipped, could exploit economies of scale and scope and become relatively big businesses in their own right. The hog-corn nexus proved to be a, perhaps the, crucial link in this chain.
Following the integration of the prairie with the eastern seaboard and from there the rest of the world, the hog farmer faced what he perceived to be an iron law of hog-corn economics, and it was this law that ultimately manifested itself in the hog cycle. The law was that a hog was nothing more than “fifteen or twenty bushels of corn,” or that a bushel of corn could be converted into ten pounds (net) of hog. Twenty bushels of corn spread over a year or so, depending on the breed, might reasonably yield 200 pounds of pork of various cuts – roughly the average for hogs slaughtered during the postbellum era (Cuff 1992). Therefore, the on-farm rule-of-thumb for profitability in raising hogs thus became: if the price of hogs in pounds was ten times the price of corn in bushels, it was profitable to feed corn to hogs.

This relationship created the hog-corn cycle. If the supply and demand for hogs were such that the price of pork was greater than (roughly) ten times that of corn, farmers would slaughter or ship mature hogs; feed all of their corn to, and if possible purchase more to be fed to, their maturing hogs; and breed more. In the absence of any other factors – such as weather shocks, which might improve or worsen the next corn crop – this behavior tended to put upward pressure on the price of corn, and productive resources that might have gone to other farm products went in search of more corn. At the same time, however, the increasing number of hogs on the market tended to depress the price of hogs. As the price ratio fell below the magic number, farmers cut back on hog production, corn inventories began to accumulate, and the cycle began again.

At this level of analysis, the hog-corn relationship appears to be a simple exercise in comparative statics: A decrease in the price of an input (corn) leads to a decrease in the marginal cost of production, and hence in the average variable cost of producing the output (hogs). Therefore, in a competitive (that is, price-taking) market, firms increase production. And as each does so, market supply increases and the result is a decrease in market price. But with respect to the hog-corn relationship in particular and agricultural commodities more generally, especially in the past before technology divorced production from the antediluvian rhythm of the seasons, the decision made today to supply a product not realized until months into the future was necessarily based on the past prices (and expectations, of course). The result was not necessarily a new set of (assumed to be) stable equilibria, but rather a series of potentially unstable disequilibria.

To see how this might occur, consider that farmers necessarily had to make a decision about corn acreage in the spring. If this year’s crop proved to be in relatively short supply as a result of planting decisions which were made in response to last year’s price before hog producers began to bid it up, then that would tend to put more upward pressure on this year’s corn price. As the increase in hog production, which itself began before the run-up in corn prices, simultaneously began to put downward pressure on hog prices, the hog-corn price ratio would fall below the crucial ten-to-one ratio. At that point, farmers would begin to slaughter increasingly younger hogs – even those well below the age of maturity – because the marginal cost of continuing
to feed them would exceed the expected price. This step, however, only exacerbated the downward swing in the cycle, and so on it went. Graphing this behavior in price and quantity space yielded the famous diagram of a series of disequilibria, and because of the diagram’s shape the underlying theory came to be called the “cobweb theorem.” Depending on the behavior in the other factors influencing the hog and corn markets, the path of this divergence might be arrested as quickly as after one or two years, or it might continue for four or five years. Eyeballing the hog and corn price data, Fred Shannon (1945, p. 167) observed that in the late nineteenth century, the peak-to-peak duration typically lasted four to six years. Of course, the advent of modern statistical techniques permits a slightly more discerning approach to the topic.

Early studies of the hog-corn cycle were implicitly based on a linear model of the relationship between the two markets (Coase and Fowler 1937, Ezekiel 1938); however, statistical techniques at the time prohibited an explicit test of the markets’ dynamics. With the evolution of econometrics, it followed that subsequent efforts to do so employed linear models (Harlow 1960, Jelavich 1973, Larson 1964). The fundamental problem associated with the use of such models in the hog-corn cycle pervades models of population dynamics more generally. Specifically, animals (porcine or otherwise) may be slaughtered, in response to market signals, literally overnight; but producing them, again in response to market signals, takes considerably longer. Therefore, one might logically expect this inherent asymmetry to be better represented by some nonlinear form. Furthermore, the very nature of these relationships, linear or otherwise, which themselves are manifestations of the underlying structure of the corn and hog markets, would be expected to change through time. Transportation improvements, refrigeration, public policies, and a host of organizational and technological innovations specific to corn and hog markets, have all been observed over time. The TV-STAR modeling framework, to which we now turn, is well designed to incorporate both nonlinearity and structural change.

STAR Models and the STAR Modeling Cycle

In this section we describe the basic modeling framework used to examine the hog-corn cycle as might be applied to a time series of monthly observations. The smooth transition autoregressive (STAR) model of Teräsvirta (1994) is the modeling framework used throughout. Accordingly, a STAR model of order \( p \) and augmented with (monthly) seasonal dummies is specified as

\[
\Delta y_t = \left( \phi_1 x_t + \delta_1 D_t \right) \left( 1 - G \left( s_t ; \gamma, c \right) \right) + \left( \phi_2 x_t + \delta_2 D_t \right) G \left( s_t ; \gamma, c \right) + \varepsilon_t, \quad (1a)
\]

or, alternatively,
\[
\Delta y_t = \phi x_t + \kappa_i D_t + (\phi_s x_t + \kappa_s D_t)G(s; \gamma, c) + \varepsilon_t,
\]

where \( y_t \) is the log-level of the hog-corn price ratio; \( \Delta \) is a first difference operator; \( x_t = (1, \hat{x}_t' \hat{x}_t) \), such that \( \hat{x}_t = (\Delta y_{t-1}, \ldots, \Delta y_{t-p}) \);

\[
D_t = (D_{1t}, D_{2t}, \ldots, D_{12t})' = (D_{1t} - D_{12t}, D_{2t} - D_{12t}, \ldots, D_{11t} - D_{12t})',
\]

where \( D_{\omega t}, \omega = 1, \ldots, 12 \), are seasonal dummy variables, with \( D_{\omega t} = 1 \) when time \( t \) corresponds to month \( \omega \) and zero otherwise; \( \phi_i = (\phi_{i0}, \phi_{i1}, \ldots, \phi_{ip})' \) and \( \delta_i = (\delta_{i0}, \delta_{i1}, \ldots, \delta_{i11})' \), \( i = 1, 2 \), are parameter vectors, and \( \phi = \phi_1, \kappa_1 = \delta_1, \phi_2 = (\phi_2 - \phi_1) \), and \( \kappa_s = (\delta_2 - \delta_1) \); and \( \varepsilon_t \) is a white noise process, \( \varepsilon_t \sim NID(0, \sigma^2) \). We also assume that \( \Delta y_t \) is a (globally) stationary and ergodic process. In (1), \( G(s; \gamma, c) \) is the so-called transition function, and by construction it is continuous and bounded between zero and one. With a view to the empirical work to come, we consider the transition function \( G(s; \gamma, c) \) as being specified according to the logistic function

\[
G(s; \gamma, c) = [1 + \exp(-\gamma (s - c))]^{-1}, \quad \gamma > 0.
\]

When combined with (1), transition function (2) leads to a logistic STAR, or LSTAR, model.²

In (2), \( s_t \) is the transition variable and \( \gamma \) and \( c \) are, respectively, slope and location parameters. For the LSTAR model, \( c \) is interpreted as the threshold between two regimes in that \( G(c; \gamma, c) = 0.5 \), with \( G(.) \) changing smoothly from zero to one (i.e., from one regime to another) as \( s_t \) increases. Here \( \gamma \) is referred to as the smoothness parameter. In the LSTAR model, as \( \gamma \to \infty \), \( G(s; \gamma, c) \) approaches a Heaviside indicator function \( I_c = \{s > c\} \), defined as \( I_c = \{A\} = 1 \) if \( A \) is true and \( I_c = \{A\} = 0 \) otherwise. In other words, as \( \gamma \to \infty \), the regime switch becomes instantaneous. Therefore, when the transition variable \( s_t \) is defined as \( s_t = \Delta y_{t-d} \), and when \( \gamma \) is very large, the LSTAR model given by (1) and (2) becomes a self-exciting threshold autoregression (SETAR). Also, as \( \gamma \to 0 \), the LSTAR model converges to an AR model of order \( p \), or AR(\( p \)).

² Alternatively, the transition function \( G(s; \gamma, c) \) could be specified as an exponential function, that is, \( G(s; \gamma, c) = 1 - \exp(-\gamma (s - c)^2) \), \( \gamma > 0 \). This transition function combined with (1) yields the exponential STAR, or ESTAR, model. As explained by van Dijk and Franses (1999), ESTAR models are not suitable candidates in multiple regime STAR (MRSTAR) models, and therefore are not considered as candidates for TV-STAR models.
Testing Linearity and Parameter Constancy

As the foregoing discussion makes clear, linear AR models are nested within the LSTAR framework. It is therefore desirable to test the LSTAR model against an AR specification. As Teräsvirta (1994) notes, a fundamental problem with using the LSTAR model to test linearity is that an AR model may be achieved in one of two ways: an AR model obtains if \( \gamma = 0 \) or, alternatively, if the restrictions \( \varphi'_1 = \varphi'_2 \) and \( \delta_1 = \delta_2 \) are imposed on (1). The problem therefore is that testing \( H_0 : \gamma = 0 \) against \( H_1 : \gamma \neq 0 \) within the LSTAR model results in a non-standard test, that is, a test for which there are unidentified nuisance parameters under the null associated with the autoregressive coefficients and the location parameter, the classic Davies (1977, 1987) problem. One approach to dealing with this problem was proposed by Luukkonen, Saikkonen, and Teräsvirta (1988), who recommend replacing the transition function \( G(s_t; \gamma, c) \) in (1) by a suitable Taylor series approximation. The reparameterized model is no longer associated with an identification problem, and linearity testing proceeds by using a standard Lagrange multiplier (LM) testing approach.

Another issue in testing linearity versus the LSTAR model is the choice of transition variable, \( s_t \). For modeling nonlinearity, Teräsvirta (1994) suggests using \( s_t = g(\Delta y_{t-d}) \), that is, the transition variable is specified to be some function of lagged dependent variables. Another possibility is to assume, as in Lin and Teräsvirta (1994), that \( s_t = \tau^*, \tau^* = t/T \), which results in an AR model with parameters that time vary in a smooth manner, i.e., the TVAR model. In any event, assuming that the delay parameter \( d \) is known, one test for linearity is obtained by replacing \( G(s_t; \gamma, c) \) in (1b) by a first-order Taylor series approximation, which yields the following artificial regression

\[
\Delta y_t = \beta_0' x_t + \pi_0' D_t + \beta_1' x_t s_t + \pi_1' D_t s_t + v_t , \tag{3}
\]

where the parameters \( \beta_i \) and \( \phi_i, i = 0,1 \), are functions of original parameters in (1b) such that when \( \gamma = 0 \), \( \beta_0 \neq 0 \) and \( \pi_0 \neq 0 \), and \( \beta_1 = 0 \) and \( \pi_1 = 0 \). In this case a test of linearity involves testing \( H_{01} : \beta_1 = 0 \) and \( \pi_1 = 0 \) against the alternative that \( H_{01} \) is not true. This test for nonlinearity, called the LM1 test, may be conducted by using either an asymptotic \( \chi^2 \) test with \( p + 1 + 11 \) degrees of freedom or an appropriate \( F \) version of the test.\(^3\) Throughout we rely on the \( F \) version of the LM test in question, which

\(^3\) The \( F \) version of the LM1 test statistic is obtained as follows. Estimate (3) by imposing the restrictions associated with \( H_{01} \). Denote the resulting sum of squared residuals by SSR\(_0\). Then, estimate (3) unrestricted and compute the sum of squared residuals, SSR\(_1\). The \( F \) version of the LM1 test
A NONLINEAR MODEL OF THE U.S. HOG-CORN CYCLE

typically has better size properties than its $\chi^2$ counterpart. See Granger and Teräsvirta (1993, p. 66).

Luukkonen, Saikkonen, and Teräsvirta (1988) observe that the LM statistic has poor power in cases where only the intercept varies across regimes. A test that does have power in this situation involves using a third-order Taylor series approximation, as opposed to a first-order approximation, for $G(s_t; \gamma, c)$ in (1b). The following artificial regression obtains:

$$\Delta y_t = \beta_0' x_t + \pi_0' D_t + \beta_1' x_t s_t + \beta_2' x_t s_t^2 + \beta_3' x_t s_t^3 + \pi_1' D_t s_t + \pi_2' D_t s_t^2 + \pi_3' D_t s_t^3 + \nu_t.$$ (4)

In this case a test of linearity involves testing $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ and $\pi_1 = \pi_2 = \pi_3 = 0$ against the alternative that $H_0$ is not true. This test, denoted the LM$_3$ test, may be conducted by using either an asymptotic $\chi^2$ test with $3(p+1+1)$ degrees of freedom or its $F$ test counterpart. Depending on lag length, $p$, (4) could involve many parameters and therefore could be quite costly in terms of degrees of freedom. Now, it happens that the expressions for $\beta_i$ and $\pi_i, i=1,2,3$, in (4) in terms of $\phi_1,\phi_2,\kappa_1,\kappa_2,\gamma$, and $c$ are such that the only parameters that depend on the intercepts $\phi_{10}$ and $\phi_{20}$ in (1b) are $\beta_{10}, \beta_{20}, \beta_{30}, \pi_{10}, \pi_{20}$, and $\pi_{30}$. Therefore, an “economy” version of the LM$_3$ statistic may be derived by including only $s_t^2$ and $s_t^3$ as additional regressors in (3). The artificial regression in this case is then

$$\Delta y_t = \beta_0' x_t + \pi_0' D_t + \beta_1' x_t s_t + \pi_1' D_t s_t + \beta_2' s_t^2 + \beta_3' s_t^3 + \nu_t.$$ (5)

and testing the null hypothesis $H_0^e: \beta_1 = 0, \pi_1 = 0, and \beta_2 = \beta_3 = 0$ yields the LM$_e$ test, which may be conducted by using either an asymptotic $\chi^2$ test with $(p+1+1+2)$ degrees of freedom or, again, by using the appropriate $F$ test counterpart.

In practice when $s_t$ is taken to be $g(\Delta y_{t-d})$ (as opposed to $\hat{r}^*$), delay parameter $d$ is seldom if ever known, and therefore must also be determined as part of the testing procedure. As in Teräsvirta (1994), $d$ is determined by repeating the LM$_1$ and LM$_3$ tests for all values of $d$ such that $1 \leq d \leq D_{\text{max}}$, $D_{\text{max}}$ being the maximal lag length.

statistic is then $\text{LM}_1 = [(\text{SSR}_0 - \text{SSR}_1)/(p+1+1)]/[\text{SSR}_1/(T-2(p+1+1))]$. Under $H_0$ the test statistic is distributed asymptotically as an $F$ distribution with $(p+1+1)$ and $T-2(p+1+1)$ degrees of freedom.

4 An additional test, the LM$_2$ test, is also described by Teräsvirta (1994). This test involves replacing $G(s_t; \gamma, c)$ in (1b) with a second-order Taylor series expansion. This test is not discussed here because it is useful only for testing for ESTAR-type nonlinearity, an option that we have precluded. In the empirical work that follows we did test initially for ESTAR-type nonlinearity (results not shown), and concluded in every case that an LSTAR specification was more appropriate.
considered. If \( H_01 \) (\( H_03 \)) is rejected for more than one value of \( d \), then \( \hat{d} \) is determined by choosing the value associated with the smallest overall \( p \)-value. On the other hand, if none of the \( p \)-values for LM1 (LM3) indicate rejection of \( H_01 \) (\( H_03 \)), then the linear AR model is not rejected.

Several additional caveats apply in the case where \( s_t = t^* \), that is, when the null hypothesis of parameter constancy (as opposed to linearity) is being tested. In this case for finite \( \gamma \) the logistic function implies monotonically smooth parameter change over time. This need not be the case. Specifically, define (2) alternately as:

\[
G_k(t^*; \gamma, c) = \left[ 1 + \exp\left\{ -\gamma \prod_{j=1}^{k} \left( t^* - c_j \right)^a \right\} \right]^{-1}, \quad \gamma > 0, c_i < c_j, \forall i < j, k = 1, 2, 3. \tag{6}
\]

Note that (6) reduces to (2) when \( k = 1 \), implying that structural change is monotonic, with a single structural break occurring when \( \gamma \to \infty \). Alternatively, for \( k = 3 \), (6) admits the possibility of non-monotonic and non-symmetric change. Therefore, (6) affords considerable flexibility in modeling structural change. The LM1 test with \( s_t = t^* \) corresponds to \( k = 1 \) in (6), while the LM3 test corresponds to \( k = 3 \) (Lin and Teräsvirta 1994).

Model Diagnostics – Autocorrelation

Once a candidate LSTAR model is chosen, parameter estimates may be obtained by using either a grid search or standard nonlinear estimation techniques. Naturally, once an LSTAR model has been estimated, its ability to adequately characterize the data should be evaluated by employing a variety of diagnostic tests. Of particular interest are tests of the hypothesis of no remaining autocorrelation in the model’s residuals and tests of hypotheses of no remaining nonlinearity or of no parameter constancy.

Turning first to a test of the hypothesis of no remaining autocorrelation, let \( F(\bar{x}_i; \theta) \) denote the skeleton of the model, where

\[
F(\bar{x}_i; \theta) = \Phi_i \bar{x}_i \left( 1 - G(s_i; \gamma, c) \right) + \Phi_i \bar{x}_i G(s_i; \gamma, c), \tag{7}
\]

and where \( \theta = (\Phi_1, \Phi_2, \delta_1, \delta_2, \gamma, c)^T \) and \( \bar{x}_i = (x_i, D_i)^T \). Eitrheim and Teräsvirta (1996) propose testing the hypothesis of no remaining autocorrelation in the model’s residuals up to and including order \( q \) by estimating the auxiliary regression

\[
\hat{\epsilon}_i = \pi_i \nabla F(\bar{x}_i; \hat{\theta})^T + \sum_{j=1}^{q} \alpha_j \hat{\epsilon}_{i-j} + \delta_i, \tag{8}
\]
where $\nabla F(\hat{\mathbf{x}}_t; \hat{\theta}) = \partial F(\hat{\mathbf{x}}_t; \hat{\theta})/\partial \theta$. The LM test statistic is computed in the usual fashion as $TR^2$, where $R^2$ is the r-squared coefficient from the auxiliary regression in (8). Under the null hypothesis of no remaining autocorrelation, that is, under $H_0: \alpha_1 = \ldots = \alpha_q = 0$, the resulting test statistic has an asymptotic $\chi^2$ distribution with $q$ degrees of freedom. An $F$-version of the test may also be constructed.

**MRSTAR and Additive STAR Models**

In many instances a simple two-regime STAR model may not adequately capture all essential nonlinear features of the data. Regarding extensions to that basic STAR model, several alternatives exist, including the multiple regime STAR (MRSTAR) model of van Dijk and Franses (1999) and the additive STAR model of Eitrheim and Teräsvirta (1996). The former is of interest because it includes as a special case the TV-STAR model.

To illustrate, an MRSTAR model with four regimes is expressed as

$$
\Delta y_t = \left( \varphi_1 x_t + \delta_1^* D_t \right) \left( 1 - G_1(s_{1t}; \gamma_1, c_1) \right) + \left( \varphi_2 x_t + \delta_2^* D_t \right) G_1(s_{1t}; \gamma_1, c_1) \left( 1 - G_2(s_{2t}; \gamma_2, c_2) \right) + \left( \varphi_3 x_t + \delta_3^* D_t \right) \left( 1 - G_3(s_{3t}; \gamma_3, c_3) \right) \left( 1 - G_2(s_{2t}; \gamma_2, c_2) \right) + \epsilon_t,
$$

or, alternatively,

$$
\Delta y_t = \varphi_1^* x_t + \delta_1^* D_t + \left( \varphi_2^* x_t + \delta_2^* D_t \right) \left( 1 - G_1(s_{1t}; \gamma_1, c_1) \right) + \left( \varphi_3^* x_t + \delta_3^* D_t \right) \left( 1 - G_2(s_{2t}; \gamma_2, c_2) \right) + \left( \varphi_4^* x_t + \delta_4^* D_t \right) G_1(s_{1t}; \gamma_1, c_1) \left( 1 - G_2(s_{2t}; \gamma_2, c_2) \right) + \epsilon_t,
$$

where $\varphi_1^* = \varphi_1$, $\varphi_2^* = \varphi_2 - \varphi_1$, $\varphi_3^* = \varphi_4 - \varphi_1$, and $\varphi_4^* = \varphi_1 + \varphi_4 - \varphi_2 - \varphi_1$, with $\delta_1^*, \delta_2^*, \delta_3^*$, and $\delta_4^*$ being similarly defined, and where $G_1(.)$ and $G_2(.)$ belong to the class of logistic functions in (2). Clearly if $\gamma_2 = 0$ or if $\varphi_4 = \varphi_1$, $\varphi_2 = \varphi_1$, $\delta_1 = \delta_3$, and $\delta_2 = \delta_4$, the two-regime LSTAR model obtains.

To test the LSTAR model versus the MRSTAR model, the same strategy used in testing for nonlinearity applies, that is, $G_2(s_{2t}; \gamma_2, c_2)$ is replaced by a suitable Taylor

---

5 As a practical matter, if the estimated STAR is poorly behaved in the sense that estimated residuals $\hat{\epsilon}_t$ are not exactly orthogonal to the gradient matrix, a situation that could arise if the likelihood function is relatively flat with respect to one or more of the parameters (e.g., the smoothness parameter, $\gamma$), then the empirical size of the test will be increased. To handle this situation, Eitrheim and Teräsvirta (1996) replace the residuals $\hat{\epsilon}_t$ by those obtained from first regressing $\hat{\epsilon}_t$ on the gradient matrix. Adding this additional step ensures that the residuals used in the diagnostic LM tests are, by construction, orthogonal to the gradient matrix.
series expansion. For example, if a third-order Taylor series is used, the approximation to (9) is

$$
\Delta y_t = \eta_1 x_t + \lambda_1 D_t + (\eta_2 x_t + \lambda_2 D_t)G_1(s_{it};\gamma_1, c_1) + \beta_1 x_t s_{2t} + \beta_2 x_t s_{3t} + \pi_1 D_t s_{1t} + \pi_2 D_t s_{2t} + \pi_3 D_t s_{3t} + \rho_{s_1} D_t s_{1t} + \rho_{s_2} D_t s_{2t} + \rho_{s_3} D_t s_{3t} + \gamma_1 s_{2t} + \gamma_2 s_{3t} + \epsilon_t \tag{10}
$$

The null hypothesis of no remaining nonlinearity is $H_0: \beta = \cdots = \beta = 0$ and $\pi = \cdots = \pi = 0$, with the LM test constructed by running a regression similar to that in (7). That is, residuals from the estimated LSTAR model are regressed on the gradient vector $\nabla F(\vec{x}; \hat{\theta})$ and additional regressors $x_t s_{2t}, x_t s_{3t}, D_t s_{1t}, D_t s_{2t}, D_t s_{3t}$, and

$$
x_t s_{2t}, \hat{G}_t, x_t s_{3t}, \hat{G}_t, D_t s_{1t}, D_t s_{2t}, \hat{G}_t, D_t s_{3t}, \hat{G}_t,
$$

where $\hat{G}_t = G_t(s_{it}; \hat{\gamma}_1, c_1)$. The LM test statistic is then constructed either as an asymptotic $\chi^2$ test with $6(p+1+11)$ degrees of freedom or as a comparably defined $F$ test. See van Dijk and Franses (1999) for details.

Eitrheim and Teräsvirta (1996) propose an alternative to the MRSTAR model: the additive STAR model. In this case (1b) is modified by appending a second additive STAR component. That is,

$$
\Delta y_t = \phi_1 x_t + \kappa_1 D_t + (\phi_2 x_t + \kappa_2 D_t)G_1(s_{it};\gamma_1, c_1) + (\phi_3 x_t + \kappa_3 D_t)G_2(s_{2t};\gamma_1, c_1) + \epsilon_t \tag{11}
$$

is an additive STAR model. The foregoing tests for remaining nonlinearity of the MRSTAR model type may be modified to test for additive STAR model effects by simply excluding regressors in artificial regression (10) that involve $\hat{G}_t$. Testing then proceeds in the usual fashion.

The TV-STAR Model

As already suggested, there may be occasions where the parameters of the LSTAR model are not constant through time. For instance, it may be that institutional or technological change has caused seasonal patterns to shift over time, a possibility that seems plausible in the context of the hog-corn ratio. In this case it is perhaps better to specify a model for $\Delta y_t$ that includes both regime-switching and non-constant parameters, a TV-STAR model. As a diagnostic test, a test of the hypothesis of parameter constancy is formulated in a manner similar to that for multiple regimes. In this case the null hypothesis is that the parameters in the LSTAR model are constant,
while the alternative hypothesis allows the parameters to vary over time. The TV-STAR model is simply obtained from (9) by setting $s_{2t} = t^*$. The test of parameter constancy then proceeds as before by using auxiliary regression (10), where $s_{2t} = t^*$.

Several interesting prospects follow from the specification in (9) in the context of a TV-STAR model. It may be, for example, that only constant terms and parameters for seasonal dummies change smoothly through time; the autoregressive parameters remain stable. This possibility was explored by Skalin and Teräsvirta (2002) in their examination of OECD (Organization for Economic Cooperation and Development) unemployment data. In this case the resulting TV-STAR model is defined as

$$
\Delta y_t = \left[ \phi_1 \bar{x}_t + \left( \phi_{10} + \delta_1 D_t \right) \left( 1 - G_1 (t^*; \gamma_1, c_1) \right) \right] + \left( \phi_{20} + \delta_2 D_t \right) G_1 (t^*; \gamma_1, c_1) \left[ 1 - G_2 (s_t; \gamma_2, c_2) \right] \\
+ \left[ \bar{\phi}_2 x_t + \left( \phi_{30} + \delta_3 D_t \right) \left( 1 - G_1 (t^*; \gamma_1, c_1) \right) \right] + \left( \phi_{40} + \delta_4 D_t \right) G_1 (t^*; \gamma_1, c_1) \left[ 1 - G_2 (s_t; \gamma_2, c_2) \right] + \epsilon_t,
$$

where $\phi_i$ and $\bar{\phi}_i$ are autoregressive terms. An interesting feature of (12) is that not only do seasonal patterns change smoothly through time, but they will also depend upon the regime in which the model happens to reside.

**Heteroskedasticity Robust Tests**

When performing LM tests of residual autocorrelation, heteroskedasticity may result in spurious rejection of the null hypothesis (Davidson and MacKinnon 1985). Consequently, it is not unreasonable to assume that ignored heteroskedasticity in LM tests for linearity, parameter constancy, and model misspecification might have similar effects. For this reason it may be desirable to have test statistics that are robust in the presence of heteroskedasticity, even when the precise form of heteroskedasticity is unknown. Fortunately, Wooldridge (1990) has developed a simple set of procedures for obtaining heteroskedasticity robust LM tests in a general setting. Details on implementing heteroskedasticity robust tests in a STAR framework are provided in van Dijk, Teräsvirta, and Franses (2002).

While it seems desirable to compute heteroskedasticity robust LM tests if there is evidence that the error variance is non-constant, a note of caution is in order. Lundbergh and Teräsvirta (1998) provide simulation evidence showing that, at least in certain instances, robustification substantially reduces the power of linearity tests. In other words, robustification may make it difficult to detect nonlinearity when in fact it truly exists. Here we simply present both standard and robustified versions of LM tests for nonlinearity. Final model specifications are then determined through careful evaluation of each candidate model’s properties at the estimation and misspecification testing stages.
**Data**

The data used in the empirical analysis consists of monthly prices of hogs relative to corn for the January 1880 (1880:01) to December 2002 (2002:12) period. To obtain a consistent series that spans this 120-year period, several data series were spliced together and some adjustments were made. Average prices received by farmers, U.S., for hogs (all grades) in dollars per cwt. are available on a monthly basis, seasonally unadjusted, from the U.S. Department of Agriculture’s (USDA) National Agricultural Statistical Service (NASS) for the January 1910 to December 2002 period. Likewise, the average price of corn (all grades) in dollars per bushel received by farmers, U.S., is also available on a monthly basis, seasonally unadjusted, from NASS for the period January 1908 to December 2002. These monthly data through 1992 were obtained from the USDA-NASS data archive at Cornell University’s Mann library (http://usda.mannlib.cornell.edu/data-sets/crops/92152/). Monthly prices through 2002 were then obtained from NASS monthly prices-received bulletins.

NASS prices received for hogs prior to 1910 and for corn prior to 1908 are not available. Chicago wholesale prices for hogs and corn are, however, part of the NBER Macrohistory Database (http://www.nber.org/databases/macrohistory/contents/). Specifically, hog price data are available from January 1880 to February 1940 and corn price data from January 1880 to December 1951. Monthly hog price data from 1880 to 1919 were reported originally in Wallace (1920). Observations from 1920 to 1940 were computed by NBER, and were derived by averaging monthly highs and lows in the annual reports of the Chicago Board of Trade. Corn price data from 1880 to 1951 were computed by NBER, again from annual reports of the Chicago Board of Trade.

In total there are 372 overlapping observations for the NBER Chicago hog price series and the NASS average farm price received series between January 1910 and February 1940. Likewise, there are 528 overlapping observations for the NBER Chicago corn price series and the NASS average farm price received between January 1908 and December 1951. These overlapping observations for hog prices and corn prices are plotted in, respectively, Figure 1a and Figure 1b. As may be deduced from the plots, the correlation in each case between the NBER series and the NASS series is not perfect, but it is evident in both instances that the covariation is extremely high. The implication is that the NBER prices may be used to interpolate average farm prices received for hogs prior to 1910 and for corn prior to 1908.

To infer average farm prices received for hogs for the 1880–1909 period, the following interpolating regression was fitted to the NASS hog price received data:

\[
HP_{t}^{NASS} = b_{0} + b_{1}HP_{t}^{NBER} + b_{2}sin_{t}I_{t} - b_{3}cos_{t}I_{t} - b_{4}sin_{2}I_{t} - b_{5}cos_{2}I_{t}, \quad (13)
\]

\[
R^{2} = 0.980, \quad \hat{R}^{2} = 0.980, \quad \hat{\sigma} = 0.429, \quad T = 372.
\]
Figure 1a. Price of Hogs Received by Farmers, U.S., and Chicago Wholesale Price of Hogs, Dollars per Cwt., 1910:01–1940:02

Figure 1b. Price of Corn Received by Farmers, U.S., and Chicago Wholesale Price of Corn, Dollars per Bushel, 1908:01–1951:12
In (13), $H_P^{NASS}$ is the NASS farm price, $H_P^{NBER}$ is the NBER Chicago wholesale price, and $\sin I$, $\cos I$, $\sin 2I$, and $\cos 2I$ are leading terms from a Fourier series included to capture six- and twelve-month seasonal cycles. Equation (13) is then used to backcast NASS farm monthly average farm price received for hogs from January 1880 through December 1908. A similar procedure is used to backcast NASS average farm price received for corn for January 1880 through December 1907. Specifically, fitting an interpolating regression to the overlapping data in the 1908–1951 period obtains the following:

$$CP_t^{NASS} = 0.057 + 0.861CP_t^{NBER} + 0.005\sin I - 0.012\cos I - 0.009\sin 2I - 0.029\cos 2I,$$

$$R^2 = 0.973, \quad \bar{R}^2 = 0.973, \quad \hat{\sigma} = 0.069, \quad T = 528.$$

Again, $CP_t^{NASS}$ is the NASS farm price, and $CP_t^{NBER}$ is the NBER Chicago wholesale price.

The primary hog price, corn price, and hog-corn price ratio data – in levels – are plotted in Figure 2. The hog-corn ratio plot in Figure 2c is suggestive – there appears to be a substantial cyclical feature to these data, and it was exactly this observation that captured the attention of Coase and Fowler (1937) and Ezekiel (1938). As the plots illustrate, over the 122-year span the hog-corn ratio has ranged from just below 6 in December 1934 to just above 40 in September 1986, the average over the entire period being 14.3. Also observed in Figure 2c is a slight upward trend in the ratio since the mid to late 1940s. Finally, the ratio appears to have become more variable since the early 1970s: the sample standard deviation for the 1880-1969 period is 3.00, while the same number for the 1970-2002 period is 5.18, a 73 percent increase. Figures 2a and 2b show that during the latter period both hog and corn prices became more variable, although hog price variability appears relatively more pronounced. In the empirical application the hog-corn ratio is converted to natural logarithms in an attempt to mitigate some of the observed heteroskedasticity in each model’s residuals.

**Modeling the Hog-Corn Ratio**

In this section we present results on estimation of a provisional linear model fitted to the hog-corn ratio data. We then present results on tests for nonlinearity, estimates of a candidate TVAR model, results of additional model misspecification tests, and finally estimates of a TV-STAR model.

**Linear Model Results**

A linear autoregressive model is first fitted to the data. To account for seasonality...
Figure 2a. Farm Price Received for Hogs, Dollars per Cwt., 1880–2002

Figure 2b. Farm Price Received for Corn, Dollars per Bushel, 1880–2002

Figure 2c. Hog-Corn Price Ratio, 1880–2002
we include eleven monthly dummy variables, as previously defined. The AIC criterion is used to choose the lag length. Allowing up to 48 lags, the AIC is minimized at lag 36, implying a total of 1,439 usable observations. To conserve space, parameter estimates for the linear model are not presented; they are, however, available upon request. Several diagnostics for the best-fitting linear AR model are reported in the left-most column of Table 1. LM test results show that even with 36 lags, the linear model apparently does not capture all of the residual autocorrelation. LM tests also reveal substantial evidence of ARCH effects. Based on the LJB test (Lomnicki 1961, Jarque and Bera 1980), the residuals associated with the AR model fail to satisfy normality. As indicated by the excess kurtosis measure reported in Table 1, the error distribution for the linear model has thicker tails than that implied by normality. An examination of the model’s residuals reveals at least nineteen outliers, with eight of these being more than four standard deviations away from the mean. These outliers are responsible for most of the excess kurtosis observed in the AR model’s residuals, and therefore for the model’s failure to satisfy the normality assumption.

**Linearity and Parameter Constancy Test Results**

Before performing linearity tests it is necessary to identify a slate of candidate transition variables. In testing for nonlinearity we use various lags of seasonal first differences of the hog-corn ratio for $s_t$ (Skalin and Teräsvirta 2002). That is, in testing for linearity we use $s_t = \Delta_{12}y_{t-d} = y_{t-1-d} - y_{t-12-d}$, $d = 1, ..., D_{\text{max}}$, where $y_t$ denotes the hog-corn ratio and $D_{\text{max}} = 8$. Seasonal first-differences are used as we are primarily interested in nonlinearities associated with the hog-corn cycle – the transition variable should reflect sustained periods of expansion or contraction. We therefore omit monthly first differences as potential transition variables in that they are normally too noisy to provide a consistent signal about the cycle’s regime. Finally, to test for parameter constancy we simply use a linear trend, $s_t = t = t/T$, $T = 1439$.

Results for the LM$_3$ and LM$_1$ linearity tests, both standard and robustified, are presented in Table 2, along with comparable results for parameter constancy. Tests were performed by using 36 lags of the hog-corn ratio along with eleven monthly dummy variables. As well, tests are performed for linearity (parameter constancy) using only monthly dummy variables and only lagged dependent variables. Focusing first on linearity test results obtained by using all regressors, the null hypothesis of

---

6 Of interest is that the largest outlier in absolute terms was the positive value associated with January 1999. Correspondingly, the hog-corn ratio in December 1998 was 7.5, the lowest point observed for the ratio in the post-war period, and the lowest point since December 1934, when it stood at 6.0, the all-time low.

7 Tests were performed initially by using $D_{\text{max}} = 12$, but all results following $d = 8$ were found to be statistically insignificant.
Table 1. Diagnostic Tests for Estimated Models for the U.S. Hog-Corn Ratio

<table>
<thead>
<tr>
<th>Measure</th>
<th>AR</th>
<th>TVAR</th>
<th>TV-STAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>1439</td>
<td>1439</td>
<td>1439</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.170</td>
<td>0.214</td>
<td>0.289</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.142</td>
<td>0.179</td>
<td>0.223</td>
</tr>
<tr>
<td>$\hat{\sigma}_e$</td>
<td>0.083</td>
<td>0.080</td>
<td>0.076</td>
</tr>
<tr>
<td>$\hat{\sigma}<em>{e,\text{NL}} / \hat{\sigma}</em>{e,L}$</td>
<td>—</td>
<td>0.957</td>
<td>0.910</td>
</tr>
<tr>
<td>AIC</td>
<td>-4.940</td>
<td>-4.975</td>
<td>-4.990</td>
</tr>
<tr>
<td>SIC</td>
<td>-4.522</td>
<td>-4.435</td>
<td>-3.909</td>
</tr>
<tr>
<td>SK</td>
<td>0.079 (0.110)</td>
<td>0.091 (0.080)</td>
<td>0.016 (0.596)</td>
</tr>
<tr>
<td>EK</td>
<td>3.383 (0.000)</td>
<td>3.724 (0.000)</td>
<td>2.517 (0.000)</td>
</tr>
<tr>
<td>LJB</td>
<td>705.242 (0.000)</td>
<td>853.115 (0.000)</td>
<td>383.700 (0.000)</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>30.702 (0.000)</td>
<td>28.281 (0.000)</td>
<td>21.200 (0.000)</td>
</tr>
<tr>
<td>ARCH(6)</td>
<td>21.110 (0.000)</td>
<td>19.406 (0.000)</td>
<td>14.478 (0.000)</td>
</tr>
<tr>
<td>LM$_{Sc}(4)$</td>
<td>S 3.469 (0.008)</td>
<td>3.986 (0.003)</td>
<td>1.021 (0.395)</td>
</tr>
<tr>
<td>LM$_{Sc}(4)$</td>
<td>R 3.149 (0.014)</td>
<td>3.369 (0.009)</td>
<td>0.826 (0.509)</td>
</tr>
<tr>
<td>LM$_{Sc}(6)$</td>
<td>S 4.162 (0.012)</td>
<td>2.921 (0.008)</td>
<td>0.155 (0.988)</td>
</tr>
<tr>
<td>LM$_{Sc}(6)$</td>
<td>R 4.279 (0.000)</td>
<td>2.509 (0.020)</td>
<td>0.177 (0.983)</td>
</tr>
<tr>
<td>LM$_{Sc}(8)$</td>
<td>S 3.635 (0.000)</td>
<td>4.279 (0.000)</td>
<td>0.982 (0.448)</td>
</tr>
<tr>
<td>LM$_{Sc}(8)$</td>
<td>R 3.560 (0.000)</td>
<td>3.807 (0.000)</td>
<td>0.883 (0.530)</td>
</tr>
<tr>
<td>LM$_{Sc}(12)$</td>
<td>S 3.000 (0.000)</td>
<td>3.018 (0.000)</td>
<td>1.593 (0.087)</td>
</tr>
<tr>
<td>LM$_{Sc}(12)$</td>
<td>R 2.521 (0.000)</td>
<td>2.604 (0.002)</td>
<td>1.509 (0.114)</td>
</tr>
</tbody>
</table>

Note: The table presents diagnostic tests for the estimated AR, TVAR, and TV-STAR models for the hog-corn ratio, 1882:02–2002:12. $T$ denotes sample size, $R^2$ and $\bar{R}^2$ the unadjusted and adjusted r-squared, and $\hat{\sigma}_e$ the residual standard error. $\hat{\sigma}_{e,\text{NL}} / \hat{\sigma}_{e,L}$ is the ratio of the residual standard error from the respective nonlinear (STAR) model relative to the linear (AR) model. SK is skewness, EK is excess kurtosis, and LJB is the Lomnicki-Jarque-Bera test of normality of the residuals. ARCH is the LM test of no autoregressive conditional heteroskedasticity (ARCH), and LM$_{Sc}(\tau)$ denotes the F variant of standard (S) and heteroskedasticity robust (R) versions of the LM test of no remaining autocorrelation in the residuals up to and including lag $\tau$. Numbers in the parentheses after values of the test statistics are $p$-values.
Table 2. Results of Standard and Heteroskedasticity Robust LM-Type Tests for STAR Model Nonlinearity for Monthly Hog-Corn Ratio

<table>
<thead>
<tr>
<th>Transition Variable, $s_i$</th>
<th>All Regressors</th>
<th>Monthly Dummies</th>
<th>Lagged Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{12}y_{t-1}$</td>
<td>0.001 0.000</td>
<td>0.134 0.180</td>
<td>0.365 0.223 0.415 0.345</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-2}$</td>
<td>0.001 0.007</td>
<td>0.284 0.365</td>
<td>0.014 0.056 0.048 0.161</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-3}$</td>
<td>0.009 0.013</td>
<td>0.423 0.189</td>
<td>0.021 0.040 0.031 0.073</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-4}$</td>
<td>0.006 0.021</td>
<td>0.142 0.283</td>
<td>0.059 0.071 0.105 0.125</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-5}$</td>
<td>0.007 0.037</td>
<td>0.035 0.173</td>
<td>0.092 0.041 0.141 0.093</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-6}$</td>
<td>0.001 0.021</td>
<td>0.033 0.051</td>
<td>0.099 0.059 0.086 0.076</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-7}$</td>
<td>0.001 0.001</td>
<td>0.176 0.010</td>
<td>0.098 0.018 0.097 0.034</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-8}$</td>
<td>0.009 0.017</td>
<td>0.199 0.057</td>
<td>0.350 0.063 0.223 0.044</td>
</tr>
<tr>
<td>$\text{i}^*$</td>
<td>0.000 0.000</td>
<td>0.000 0.000</td>
<td>0.000 0.000 0.000 0.000</td>
</tr>
</tbody>
</table>

Note: The tabulated numbers are $p$-values of $F$ variants the LM-type tests for specification of STAR-type models described by Teräsvirta (1994) applied to the U.S. hog-corn ratio, 1883:02–2002:12. The tests are applied to an AR model with 36 lags of first differences and seasonal dummies. LM$_3$ denotes the linearity test based on a third-order Taylor series, while LM$_1$ is the linearity test based on a first-order Taylor series.
linearity is convincingly rejected against the STAR model for transition variables $s_t = \Delta_{12} y_{t-d}$ associated with delay parameter values $d = 1, \ldots, 8$ when standard tests are employed. Results for robustified LM tests are less convincing. In this case linearity is rejected in favor of the STAR model specification at the 10-percent level for $d = 5, 6, 7$, and 8, depending on whether an LM$_1$ or LM$_3$ test is used (although not shown in order to conserve space, comparable results were obtained for the LM$_3^*$ test). Bearing in mind the Lundbergh-Teräsvirta caveat regarding robustification, results in Table 2 suggest that at least some of the evidence in favor of STAR models might be due to neglected heteroskedasticity.

Linearity tests for only monthly dummy variables are, across the board, less indicative of STAR-type nonlinearity than the comparable results obtained when all regressors are included (Table 2). There is, however, evidence of significant nonlinearities at the 10-percent level for $d = 2, 3, 6$, and 7. In this instance, however, there is generally less discrepancy between standard and robust test results. Turning to the case where linearity is tested for against lagged dependent variables, the results are quite comparable to those obtained when all regressors are included.

The most striking test results in Table 2 are those for parameter constancy. Regardless of the test used (standard or robust, LM$_1$ or LM$_3$), the null hypothesis of parameter constancy is soundly rejected when all regressors are included. An essentially identical result is obtained when only seasonal dummy variables are included. Alternatively, there is little evidence that coefficients on lagged dependent variables are nonconstant (Table 2). The overall picture that emerges from Table 2 then is that of some support for STAR-type nonlinearity, but overwhelming support for the notion that seasonal patterns in the hog-corn ratio have not remained constant through time. For many of the reasons mentioned in the introduction, this result is not surprising.

A Provisional TVAR Model

Based on results in Table 2, it seems reasonable to first estimate a TVAR model wherein only seasonal effects and the constant term vary smoothly through time. To this end a TVAR model that uses (6) with $k = 1$ was fitted to the data by using nonlinear least squares.$^8$ While to preserve space the autoregressive parameter esti-

---

$^8$ In estimation we follow Teräsvirta (1994) in normalizing the slope parameter $\gamma$ in the logistic transition function in (2) by the standard deviation of $s_t$, the transition variable. That is, the transition function $G(s; \gamma, c) = [1 + \exp\{-\gamma(s - c)/\hat{\sigma}_s\}]^{-1}$ is the one actually used in estimation. This is done to make $\gamma$ approximately scale-free, a useful property when choosing starting values for the slope parameter.

$^9$ Results in Table 2 suggest that $k = 3$, a model in which nonsymmetric and nonmonotonic change in seasonal effects is admitted, might be preferred to the case where $k = 1$. Such a model was estimated, but results showed that there was nearly perfect collinearity in the derivatives of the model with respect
mates are not presented, the point estimate of \( c_1 \), the location parameter, is 0.72, indicating that 50 percent of the adjustment in seasonal change does not occur until early 1969. Results of various standard residual diagnostic tests applied to this model are recorded in the center column of Table 1. Results show there is an improvement in fit for the TVAR model relative to the linear AR specification: the adjusted r-squared for the former is 0.18, versus 0.14 for the latter. As well, the standard deviation of the residuals from the TVAR model is about four percent smaller than that of the AR model. The AIC for the TVAR model (-4.98) is also smaller than that for the linear AR model (-4.94). Taken together these results suggest that the TVAR model, where time variation occurs only for the constant term and seasonal dummies, fits the data better than does the constant parameter AR model, even after accounting for the additional parameters required to fit the TVAR model. Based on the LJB test, the residuals associated with the TVAR model also fail to satisfy normality – again due to excess kurtosis. And like the linear model, the TVAR model is also associated with conditionally heteroskedastic errors. Most disturbing, the TVAR model, like its constant parameter counterpart, has remaining residual autocorrelation at lags four, six, eight, and twelve.

Diagnostic tests for remaining nonlinearity \((d = 1,\ldots,8)\) and for parameter constancy for the TVAR model, notably the LM\(_3\) and LM\(_1\) tests, are presented in Table 3. When all regressors are included there is evidence of both STAR-type nonlinearity and additional parameter nonconstancy. The nonlinearity seems most relevant for delay values of \( d = 1, 6, \) and 8. And unlike the initial linearity tests, results in Table 3 indicate substantial evidence of STAR-type nonlinearity in the seasonal terms for all values of \( d \) between one and seven, as well as remaining parameter nonconstancy. Considering only the lagged dependent variables, \( p \)-values recorded in Table 3 suggest the presence of STAR-type nonlinearity at the 10-percent level for \( d = 1, 3, \) and 6. There is also some evidence that the autoregressive parameters are not constant, depending on whether the standard or robust tests are consulted.

**A TV-STAR Model**

Results in Table 3 suggest several possibilities. First, it seems reasonable to fit a second time-varying component to the model, so that the resulting encapsulated TVAR model is consistent with the specification in (12) where \( s_t = t^2 \). Alternatively, results also suggest that a TV-STAR model similar to (12), and effectively identical to those employed by Skalin and Teräsvirta (2002) and Lundberg, Teräsvirta, and van Dijk (2003), may be appropriate, where \( s_t = \Delta_{et} y_{t-d} \) and \( d \) is set equal to either of 1, 3, or 6. The first alternative, the encapsulated TVAR model, was estimated and found to

\[ \text{to the } c_1 \text{ and } c_3 \text{ parameters. Furthermore, the fit was only nominally better than the TVAR model with } k = 1, \text{ where parameter change is restricted to be monotonic.} \]
Table 3. Results of Standard and Heteroskedasticity Robust LM-type Diagnostic Tests for TVAR Model Estimated for Monthly Hog-Corn Ratio

<table>
<thead>
<tr>
<th>Transition Variable, $s_i$</th>
<th>All Regressors</th>
<th>Monthly Dummies</th>
<th>Lagged Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$LM_3^e$</td>
<td>$LM_1$</td>
<td>$LM_3^e$</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-1}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.046</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-2}$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.284</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-3}$</td>
<td>0.002</td>
<td>0.002</td>
<td>0.160</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-4}$</td>
<td>0.005</td>
<td>0.005</td>
<td>0.115</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-5}$</td>
<td>0.034</td>
<td>0.033</td>
<td>0.140</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-6}$</td>
<td>0.002</td>
<td>0.003</td>
<td>0.051</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-7}$</td>
<td>0.006</td>
<td>0.006</td>
<td>0.083</td>
</tr>
<tr>
<td>$\Delta_{12}y_{t-8}$</td>
<td>0.053</td>
<td>0.055</td>
<td>0.040</td>
</tr>
<tr>
<td>$t^*$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Note: Numbers are $p$-values of LM-type tests for model misspecification of LSTAR-type models described by Eitrheim and Teräsvirta (1996) and van Dijk and Franses (1999) and applied to the U.S. hog-corn ratio, 1883:02–2002:12. The first eight rows denote tests for remaining nonlinearity, and the final row reports tests for parameter constancy. $LM_3^e$ denotes an economy version of the $LM_3$ test (i.e., a third-order Taylor series expansion with interactions omitted for second- and third-order terms) for remaining nonlinearity (parameter non-constancy). $LM_1$ is analogously defined, but for a first-order Taylor series expansion.
provide a good fit to the data. One drawback with this model, however, was evidence of remaining residual autocorrelation at various lags. The TV-STAR model in (12) was also fitted to the data with \( s_t = \Delta_{12} y_{t-d} \), \( d = 1, \ldots, 6 \). Results indicated that a TV-STAR model with \( d = 1 \) performed best overall. Interestingly, the explanatory power of this model was essentially identical to that of the encapsulated TVAR model. We therefore focus our remaining attention on results for the TV-STAR model with transition variable \( s_t = \Delta_{12} y_{t-1} \). Parameter estimates along with heteroskedasticity-robust standard errors for this model are presented in Table 4. Results for several standard misspecification tests applied to this model are recorded in the right-most column of Table 1.

Results in Table 1 show that, based on the adjusted \( r \) squared and the AIC, the TV-STAR model represents an improvement over either the AR or TVAR specifications. Based on the LJB statistic there is evidence that this model also fails the normality assumption; however, excess kurtosis has now been reduced substantially relative to the other models. Evidence of significant ARCH effects remains. But of considerable interest are the LM test results for remaining residual autocorrelation. In contrast to the AR and TVAR models, the TV-STAR model is associated with no significant autocorrelation at any of the lags considered. In this regard the TV-STAR model apparently does a better job of explaining the dynamics intrinsic to the hog-corn ratio than either its AR or TVAR model counterparts.

Diagnostic tests for remaining nonlinearity (both additive and MRSTAR) and for parameter constancy are presented in Table 5. Results show there is little evidence of remaining nonlinearity of the additive type when either standard or robust results are considered. There is some evidence of remaining MRSTAR nonlinearity, but this conclusion depends on whether standard or robust versions of the test are employed. Finally, there is evidence of remaining parameter instability (both of the additive and TV-STAR model types), although this possibility has not been followed up on.

The estimate of the location parameter \( c_2 \) is reasonably close to zero, implying that regimes where \( G_2(\Delta_{12} y_{t-1}) = 0 \) and \( G_2(\Delta_{12} y_{t-1}) = 1 \) are associated with positive and negative changes in the hog-corn cycle over the past twelve months. As illustrated in Figure 3, where each circle denotes at least one observation, the transition between the two regimes is rather smooth, with a number of intermediate points residing between full expansion and contraction. Because \( G_2(\Delta_{12} y_{t-1}) \) is simply a monotonic transformation of \( \Delta_{12} y_{t-1} \), it follows that periods for which \( G_2(\Delta_{12} y_{t-1}) = 0 \) (1) are roughly associated with troughs (peaks) in the hog cycle.\(^{10}\) The transition function

\(^{10}\) The usual interpretation in a standard business cycle setting is that \( G_1(\Delta_{12} y_{t-1}; \hat{\gamma}_2, \hat{c}_2) = 0 \) is associated with a recession and \( G_1(\Delta_{12} y_{t-1}; \hat{\gamma}_2, \hat{c}_2) = 1 \) with an expansion. This interpretation does not necessarily apply here, however, because we are dealing with prices and not with inventories of hogs. \( G_1(\Delta_{12} y_{t-1}; \hat{\gamma}_2, \hat{c}_2) = 0 \) would typically be associated with large hog numbers, and could therefore be thought of as being associated with the peak of an expansionary phase.
### Table 4. TV-STAR Model Estimates for the Monthly U.S. Hog-Corn Ratio

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_{t-1}$</td>
<td>-0.128</td>
<td>0.097</td>
<td>$\Delta y_{t-1}$</td>
<td>0.161</td>
<td>0.050</td>
<td>$\Delta y_{t-25}$</td>
<td>-0.193</td>
<td>0.062</td>
<td>$\Delta y_{t-25}$</td>
<td>-0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>$\Delta y_{t-2}$</td>
<td>-0.164</td>
<td>0.091</td>
<td>$\Delta y_{t-2}$</td>
<td>-0.061</td>
<td>0.046</td>
<td>$\Delta y_{t-26}$</td>
<td>-0.099</td>
<td>0.056</td>
<td>$\Delta y_{t-26}$</td>
<td>0.053</td>
<td>0.027</td>
</tr>
<tr>
<td>$\Delta y_{t-3}$</td>
<td>-0.231</td>
<td>0.066</td>
<td>$\Delta y_{t-3}$</td>
<td>0.039</td>
<td>0.041</td>
<td>$\Delta y_{t-27}$</td>
<td>-0.124</td>
<td>0.063</td>
<td>$\Delta y_{t-27}$</td>
<td>0.040</td>
<td>0.019</td>
</tr>
<tr>
<td>$\Delta y_{t-4}$</td>
<td>-0.067</td>
<td>0.070</td>
<td>$\Delta y_{t-4}$</td>
<td>-0.013</td>
<td>0.046</td>
<td>$\Delta y_{t-28}$</td>
<td>-0.077</td>
<td>0.049</td>
<td>$\Delta y_{t-28}$</td>
<td>0.035</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Delta y_{t-5}$</td>
<td>-0.083</td>
<td>0.086</td>
<td>$\Delta y_{t-5}$</td>
<td>-0.090</td>
<td>0.040</td>
<td>$\Delta y_{t-29}$</td>
<td>-0.099</td>
<td>0.054</td>
<td>$\Delta y_{t-29}$</td>
<td>-0.037</td>
<td>0.020</td>
</tr>
<tr>
<td>$\Delta y_{t-6}$</td>
<td>-0.131</td>
<td>0.073</td>
<td>$\Delta y_{t-6}$</td>
<td>0.015</td>
<td>0.043</td>
<td>$\Delta y_{t-30}$</td>
<td>0.001</td>
<td>0.058</td>
<td>$\Delta y_{t-30}$</td>
<td>-0.086</td>
<td>0.035</td>
</tr>
<tr>
<td>$\Delta y_{t-7}$</td>
<td>-0.186</td>
<td>0.077</td>
<td>$\Delta y_{t-7}$</td>
<td>-0.001</td>
<td>0.041</td>
<td>$\Delta y_{t-31}$</td>
<td>-0.192</td>
<td>0.064</td>
<td>$\Delta y_{t-31}$</td>
<td>-0.037</td>
<td>0.026</td>
</tr>
<tr>
<td>$\Delta y_{t-8}$</td>
<td>-0.071</td>
<td>0.083</td>
<td>$\Delta y_{t-8}$</td>
<td>-0.007</td>
<td>0.040</td>
<td>$\Delta y_{t-32}$</td>
<td>-0.167</td>
<td>0.057</td>
<td>$\Delta y_{t-32}$</td>
<td>-0.024</td>
<td>0.031</td>
</tr>
<tr>
<td>$\Delta y_{t-9}$</td>
<td>-0.063</td>
<td>0.067</td>
<td>$\Delta y_{t-9}$</td>
<td>0.015</td>
<td>0.039</td>
<td>$\Delta y_{t-33}$</td>
<td>-0.079</td>
<td>0.054</td>
<td>$\Delta y_{t-33}$</td>
<td>-0.007</td>
<td>0.022</td>
</tr>
<tr>
<td>$\Delta y_{t-10}$</td>
<td>-0.193</td>
<td>0.066</td>
<td>$\Delta y_{t-10}$</td>
<td>-0.016</td>
<td>0.038</td>
<td>$\Delta y_{t-34}$</td>
<td>-0.104</td>
<td>0.055</td>
<td>$\Delta y_{t-34}$</td>
<td>0.014</td>
<td>0.026</td>
</tr>
<tr>
<td>$\Delta y_{t-11}$</td>
<td>-0.034</td>
<td>0.068</td>
<td>$\Delta y_{t-11}$</td>
<td>0.107</td>
<td>0.037</td>
<td>$\Delta y_{t-35}$</td>
<td>-0.048</td>
<td>0.052</td>
<td>$\Delta y_{t-35}$</td>
<td>0.029</td>
<td>0.025</td>
</tr>
<tr>
<td>$\Delta y_{t-12}$</td>
<td>-0.126</td>
<td>0.078</td>
<td>$\Delta y_{t-12}$</td>
<td>-0.084</td>
<td>0.041</td>
<td>$\Delta y_{t-36}$</td>
<td>-0.007</td>
<td>0.051</td>
<td>$\Delta y_{t-36}$</td>
<td>0.014</td>
<td>0.032</td>
</tr>
<tr>
<td>$\Delta y_{t-13}$</td>
<td>-0.176</td>
<td>0.070</td>
<td>$\Delta y_{t-13}$</td>
<td>-0.106</td>
<td>0.034</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-14}$</td>
<td>-0.163</td>
<td>0.059</td>
<td>$\Delta y_{t-14}$</td>
<td>-0.059</td>
<td>0.035</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-15}$</td>
<td>-0.147</td>
<td>0.066</td>
<td>$\Delta y_{t-15}$</td>
<td>-0.044</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-16}$</td>
<td>-0.127</td>
<td>0.053</td>
<td>$\Delta y_{t-16}$</td>
<td>-0.056</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-17}$</td>
<td>-0.207</td>
<td>0.062</td>
<td>$\Delta y_{t-17}$</td>
<td>-0.073</td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-18}$</td>
<td>-0.112</td>
<td>0.053</td>
<td>$\Delta y_{t-18}$</td>
<td>-0.040</td>
<td>0.040</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-19}$</td>
<td>-0.076</td>
<td>0.057</td>
<td>$\Delta y_{t-19}$</td>
<td>-0.052</td>
<td>0.033</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-20}$</td>
<td>-0.063</td>
<td>0.057</td>
<td>$\Delta y_{t-20}$</td>
<td>-0.020</td>
<td>0.039</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-21}$</td>
<td>-0.027</td>
<td>0.057</td>
<td>$\Delta y_{t-21}$</td>
<td>-0.017</td>
<td>0.036</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-22}$</td>
<td>-0.082</td>
<td>0.056</td>
<td>$\Delta y_{t-22}$</td>
<td>-0.018</td>
<td>0.036</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-23}$</td>
<td>-0.103</td>
<td>0.061</td>
<td>$\Delta y_{t-23}$</td>
<td>-0.024</td>
<td>0.038</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta y_{t-24}$</td>
<td>-0.138</td>
<td>0.062</td>
<td>$\Delta y_{t-24}$</td>
<td>-0.019</td>
<td>0.038</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4. TV-STAR Model Estimates for the Monthly U.S. Hog-Corn Ratio (cont’d.)

\[
\begin{align*}
(1 - G_2(\Delta_{12}y_{t-1})) & (1 - G_i(t')) \\
(1 - G_2(\Delta_{12}y_{t-1})) & G_i(t') \\
G_2(\Delta_{12}y_{t-1}) & (1 - G_i(t')) \\
G_2(\Delta_{12}y_{t-1}) & G_i(t')
\end{align*}
\]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
<th>Variable</th>
<th>Coef</th>
<th>HCSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.128</td>
<td>0.097</td>
<td>Constant</td>
<td>0.161</td>
<td>0.050</td>
<td>Constant</td>
<td>-0.193</td>
<td>0.062</td>
<td>Constant</td>
<td>-0.011</td>
<td>0.006</td>
</tr>
<tr>
<td>D1</td>
<td>-0.164</td>
<td>0.091</td>
<td>D1</td>
<td>-0.061</td>
<td>0.046</td>
<td>D1</td>
<td>-0.099</td>
<td>0.056</td>
<td>D1</td>
<td>0.053</td>
<td>0.027</td>
</tr>
<tr>
<td>D2</td>
<td>-0.231</td>
<td>0.066</td>
<td>D2</td>
<td>0.039</td>
<td>0.041</td>
<td>D2</td>
<td>-0.124</td>
<td>0.063</td>
<td>D2</td>
<td>0.040</td>
<td>0.019</td>
</tr>
<tr>
<td>D3</td>
<td>-0.067</td>
<td>0.070</td>
<td>D3</td>
<td>-0.013</td>
<td>0.046</td>
<td>D3</td>
<td>-0.077</td>
<td>0.049</td>
<td>D3</td>
<td>0.035</td>
<td>0.025</td>
</tr>
<tr>
<td>D4</td>
<td>-0.083</td>
<td>0.086</td>
<td>D4</td>
<td>-0.090</td>
<td>0.040</td>
<td>D4</td>
<td>-0.099</td>
<td>0.054</td>
<td>D4</td>
<td>-0.037</td>
<td>0.020</td>
</tr>
<tr>
<td>D5</td>
<td>-0.131</td>
<td>0.073</td>
<td>D5</td>
<td>0.015</td>
<td>0.043</td>
<td>D5</td>
<td>0.001</td>
<td>0.058</td>
<td>D5</td>
<td>-0.086</td>
<td>0.035</td>
</tr>
<tr>
<td>D6</td>
<td>-0.186</td>
<td>0.077</td>
<td>D6</td>
<td>-0.001</td>
<td>0.041</td>
<td>D6</td>
<td>-0.192</td>
<td>0.064</td>
<td>D6</td>
<td>-0.037</td>
<td>0.026</td>
</tr>
<tr>
<td>D7</td>
<td>-0.071</td>
<td>0.083</td>
<td>D7</td>
<td>-0.007</td>
<td>0.040</td>
<td>D7</td>
<td>-0.167</td>
<td>0.057</td>
<td>D7</td>
<td>-0.024</td>
<td>0.031</td>
</tr>
<tr>
<td>D8</td>
<td>-0.063</td>
<td>0.067</td>
<td>D8</td>
<td>0.015</td>
<td>0.039</td>
<td>D8</td>
<td>-0.079</td>
<td>0.054</td>
<td>D8</td>
<td>-0.007</td>
<td>0.022</td>
</tr>
<tr>
<td>D9</td>
<td>-0.193</td>
<td>0.066</td>
<td>D9</td>
<td>-0.016</td>
<td>0.038</td>
<td>D9</td>
<td>-0.104</td>
<td>0.055</td>
<td>D9</td>
<td>0.014</td>
<td>0.026</td>
</tr>
<tr>
<td>D10</td>
<td>-0.034</td>
<td>0.068</td>
<td>D10</td>
<td>0.107</td>
<td>0.037</td>
<td>D10</td>
<td>-0.048</td>
<td>0.052</td>
<td>D10</td>
<td>0.029</td>
<td>0.025</td>
</tr>
<tr>
<td>D11</td>
<td>-0.126</td>
<td>0.078</td>
<td>D11</td>
<td>-0.084</td>
<td>0.041</td>
<td>D11</td>
<td>-0.007</td>
<td>0.051</td>
<td>D11</td>
<td>0.014</td>
<td>0.032</td>
</tr>
</tbody>
</table>

\[
G_i(t^*; \hat{\theta}_1, \hat{\theta}_2) = [1 + \exp\{-(1.964 (t^* - 0.685)/\hat{\sigma}_{t^*})\}]^{-1} \\
(0.858) (0.124) \\
[2.553] [0.265]
\]

\[
G_2(\Delta_{12}y_{t-1}; \hat{\theta}_2, \hat{\theta}_2) = [1 + \exp\{-(37.306 (\Delta_{12}y_{t-1} - 0.084)/\hat{\sigma}_{\Delta_{12}y_{t-1}})\}]^{-1} \\
(30.789) (0.043) \\
[31.551] [0.006]
\]

Note: The table presents TV-STAR model estimates for the hog-corn ratio model, 1882:02–2002:12, where \( t^* = t/1439 \) is the transition variable for \( G_i(.) \) and \( s_t = \Delta_{12}y_{t-1} \) is the transition variable for \( G_2(.) \). HCSE denotes heteroskedasticity robust standard errors and D1-D11 denote seasonal dummy variables. Values in parentheses (brackets) are ordinary (heteroskedasticity robust) standard errors.
Table 5. Results of Standard and Heteroskedasticity Robust LM-Type Diagnostic Tests for TV-STAR Model Estimated for Monthly Hog-Corn Ratio

<table>
<thead>
<tr>
<th>Transition Variable $s_{2j}$</th>
<th>Remaining Additive Nonlinearity</th>
<th>Remaining MRSTAR Nonlinearity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standard</td>
<td>Robust</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-1}$</td>
<td>0.237</td>
<td>0.270</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-2}$</td>
<td>0.172</td>
<td>0.151</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-3}$</td>
<td>0.145</td>
<td>0.179</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-4}$</td>
<td>0.080</td>
<td>0.091</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-5}$</td>
<td>0.121</td>
<td>0.120</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-6}$</td>
<td>0.296</td>
<td>0.263</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-7}$</td>
<td>0.051</td>
<td>0.047</td>
</tr>
<tr>
<td>$\Delta_1 y_{t-8}$</td>
<td>0.117</td>
<td>0.107</td>
</tr>
<tr>
<td>$t$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Numbers are $p$-values of LM-type tests for model misspecification in the form of remaining nonlinearity additive described by Eitrheim and Teräsvirta (1996) (the first four columns) and remaining MRSTAR nonlinearity described by Lundbergh, Teräsvirta, and van Dijk (2003) (the last two columns), and applied to the U.S. hog-corn ratio, 1883:02–2002:12. The first eight rows denote tests for remaining nonlinearity and the final row reports tests for parameter constancy. $\text{LM}_3^e$ denotes an economy version of the $\text{LM}_3$ test (i.e., a third-order Taylor series expansion with interactions omitted for second- and third-order terms) for remaining nonlinearity (parameter non-constancy). $\text{LM}_1$ is analogously defined, but for a first-order Taylor series expansion.
Figure 3. Estimated Transition Function, $G_2(\Delta_{12}y_{t-1})$, as a Function of the Transition Variable, $\Delta_{12}y_{t-1}$

$G_2(\Delta_{12}y_{t-1})$ plotted against time is shown in Figure 4. Results indicate that, with the exception of the mid-1880s and the late 1920s and early-to-mid 1930s, there has been roughly a two-to-five year hog-corn cycle, a result that is consistent with some previous research (e.g., Jelavich 1973). There is also some evidence that the duration of the cycle has increased since the late 1960s.

The estimate of location parameter $c_1$ suggests that structural change is centered around $t^* = 0.68$, which corresponds with April 1965. This result is not surprising, as twice-per-year farrowing cycles did not begin to disappear until the late 1950s and early-to-mid 1960s. This move has unquestionably had a large impact on seasonality of hog production, and therefore the seasonality of hog prices. The structural change is quite smooth, as illustrated in Figure 4. Moreover, the change is not entirely complete by the end of the sample period.

To gain further insights into the nature of the structural change, the estimated seasonal effects for March, June, September, and December are plotted in Figure 5. Estimated seasonal effects for the same months, plotted against transition function $G_2(\Delta_{12}y_{t-1})$, are depicted in Figure 6. The results for March show that the associated
seasonal effect has gone from a range of (+4, -1) percent to (-2, -5) percent during the sample period, with the switch occurring in the mid-1950s. As depicted in Figure 6a, the responsiveness to peaks in March is overall much larger than that for troughs. This result is consistent with the estimated seasonal effects reported in Table 4. Similar seasonal time patterns are noted for September and December (Figures 5c and 5d, respectively), although the overall effect for September is much larger than for the other months, with a range of (+1.5, 8) percent early in the sample to (-4, -7.5) percent late in the sample. As well, unlike the other months considered, there is a greater range of response in September during troughs than there is during peaks (Figure 6c). The seasonal effect for June, on the other hand, has gone from a range of (-3.5, -5) percent early in the sample to a range of (0, 6) percent late in the sample, with the switch again occurring in the mid-1950s (Figure 5b). The same information plotted against transition function $G_2(\Delta_{12}y_{t-1})$ shows a rather tight seasonal response in both regimes, but with the overall June effect being somewhat larger during peaks (Figure 6b). Overall, Figure 5 illustrates that seasonal patterns for the hog-corn cycle have changed over time. Figure 6 shows that seasonal responses to the current state of the hog-corn cycle are also potentially important and, moreover, vary with the specific month considered.
Figure 5. Estimated Seasonal Effects for the TV-STAR Model of the Hog-Corn Ratio over the Sample Period for Select Months: (a) March, (b) June, (c) September, and (d) December
Figure 6. Estimated Seasonal Effects for the TV-STAR Model of the Hog-Corn Ratio over the Sample Period for Select Months versus Transition Function $G_z(\Delta z_{t-1})$: (a) March, (b) June, (c) September, and (d) December
Model Dynamics

As the foregoing makes clear, there are features of the hog market consistent with both nonlinear dynamics and structural change. It is therefore desirable to characterize the dynamic behavior of the estimated TV-STAR model in some consistent and reasonably transparent ways, the focus of this section.

Deterministic Extrapolation

We consider first a deterministic extrapolation of the model to obtain insights into the behavior implied by the model. This is done by iterating the skeleton of the model, that is, the deterministic part of the model, ahead without introducing stochastic shocks. We start the extrapolation of the skeleton by using the final values of the sample data as initial values. Iterating the model ahead for 33 years, we find that the realizations converge to something very similar to a limit cycle with a period of three to four years. The results are plotted in Figure 7. Indeed, the periodicity of the cycle is very similar to that observed in the data. The cycle is also asymmetric, as downturns tend to be sharper and happen more quickly than upturns. At the least we may therefore conclude that the hog-corn ratio is a highly persistent series.

Figure 7. Deterministic Extrapolation of the TV-STAR Model of the U.S. Hog-Corn Ratio (horizon 33 years)
Generalized Impulse Response Functions

To obtain additional information about the dynamic properties of the model, shock propagation is examined by computing generalized impulse response functions (GIs), as proposed by Koop, Pesaran, and Potter (1996). The GI is useful for assessing the properties of nonlinear models because it may be used to average over “histories,” “shocks,” and “futures.” Let \( \varepsilon_t = \delta \) denote a specific shock and \( \Omega_{t-1} = \omega_{t-1} \)
a particular history. The GI is then defined as

\[
GI_{\Delta t} (h, \delta, \omega_{t-1}) = E \left[ \Delta y_{t+h} \left| \varepsilon_t = \delta, \omega_{t-1} \right. \right] - E \left[ \Delta y_{t+h} \left| \omega_{t-1} \right. \right], \quad h = 0, 1, 2, \ldots
\]

In (13), the expectation of \( \Delta y_t \) is conditional only with respect to the shock and the history – all shocks that might occur in intermediate periods (futures) are, in effect, averaged out. The GI is therefore a function of \( \delta \) and \( \omega_{t-1} \), which in turn are realizations of the random variables \( \varepsilon_t \) and \( \Omega_{t-1} \). The implication is that \( GI_{\Delta t} (h, \delta, \omega_{t-1}) \), defined as,

\[
GI_{\Delta t} (h, \delta, \omega_{t-1}) = E \left[ \Delta y_{t+h} \left| \varepsilon_t = \delta, \Omega_{t-1} \right. \right] - E \left[ \Delta y_{t+h} \left| \omega_{t-1} \right. \right], \quad (14)
\]

is itself a random variable. The GI defined in (14) also has several conditional versions of potential interest. For example, only a particular history \( \omega_{t-1} \) might be considered, and the GI taken as a random variable only in the shock \( \varepsilon_t \). Alternatively, the shock might be fixed at \( \varepsilon_t = \delta \) and the GI treated as a random variable with respect to the history \( \Omega_{t-1} \). Finally, it is possible to consider some subset of shocks and/or histories, defined as \( \delta \) and \( \mathcal{H} \) respectively, so that the conditional GI is given by \( GI_{\Delta t} (h, \delta, \mathcal{H}) \). In the case of the TV-STAR model, this latter property is useful for considering all histories in a particular regime associated with, say, either a positive or negative shock.

Regarding the TV-STAR model considered here, we compute the GI in (13) in the following manner. First, we draw a random sample of 360 “histories,” that is, initialization values, from the data used to estimate the model.\(^{11}\) Values of the normalized initial shock are set equal to \( \delta/\hat{\sigma}_{\varepsilon} = \pm 3, \pm 2.8, \ldots, \pm 0.2 \), where \( \hat{\sigma}_{\varepsilon} \) is the estimated standard deviation of the residuals from the TV-STAR model. The maximum forecast horizon is set at 96, that is, \( h = 0, \ldots, 96 \). Therefore, for each combination of history and initial shock, we compute \( GI_{\Delta t} (h, \delta, \omega_{t-1}) \) for \( h = 0, \ldots, 96 \). An analytical expression for the conditional expectation in (13) is not available for \( h > 0 \) for the TV-STAR model. Here the expectations are evaluated numerically by using 800 bootstrap simulations and taking the sample means. To summarize, the

\(^{11}\) Note that the number of histories, 360, is close to 25 percent of the total number of observations (histories) available.
conditional expectation in (13) is estimated as the means over 800 realizations of \( \Delta y_{t+h} \), obtained by iterating the TV-STAR model, with and without the initial shock used in the calculation of \( \Delta y_t \), and by using 800 TV-STAR residuals sampled with replacement. With 30 shocks and 360 histories, this implies that 10,800 GI response vectors of length 96 are calculated. Impulse responses for the level of the hog-corn ratio are constructed by totaling the impulse responses for the first differences, that is, 

\[
G(y_t \delta, \omega_{t, j}) = \sum_{j=0}^{\hat{n}} G(y_{t+j} \delta, \omega_{t, j}).
\]

We compute GIs for three STAR models: the one observed prior to the structural change \( G_t(t*) = 0 \); the one observed when structural change is at the midway point \( G_t(t*) = 0.5 \); and the one obtained when structural change is complete \( G_t(t*) = 1 \).

Mean paths for the GIs, conditional on histories for \( G_1(\Delta_{12}y_{t-1}; \hat{y}_2, \hat{e}_2) > 0.5 \) (peaks) and for \( G_2(\Delta_{12}y_{t-1}; \hat{y}_2, \hat{e}_2) \leq 0.5 \) (troughs), and conditional on positive and negative shocks, are presented in Figure 8. Several features of interest are revealed in these plots. First, there is considerable persistence in the shocks, a result that is consistent with other observed features of the hog-corn ratio data and with the deterministic extrapolation. Furthermore, the persistence seems to be more highly amplified at the end of the structural change (Figure 8c) than at the beginning (Figure 8a). Second, there is little evidence of asymmetric response to positive versus negative shocks in the mean paths of the GIs, at least for the shocks occurring at the beginning and midpoint of structural change. This is certainly the case for the regime where \( G_2(\Delta_{12}y_{t-1}; \hat{y}_2, \hat{e}_2) \leq 0.5 \), although somewhat less so in the case where \( G_2(\Delta_{12}y_{t-1}; \hat{y}_2, \hat{e}_2) > 0.5 \) and where \( G_t(t*) = 1 \) (Figure 8c). Third, and most importantly, there is a distinct difference in shock transmission between the two regimes, which becomes more pronounced with structural change. Shocks during the “trough” regime are larger in magnitude and have more persistence, at least initially, than are shocks during the “peak” regime. This effect is especially noticeable for negative shocks at the end of the structural change (Figure 8c), where there are noticeable differences in the mean paths even at horizons of up to 60 months or more.
Figure 8. Mean Paths for Generalized Impulse Response Functions of the TV-STAR Model for the Hog-Corn Ratio, (a) $G_1(t^*) = 0$, (b) $G_1(t^*) = 0.5$, and (c) $G_1(t^*) = 1$
Conclusions

In this chapter we have explicitly modeled potential nonlinear features of the U.S. hog-corn cycle in combination with structural change. While previous research has found evidence of nonlinearity in the hog-corn cycle, no prior attempts have been made to explicitly model the implied nonlinearity. We do so here by using a class of endogenous regime-switching models belonging to the family of smooth transition autoregression (STAR) models. The time series of monthly observations on the hog-corn ratio used in the empirical analysis spans the 1880–2002 period. Not only does this period include a number of complete cycles, but it also encompasses many historical and institutional changes that might lead to structural instability. At the beginning of the sample period a national cycle was just emerging as local and regional markets became more integrated. Moreover, breeding cycles were such that a new pig crop would typically be produced only once or at most twice a year. This situation began to change rather rapidly in the post-war period as producers switched to total confinement operations and to nearly continuous breeding-production cycles. Consolidation of this sort was especially prevalent in the 1950s and 1960s. We might therefore suspect that these and other effects would have a substantial impact on seasonal production patterns, and therefore on seasonal price patterns.

In modeling the data we followed the basic testing, estimation, and evaluation cycle proposed initially by Teräsvirta (1994). The results of various linearity tests suggested that a time-varying STAR, or TV-STAR, model – wherein structural change occurs only for the intercept and seasonal dummy terms – is appropriate for modeling the hog-corn cycle. A TV-STAR model that uses \( \Delta_{12} y_{t-4} \) as a transition variable was subsequently fitted to the data, and found – based on comparisons with linear AR and TVAR models and, as well, model diagnostics – to be a suitable specification in most respects. We proceeded to analyze various features of this model.

A careful examination of the TV-STAR model’s properties yielded several interesting features of the hog-corn cycle. First, the cycle appears to have occurred with a somewhat regular three-to-five year frequency during the sample period. The late 1920s and early 1930s emerged as a time of high activity, as did the early part of the sample. Structural change, most notably in the form of evolving seasonal effects, appears to have occurred rapidly since the mid 1950s. When the model’s skeleton was iterated ahead, the series approached something that appears to be very much like a limit cycle. Lastly, calculation of generalized impulse response functions showed that the response of the model to a shock is quantitatively and qualitatively different in the two regimes, especially after the structural change is essentially complete. It therefore seems likely that smooth transition models would prove useful in modeling nonlinearities and structural change in other commodity price relationships, a topic that remains for future investigation.
References


