COMMENTARY ON "RATIONAL EXPECTATIONS, INFORMATION ACQUISITION, AND COMPETITIVE BIDDING"

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My paper was initially inspired by Wilson's program in the late 1970s, which included setting rigorous foundations for notions like *perfect competition* and *rational expectations equilibrium* using game theory. His 1977 paper "A Bidding Model of Perfect Competition" pioneered this area. In it, he used a limiting argument to show that the winning bid in an auction could be a consistent estimator of the value of the good being sold, even when no single bidder's information could lead to a similarly accurate estimate.

Others were being inspired by Wilson's program as well. In particular, I remember meeting in the Stanford GSB courtyard with Bob and Roger Myerson in the summer of 1978. After talking briefly about my research, Bob speculated that a standard auction might actually be a revenue-maximizing process. Roger announced, "I know how to settle that!" and immediately undertook his famous optimal auction analysis.

In my own paper, reprinted in this volume, I tried to use auction theory to analyze the rational expectations equilibrium "paradox" — according to which a price-taker whose information is "revealed" in the price has no reason to invest in collecting information. My strategy was to introduce an auction model in which information gathering was endogenous. To capture price-taking behavior, I used a second-price auction, because the price a winning bidder pays then does not depend on its bid and fixes only the quantity — zero or one — that it receives at the auction-determined price.

My first challenge was the technical one of proving that an equilibrium exists in this model with correlated values. I adopted Wilson's assumption of *conditionally independent types*, adding the assumption that the family of conditional distributions satisfied a *monotone likelihood ratio* (MLR) property. I showed that the MLR property had three important consequences. One was that it implied a certain *single crossing condition* in the bidder's decision problem, leading bidders to select monotonic bidding strategies. Second, I showed that this single crossing together with the first-order conditions implied that the bid functions were equilibrium bid functions. Finally, it enabled me to establish a "no regret" condition that might now be called ex post equilibrium.

What I couldn't have known at the time was that the technical issues I began to study in this paper would become be a recurrent research theme for me. This next development for me began at a seminar dinner, when Eric Maskin asked me what my model would look like in a Harsanyi-like formulation, in which the only random variables in the models were ones that some player observed. Like Wilson, I had used an unobserved "common value" element both to characterize the distribution of the conditionally independent types and as part of the specification of the individual bidder payoffs. Answering Maskin's question fed into my paper with Robert Weber (1982), in which we developed our *"general symmetric model,"* which put all the bidders' types directly into a valuation function. We introduced the notion of *affiliation* of the joint density function $f(x_1, \ldots, x_N, v)$ to replace the twin assumptions about conditional independence and the MLR property of $f(x_i|v)$.

Some time after that — I can't recall exactly when, Wilson explained Tarski's fixed point theorem to me. He showed his trademark excitement, suggesting that because lattices played a role in affiliation as well as in Tarski's theorem, there might be a connection. Affiliation is indeed a lattice condition, equivalent to the condition that the logarithm of the joint density, $\ln\big(f(\cdot)\big)$, is *supermodular*. Later, these lattice concepts would support analyses with John Roberts (1990a,b) of manufacturing systems and strategic complementarity, with Bengt Holmstrom (1994) of incentive systems, and with Chris Shannon (1994) of generalized comparative statics methods.

Quite mysteriously, auction and lattice ideas came together again when Wilson and I worked together on the FCC spectrum auction design. What we had called "back-up strategies" in our original presentations to the FCC wound up corresponding formally to the case of auctions when goods are substitutes. The condition of "substitutes" is mathematically identical to the condition that the bidder expenditure functions are supermodular. The monotonic auction process, with its monotonically rising prices, is simply a process to maximize the supermodular net expenditure function, in that way finding a competitive equilibrium.

There was something quite profound in Bob's advice and direction focusing on market design issues and lattice techniques. Twenty five years later, after spending so much of my career building on these few ideas, it is hard to imagine what my scholarly life would have been like without Bob's influence.

References

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