BARGAINING WITH IMPERFECT COMMITMENT

Shinsuke Kambe*

Abstract

We study the effect of imperfect commitment in non-cooperative two-person bargaining games. By establishing the reputation for being stubborn, a player sometimes commits to her initial demand, becoming unable to change her demands or to accept an inferior offer from her opponent. When the probability of being stubborn is small, the set of equilibria is shown to be small and agreement may be reached immediately despite the possibility of stubbornness. A player has greater bargaining power when she is more patient and/or is more likely to be stubborn.

1 Introduction

When a rational player is faced with a stubborn negotiator who never concedes, he will accommodate her demand since the rejection will only cause delay. Considering this effect, even a flexible player may pretend to be stubborn if her opponent is uncertain whether she is flexible or not. Our goal is to show that this type of strategy increases her bargaining power and, at the same time, that it does not necessarily lead to delay.

Two players negotiate how to share one dollar. In the beginning, they propose to each other how much they want for themselves. If the sum of their demands is not greater than one, then they will split midway between their demands. Otherwise, they keep negotiating until one player lowers her demand and their demands become compatible.

With small probabilities, we assume that the players become stubborn after making initial demands. A stubborn type commits herself to her initial demand and waits for her opponent to accept it. This assumption is motivated by an observation that negotiation sometimes breaks down because both negotiators

*I had valuable comments from Faruk Gul as well as the seminar participants at Bristol, Keio, Southampton, the TCER conference in Kyoto, and Warwick. I also thank an associate editor and a referee for their detailed comments.

insist that their own offers should be accepted. (See Crawford, 1982 for some discussion about this type of imperfect commitment.) A player's nature is known only to herself; her opponent knows only its likelihood. The stubbornness may be psychological. For example, a player may become too attached to her own proposal to accept her opponent's offer. Or it may be economical. A player tells her business associates that her reputation is at stake. She may sometimes convince them that this is so. Then backing off damages her reputation, which she thinks to be more important than any agreement.

One characteristic of equilibrium play is that, given incompatible initial demands, one player may accept the offer of the other with a positive probability in the beginning (initial mass acceptance). Observe that the payoff of a player is no more than what her opponent offers once the war of attrition starts. Hence, it is strategically important for her to cause her opponent to make the initial mass acceptance. She can do that when she has a lower offer, a lower discount rate, and/or a higher probability of being stubborn, given that other things are equal. The latter two factors are exogenous, which determine her natural bargaining power. On the other hand, she can choose the first variable strategically. If she expects her opponent to demand a large share, she can lower her initial demand and can cause her opponent to make the initial mass acceptance. Because of this, excessive demands cannot be sustained in equilibrium and the equilibrium demands tend to be near the just compatible demands that are proportional to the players' natural bargaining powers. When the probability of stubborn types tends to zero, equilibrium outcomes converge to immediate settlement at these demands.

Commitment has long been recognized as an important factor in determining bargaining outcomes. Schelling (1960) points this out clearly. Since then, many works have attempted to understand the exact role that commitment plays in bargaining games.[1] Crawford (1982) formulated imperfect commitment in the way that players become aware whether they have committed after making their initial demands. Then the players decide either to accept or to reject once and for all. In his model, occasionally both players reject and they reach an impasse. Thus he concluded that the possibility of commitment, which is an important source of bargaining power, can cause inefficient outcomes. Our model starts from the same imperfect commitment but allows players to keep negotiating until an agreement is reached. With this change, the possibility of commitment, though still serving as a source of bargaining power, becomes compatible with immediate and efficient settlement. The logic that drives our result is that a small probability of (irrational) players who behave differently from the other (normal) players can cause a big change in equilibrium outcomes. This idea was first formulated in the seminal paper by Kreps *et al.* (1982). They applied it to the finitely repeated prisoners' dilemma and showed that cooperation arises when there is a small possibility of tit-for-tat players.[2]

---

[1] As a recent contribution to this literature, see Fershtman and Seidmann (1993) and Muthoo (1996) among others.

[2] See also Kreps and Wilson (1982) and Milgrom and Roberts (1982). Some of the recent

In the paper that stimulated this article, Abreu and Gul (1997) applied this idea to a non-cooperative, two-person bargaining game.[3] (We extensively use their analyses for the war of attrition stage.) They assume that some players cannot accept anything less than what these players think to be fair. Then they use it to explain the possibility of delays in agreement. Since players mimic various kinds of irrational types with different notions for fair settlements, there is a distribution of demands in the beginning. Thus, incompatible demands are often made and then a war of attrition arises. In some sense, our model endogenizes the choice of fair settlements. A player is not necessarily attached to a particular amount. However, once a demand has been made, it sometimes becomes the fair demand for that player and anything less becomes unacceptable. Although this behavior is still not completely rational, it may not cause delays and our model has an equilibrium with an immediate settlement. When players can choose their initial demands, they tend to choose them to minimize inefficiency.

In the next section, we describe the basic model. The third section establishes the main results that characterize the set of equilibria and identify the possibility of being stubborn as being a source of bargaining power. Section 4 studies two extensions. The first extension studies risk-averse players and relates our result to the asymmetric Nash solution. The other extension treats the case of inborn stubbornness. The last section makes some concluding remarks.

2 The Basic Model

There are two players, $i = 1, 2$, who bargain over the partition of one dollar. (When we say player j, we implicitly refer to the player other than player i. Player i as well as a generic player takes the female identity while player j as well as an opponent takes the male identity for ease of exposition.) The negotiation proceeds in infinite horizon: $\mathcal{T} = [0, \infty)$.

When a settlement is reached at time t and player i's share is y, her utility is given by $u_i(y)e^{-r_i t}$, where r_i is her discount rate. For the analyses of the next section, we assume risk neutrality: $u_i(y) \equiv y$. (We treat risk aversion in Section 4.) In the case of perpetual disagreement, her utility is given by zero. The players try to maximize their expected utilities.

In the beginning ($t = 0$), both players announce simultaneously what they want for themselves. We call this *the demand stage*. Player i demands her share[4] to be $x_i \in [0, 1)$, offering $1 - x_i$ to player j. If $x_i + x_j \leq 1$, the demands

development in this line of research is found in Fudenberg and Levine (1989), Schmidt (1993a), and Watson (1993). The idea has been applied to the theory of bargaining, e.g., Kornhauser *et al.* (1989) and Schmidt (1993b).

[3]Abreu and Gul (1997), using the idea developed in this paper, extended their analyses and study the limit of equilibria in their model when the probabilities to be stubborn converge to zero. They obtain the results similar to Proposition 3 in our paper. See also Compte and Jehiel (1996) on this subject.

[4]We assume that the players do not demand the entire share. We can show that the players

are called compatible and the players will split the money at the middle of their proposals, i.e., giving $x_i + (1 - x_i - x_j)/2$ to player i. (If the sum is exactly one, the demands are said to be just compatible.)

When the demands are incompatible, the players proceed to *the war of attrition stage*. Every δ-period, the players decide simultaneously[5] what to do out of two options:

(a) reiterating her previous demand and waiting for her opponent to lower his demand, or

(b) lowering her own demand.

We assume that the players cannot rescind earlier proposals. Thus a new demand must be lower than any previous ones.[6] When at least one player chooses the second option and the sum of demands becomes no greater than one, the game ends and they split the money at the middle of their proposals. Otherwise the game continues. Without a settlement, the process continues forever (perpetual disagreement). Note that lowering one's demand to the just compatible level gives one at least what the opponent offers. Slightly abusing the word, we call this acceptance in this model.

The key assumption of the model is that, with small positive probabilities, the players commit themselves to their initial demands[7] before the war of attrition stage starts. The committed player is called a *stubborn* type. This type simply waits for her opponent to accept her offer. Those who are not stubborn are called *flexible*. The players know only the probability of their opponents becoming stubborn and cannot tell whether their opponents are really stubborn or not. (Actually, if a player knows that her opponent is stubborn, the strategy in the war of attrition will become trivial. The one who is not stubborn and knows that her opponent is will accept the opponent's offer immediately. If both know that both are stubborn, no agreement will be possible.)

Formally, we assume that player i becomes stubborn with probability z_i. We assume that $0 < z_i < 1$ and that it is independent from that for player j. This event happens between the demand stage and the war of attrition stage. Thus, when they make their initial demands, the players do not know whether they will become stubborn. In Section 4, we study the case where they do know their types before the demand stage.

do not want to demand the entire share in equilibrium and thus can prove all the propositions without this assumption. However, when they do demand the entire share, Lemma 0 needs to be modified, which adds extra complexity without increasing insight. Hence, we impose this assumption.

[5]The simultaneity of moves is assumed for the sake of specificity. As shown in Abreu and Gul (1997), the equilibrium behaviors in the war of attrition stage do not depend on the timing or the order of proposals when the interval between offers is sufficiently short.

[6]If the players, especially committed ones, could increase their demands, then there would be multiple equilibria in the war of attrition stage. This would undermine most of our results. We thank the editor and the referee for pointing this out.

[7]We introduce only this type of irrational behavior, on which our results substantially depend. The concluding remarks briefly discuss what happens when other kinds of irrationality exist. We thank the associate editor for this observation.

Let N be the set of natural numbers. Denote by x_i^t player i's demand at time t for $t \in \{0, \delta, 2\delta, \ldots\}$. The history at time $t = n\delta$ for $n \in N$ is $h^t = h^{t-\delta} \times (x_1^{t-\delta}, x_2^{t-\delta})$ and $h^0 = \emptyset$. Player i's strategy at the demand stage is $\sigma_i^0 : h^0 \to [0, 1]$. At time $t = n\delta$ for $n \in N$, only when player i is flexible does she have a meaningful strategy: $\sigma_i^t : h^t \to [0, x_i^{t-\delta}]$.

Our equilibrium concept is a version of the sequential equilibrium adapted for our dynamic model. (We introduce a stronger refinement in Section 4 for the analysis of inborn stubbornness.) We require that player i believes player j to be flexible when he lowered his demand in any previous period. Otherwise, she updates her belief by Bayes' rule given that the *ex ante* probability that player j becomes stubborn is z_j and that the stubborn type of player j commits himself to his initial demand. This is our consistency condition on beliefs. Given the beliefs, we require that the players choose the best responses on any decision nodes, including the ones off the equilibrium paths. This is our sequential rationality condition. We require these conditions to be satisfied in any equilibrium.

We actually want to study the case where the players can lower their demands at any moment and thus we regard the above model as an approximation. In the following analyses, we look at the limit of equilibria as $\delta \to 0$.

3 Equilibria

Although the negotiation can be concluded quickly in equilibrium, what happens when it prolongs is crucial in determining equilibrium outcomes. Hence, we start from the analysis of the war of attrition and then evaluate the players' strategies at the demand stage.

3.1 War of Attrition Stage

Abreu and Gul (1997) have already studied the war of attrition with the possibility of stubborn types. We borrow their results extensively in this subsection although we will rederive the dynamic path explicitly for the sake of exposition. Please refer to their paper, especially for the proof of Lemma 0.

Lemma 0 (Abreu and Gul, 1997). *Suppose that the initial demands x_1 and x_2 are incompatible.*

(1) Suppose that player j keeps his initial demand and that player i lowers for the first time at time $t(> 0)$. If the demands are still incompatible, in any continuation equilibrium, player i's expected payoff at time t converges to $1 - x_j$ while that of player j converges to x_j, when δ converges to zero.

(2) As δ converges to zero, the equilibrium outcome given the incompatible initial demands converges to the unique one described below.

($t = 0+$): One player may accept her opponent's offer with a positive probability.

($0+ < t < T$): The flexible type of player i randomly accepts player j's offer at some rate. [Take t and t' such that $0 < t < t' < T$. The probability of settlement in the interval (t, t') is positive and it converges to zero when $t' \to t$.]

($t \geq T$): By some time T, any flexible types of both players accept their opponent's offer with probability of one. There is no settlement after time T.

The first part of this lemma implies that, when the interval between periods δ is sufficiently small, the revelation of a player's type as being flexible is virtually equivalent to accepting the opponent's offer in terms of payoffs. Hence, the decision of flexible types essentially becomes whether to accept now or to wait for a bit longer. Therefore, when we analyze the limit of equilibria as $\delta \to 0$, we simply need to look at the stopping time (i.e., when to accept) as the strategy of a flexible type, instead of looking at the demand strategy that depends on history. Using this convention, we compute the limit of the equilibrium path given incompatible initial demands.

A player wants her opponent to accept her offer, but if he does not, she wants to accept his offer herself. This situation is commonly known as a war of attrition. Between time 0+ and time T, a flexible type gradually accepts her opponent's offer by choosing the timing of acceptance randomly. Let a_i be the instantaneous rate of acceptance by player i conditional on the fact that nobody has accepted.[8] Given this behavior of player i, player j feels indifferent between accepting player i's offer and waiting. Let V_j be the expected payoff of the flexible type of player j while playing in the war of attrition. Acceptance gives him $1 - x_i$. If he waits for an infinitesimal period $\triangle$, player i will accept his offer with probability $a_i \triangle$. Otherwise he will face the same war of attrition and thus he will expect the same expected payoff V_j at that point. Summarizing the argument,

$$\begin{aligned} &\text{accepting:} \qquad V_j = 1 - x_i, \\ &\text{waiting:} \qquad V_j = a_i \triangle x_j + (1 - a_i \triangle) e^{-r_j \triangle} V_j. \end{aligned}$$

Solving this system of equations and taking $\triangle$ to 0, L'Hospital's theorem shows that

$$a_i = \frac{r_j (1 - x_i)}{x_i + x_j - 1}.$$

A flexible type waits since there is a chance that her opponent is flexible and will accept before she does. Thus, the flexible types of both players keep accepting with a positive probability until the same time T, by which time all of them accept for sure. Define $G_i(t; T)$ to be the probability that player i is flexible and, in addition, she does not accept by time t (when her opponent does not accept by that time). Since the flexible type of player i accepts at the rate

[8] Many variables introduced in this section, such as a_i here, are in fact functions of x_1 and x_2. For the ease of exposition, we generally omit their dependence.

of a_i and since she accepts by the time T for sure, we have

$$\begin{aligned} -\frac{1}{G_i(t;T)+z_i}\frac{\partial G_i(t;T)}{\partial t} &= a_i, \text{ and} \\ G_i(T;T) &= 0. \end{aligned}$$

Solving these two equations, we get

$$G_i(t;T) = z_i(e^{a_i(T-t)} - 1).$$

Let T_i be the time needed for the flexible type to accept her opponent's offer with probability of one when she does not accept at all in the beginning and when her opponent does not accept her offer. We call it *the potential exhaustion time*. (When time T_i is reached without acceptance, player j believes that player i is stubborn for sure.) Since the initial probability of player i being flexible is $1 - z_i$, T_i satisfies $1 - z_i = G_i(0; T_i)$. Hence,

$$T_i = \frac{1}{a_i}\log\frac{1}{z_i} = \frac{x_i + x_j - 1}{r_j(1 - x_i)}\log\frac{1}{z_i}.$$

Suppose that, given the initial offers, player i has a shorter potential exhaustion time: $T_i < T_j$. We know from the discussion above that the flexible types of both players will accept for sure by the same time. To achieve this, the flexible type of player j must accept player i's offer with a mass probability in the beginning of the war of attrition ($t = 0+$). This phenomenon is called *initial mass acceptance*. (The initial mass acceptance is made by at most one player. Otherwise, both would have an incentive to delay their acceptance infinitesimally and to wait for the opponent's acceptance.)

Let P^{ma} be the probability that the initial mass acceptance occurs. When $T_i < T_j$, by time T_i, the flexible types of both players accept their opponent's offer. Since the flexible type of player j accepts only with probability $G_j(0; T_i)$ during the period, he has to accept player i's offer at time 0 with probability $(1 - z_j) - G_j(0; T_i)$. Using the above analyses, we get

$$\begin{aligned} P^{ma} &= (1 - z_j) - G_j(0; T_i) \\ &= 1 - z_j - z_j(e^{a_j T_i} - 1) \\ &= 1 - z_j^{(1-(T_i/T_j))}. \end{aligned}$$

When $T_i = T_j$, no player makes the initial mass acceptance, and $T = T_i = T_j$.

3.2 Demand Stage

The critical strategic decision in this game is the choice of the players' initial demands. Even the one who becomes stubborn in the war of attrition stage can choose her initial demand. This is the departure from the model of Abreu and Gul (1997).

First, we compute the expected payoffs when the initial demands are incompatible and when player j makes the initial mass acceptance. As a result of player j's initial mass acceptance, player i expects to obtain $P^{ma}x_i$. As soon as the war of attrition starts ($t > 0+$), the flexible type's expected payoff is given by the opponent's offer. The stubborn type loses the chance of settlement when her opponent is also stubborn. Since the probability that both are stubborn is $z_1 z_2$, the expected loss for player i in this case is given by $z_1 z_2 e^{-r_i T_i}(1 - x_j)$. Combining these, the expected payoff of player i is given by

$$\begin{aligned} EP_i &= P^{ma}x_i + (1 - P^{ma})(1 - x_j) - z_1 z_2 e^{-r_i T_i}(1 - x_j) \\ &= (1 - x_j) + P^{ma}(x_i + x_j - 1) - z_1 z_2 e^{-r_i T_i}(1 - x_j). \end{aligned}$$

When the flexible type of player j makes the initial mass acceptance, his payoff is $1 - x_i$, which is identical to what he expects at $t > 0+$. Thus, the expected payoff of player j is

$$EP_j = (1 - x_i) - z_1 z_2 e^{-r_j T_i}(1 - x_i).$$

(When $T_1 = T_2$, there is no initial mass acceptance. We regard this as a special case of the above where $P^{ma} = 0$.)

The comparison of the above payoffs illustrates that the opponent's initial mass acceptance is essential for a player to obtain more than what her opponent offers. From the analysis in the war of attrition stage, the ratio of potential exhaustion times is given by

$$\frac{T_i}{T_j} = \frac{a_j}{a_i}\frac{\log(1/z_i)}{\log(1/z_j)} = \frac{\log z_i}{\log z_j}\frac{r_i}{r_j}\frac{1 - x_j}{1 - x_i}.$$

Since this is increasing in x_i, decreasing her own demand tends to cause her opponent to make the initial mass acceptance. This gives players an incentive not to make excessive demands.

We define the just compatible demands $\langle x_1^*, x_2^* \rangle$ such that the potential exhaustion times of the two players are the same. Namely,

$$\begin{aligned} \frac{T_i}{T_j} = \frac{\log z_i}{\log z_j}\frac{r_i}{r_j}\frac{1 - x_j^*}{1 - x_i^*} &= 1, \text{ and} \\ x_i^* + x_j^* &= 1. \end{aligned}$$

Solving this system of equations,

$$x_i^* = \frac{r_j \log z_j}{r_i \log z_i + r_j \log z_j}.$$

In the sense that the relative length of potential exhaustion times determines who makes the initial mass acceptance, the players' natural bargaining powers are reflected in $\langle x_1^*, x_2^* \rangle$. The next proposition shows that the immediate settlement at these demands is an equilibrium outcome.

Proposition 1. *The immediate settlement at $\langle x_1^*, x_2^* \rangle$ is an equilibrium outcome.*

Proof: We examine the best response of player j at the demand stage given player i's initial demand x_i^*.

Player j has no incentive to demand less than x_j^* since $\langle x_1^*, x_2^* \rangle$ is just compatible.

Suppose that player j makes a demand $x_j (> x_j^*)$. From the analysis above, we have $(\partial(T_i/T_j))/\partial x_j < 0$. Since $T_i/T_j = 1$ at $\langle x_1^*, x_2^* \rangle$, $T_j > T_i$ holds for $x_j > x_j^*$, and player j will have to make the initial mass acceptance. Then the above analysis of the expected utilities shows that his payoff is less than $1 - x_i^* = x_j^*$.

Therefore, when player i's initial demand is x_i^*, demanding x_j^* is the best response for player j at the demand stage. By symmetry, the outcome in the proposition is supported in an equilibrium. Q.E.D.

There is no loss of efficiency in the outcome described in Proposition 1 despite the possibility of stubborn types. As a matter of fact, the players reach a settlement immediately when they do not choose their initial demands randomly.

Proposition 2. *In any equilibrium where the players do not randomly choose their initial demands, the settlement is immediate.*

Proof: Let x_i be the initial demand of player i and let x_j be that of player j. Supposing that $x_i + x_j > 1$, we derive a contradiction as follows. Without loss of generality, suppose that $T_i \leq T_j$. Then, the initial expected payoff of player j is given by

$$EP_j = (1 - x_i) - z_1 z_2 e^{-r_j T_i}(1 - x_i).$$

If he changed his initial demand to the just compatible one $(1 - x_i)$, he could avoid the chance of impasse and could obtain the payoff $(1 - x_i)$, which is higher than EP_j. This is a contradiction. Therefore, $x_i + x_j \leq 1$, and the negotiation ends immediately. Q.E.D.

The possibility of immediate settlement is a clear contrast to other studies on stubborn negotiators. In Abreu and Gul (1997), prolonged delays occur, involving the war of attrition. Crawford (1982) shows that, in a single-period game, players try to commit to incompatible demands. In the former formulation, the stubborn types cannot choose their initial demands, and this creates a distribution of initial demands, leading to delays. In the latter, demanding more increases a player's expected payoff, due to the specification of the players' payoffs at a simultaneous concession. (In our dynamic model, there is no simultaneous concession in equilibrium.) Thus the players may have an incentive to make incompatible demands. On the other hand, in our model, a player who is expected to make the initial mass acceptance is better off accepting her opponent's offer initially and avoiding the chance of impasse. (When the players choose their initial demands randomly, they may not know who will make the initial mass acceptance. As a result, there can be a failure of coordination and the players may make incompatible demands.)

The equilibrium payoffs are close to those specified in Proposition 1 when the chance of becoming stubborn is small. To understand this intuitively, suppose that $z_i = \alpha_i^{1/z}$ for some $\alpha_i > 0$ and for some $z > 0$ $(i = 1, 2)$. We consider the limit as z goes to zero. Then, z_i converges to zero while x_i^* remains constant. Suppose that player i demands x_i^*. As analyzed above, if player j demands more than $1 - x_i^* = x_j^*$, he will make the initial mass acceptance. The probability of the initial mass acceptance is given by

$$P^{ma} = 1 - z_j^{(1-(T_i/T_j))}.$$

It converges to one as z_j converges to zero. Hence,

$$EP_i \approx (1 - x_j) + (x_j + x_i^* - 1) - z_1 z_2 e^{-r_i T_i}(1 - x_j) \approx x_i^*.$$

Thus, the equilibrium payoff of player i is close to x_i^* when z_i and z_j are small. The next proposition gives the exact bound by using the same idea.

Proposition 3. *In any equilibrium,*

$$-\left(z_1 z_2 + \frac{1}{e(-\log z_j)}\right) x_i^* < EP_i - x_i^* < \left(z_1 z_2 + \frac{1}{e(-\log z_i)}\right) x_j^*.$$

Proof: We consider player i's strategy for initially demanding x_i^*.

When player j initially demands no more than $x_j^* = 1 - x_i^*$, player i obtains a payoff no less than x_i^*.

Now suppose that player j initially demands $x_j^* + \epsilon$ for some $\epsilon > 0$. Since player j demands more than x_j^*, player i can expects player j to make the initial mass acceptance. The expected payoff of player i is

$$\begin{aligned} EP_i &= (1 - x_j^* - \epsilon) + (x_j^* + \epsilon + x_i^* - 1)\left(1 - z_j^{1-T_i/T_j}\right) \\ &\quad - z_1 z_2 e^{-r_i T_i}(1 - x_j^* - \epsilon) \\ &= (1 - x_j^* - \epsilon) + \epsilon - \epsilon z_j^{1-(\log z_i/\log z_j)(r_i/r_j)((1-x_j^*-\epsilon)/(1-x_i^*))} \\ &\quad - z_1 z_2 e^{-r_i T_i}(1 - x_j^* - \epsilon) \\ &= (1 - x_j^*) - \epsilon z_j^{(\log z_i/\log z_j)(r_i/r_j)(\epsilon/(1-x_i^*))} - z_1 z_2 e^{-r_i T_i}(1 - x_j^* - \epsilon). \end{aligned}$$

Here, we evaluate each term. Since $x_j^*/x_i^* = (\log z_i/\log z_j)(r_i/r_j)$ and $x_j^* + x_i^* = 1$, the second term is $-\epsilon z_j^{\epsilon/x_i^*}$. Regarded as a function of ϵ, it takes the smallest value $-x_i^*/e(-\log z_j)$ when $\epsilon = x_i^*/(-\log z_j)$. The third term, which is the expected loss from the impasse, is greater than $-z_1 z_2(1 - x_j^*) = -z_1 z_2 x_i^*$. Combining these with the above, we get

$$\begin{aligned} EP_i &> (1 - x_j^*) - \frac{x_i^*}{e(-\log z_j)} - z_1 z_2 x_i^* \\ &= x_i^* - \left(\frac{1}{e(-\log z_j)} + z_1 z_2\right) x_i^*. \end{aligned}$$

Therefore, the expected payoff of player i from the specified strategy is no less than the stated bound, no matter what player j does in the demand stage. Since the sum of the players' payoffs must not exceed one, we can get the upper bound of player i's expected payoff by subtracting her opponent's lower bound from one.

This concludes the proof. Q.E.D.

Observe that both bounds converge to zero when both z_1 and z_2 converge to zero. Hence, for small z_1 and z_2, the payoff of player i in any equilibrium must be close to x_i^*. (Even when the initial demands are chosen randomly and thus may cause some delays, the expected efficiency loss must be small when both z_1 and z_2 are small.)

The multiplicity of equilibria occurs in this model. When the initial demands are incompatible and when both players become stubborn, agreement becomes unattainable. For player i, this reduces the expected payoff by $z_1 z_2 e^{-r_i T}(1 - x_j)$. Hence, when player j's demand is no greater than x_j^* plus this amount, accommodating player j's demand is better for player i. This illustrates why the size of the bounds is asymptotically of the order of $z_1 z_2$. (Note that the term $1/(-\log z_i)$ becomes much smaller than the term $z_1 z_2$ when z_i is sufficiently small.)

4 Extensions

This section briefly explains two extensions of our basic model. The first extension introduces risk aversion and relates our result to the asymmetric Nash bargaining solution. The second extension analyzes inborn stubbornness.

4.1 Nash Program

Proposition 3 has shown that bargaining outcomes are approximated by the immediate settlement at $\langle x_1^*, x_2^* \rangle$ when the probabilities of being stubborn are small. Observe that

$$\begin{aligned}\langle x_1^*, x_2^* \rangle \quad = \quad \arg\max_{x_1, x_2} \quad & x_1^{1/(-r_1 \log z_1)} x_2^{1/(-r_2 \log z_2)} \\ \text{s.t. } & x_1 + x_2 = 1 \\ & x_1 \in [0, 1].\end{aligned}$$

Namely, $\langle x_1^*, x_2^* \rangle$ can be regarded as the asymmetric Nash solution where player i's weight is given by $1/(-r_i \log z_i)$ for $i = 1, 2$. A player who is more patient and/or who is more likely to be stubborn gets a higher weight and thus has greater bargaining power. In this sense, our model offers yet another interpretation of the asymmetric Nash bargaining solution, linking the reputation for being stubborn to the bargaining power in the axiomatic approach.[9]

[9]The approach of relating a non-cooperative model to a cooperative solution is often called the Nash program, since it was originally proposed by Nash (1953). For more on this subject,

Our results, including the above interpretation, can be easily extended to the case of risk-averse negotiators. Suppose that player i's utility function is given by $u_i(y)e^{-r_i t}$, where $u(y)$ is twice continuously differentiable, $u'(y) > 0$ and $u''(y) \leq 0$ for any $y \in [0, 1]$. For normalization, we assume that $u_i(0) = 0$ and $u_i(1) = 1$.

The Appendix shows that, for small probabilities of stubborn types, bargaining outcomes are approximated by the immediate settlement at $\langle x_1^{**}, x_2^{**} \rangle$, which is the asymmetric Nash solution where player i's weight is given by $1/(-r_i \log z_i)$ for $i = 1, 2$.

$$\begin{aligned} \langle x_1^{**}, x_2^{**} \rangle \quad = \quad \arg\max_{x_1, x_2} \quad & u_1(x_1)^{1/(-r_1 \log z_1)} u_2(x_2)^{1/(-r_2 \log z_2)} \\ \text{s.t. } & x_1 + x_2 = 1 \\ & x_1 \in [0, 1]. \end{aligned}$$

Proposition 4. *Suppose that player i's utility function is given by* $u_i(y)e^{-r_i t}$. *Let* $\langle x_1^{**}, x_2^{**} \rangle$ *be the asymmetric Nash bargaining solution as defined above.*

(1) The immediate settlement at $\langle x_1^{**}, x_2^{**} \rangle$ *is an equilibrium outcome.*

(2) Suppose that $z_i = \alpha_i^{1/z}$ *and* $z_j = \alpha_j^{1/z}$, *where* $0 < \alpha_i < 1$, $0 < \alpha_j < 1$ *and* $0 < z < \infty$. *For any* $\epsilon > 0$, *there exists a positive* $\underline{z}$ *such that, for any* $0 < z < \underline{z}$, *player i's payoff is between* $u_i(x_i^{**}) - \epsilon$ *and* $u_i(x_i^{**}) + \epsilon$ *in any equilibrium.*

4.2 Inborn Stubbornness

This subsection studies a different scenario where the players know their own types before they make their initial demands. Our results here are similar to but weaker than the ones obtained in Section 3 since we need to restrict the strategy spaces (no random choice of initial demands) and have to employ a stronger refinement (the semiperfect sequential equilibrium).

Formally, we modify the model stated in Section 2 so that the players privately learn their types before rather than after making their initial demands. Player i is stubborn with probability z_i. We assume that even the stubborn types can choose their initial demands. The rest of the model is unchanged.

In this formulation, how to assign a reasonable belief to deviant initial demands becomes a difficult issue. Although a flexible type generally wants to mimic the stubborn type by making the same demand, the sequential equilibrium concept is too weak to restrict beliefs in this way. A player may believe that a deviation is carried out mainly by a flexible type and may respond in an aggressive way, discouraging even deviations that benefit a stubborn type. To exclude beliefs of this type, we resort to a stronger refinement, in this case, to a

---

see Binmore's articles in Binmore and Dasgupta (1987), Binmore *et al.* (1986), and Osborne and Rubinstein (1990).

weakened version of the perfect sequential equilibrium by Grossman and Perry (1986).

DEFINITION (semiperfect sequential equilibrium). For an equilibrium, let EP_{if} be the payoff of the flexible type of player i and let EP_{is} be that of the stubborn type. Given a deviant initial demand x'_i, let EP'_{if} be the expected payoff of the flexible type and let EP'_{is} be that of the stubborn one in a continuation equilibrium where player j uses the prior belief z_i. When $EP'_{if} \geq EP_{if}$ and $EP'_{is} \geq EP_{is}$ and when at least one inequality holds strictly, we say that the deviation is successful and that the examined equilibrium is not semiperfect sequential.

In perfect sequential equilibria, a deviation is considered to be carried out by those who can benefit if they as a group are believed to have deviated. We borrow this idea and define semiperfect sequential equilibria by assigning the prior belief to a deviant initial demand as long as it benefits both types. (We examine only the successful deviation by both types. In this sense, our refinement is weaker than the perfect sequential equilibrium as originally defined by Grossman and Perry, 1986.)

When the players randomly choose their initial demands, we need to assign different beliefs to different demands, which makes the analysis rather complex. Because of this difficulty, the following analysis focuses on non-random initial demands.

When the players do not randomly choose their initial demands and when we look at the semiperfect sequential equilibria, we can show that the players settle immediately and that, given an excessive demand by player j, player i's demand of x_i^* would cause player j to make the initial mass acceptance. This implies that the situation is similar to what we had in Section 3. (The proof of the next proposition is relegated to the Appendix.)

Proposition 5. *Suppose that the players know their own types before they make initial demands. We apply the semiperfect sequential equilibrium and look only at the equilibria where the players do not randomly choose their initial demands. Then,*

(1) the immediate settlement at $\langle x_1^*, x_2^* \rangle$ is an equilibrium outcome, and

(2) in any equilibrium, the players settle immediately and

$$-\left(z_j + \frac{1}{e(-\log z_j)}\right) x_i^* < EP_{if} - x_i^* = EP_{is} - x_i^* < \left(z_i + \frac{1}{e(-\log z_i)}\right) x_j^*.$$

5 Concluding Remark

This paper has demonstrated that the reputation for being stubborn is a source of bargaining power. It does not, however, explain the origin or formation of the reputation itself. That will be left for future research. Another issue that this paper does not fully address is the effect of other kinds of irrational behaviors in bargaining. The rest of this section is devoted to a brief discussion of this.

We confine our attention to a special class of stubbornness, namely, the commitment to one's initial demand (the commitment type). This is partly because it is a rather common type of stubbornness observed in daily negotiation and partly because it can demonstrate how the reputation for being stubborn enhances bargaining power most clearly. There are other types of irrational behaviors that can be observed in real negotiations. Some players give in no matter what the offer is (the chicken type). Some other players always insist on certain divisions, (the fixed-share types).

Comparison with the results of Abreu and Gul (1997) hints at what happens when there is a possibility of the chicken type and/or various fixed-share types as well as the commitment type. It is likely that there will be no equilibrium where immediate settlement occurs with probability of one. The possibility of the chicken type causes players to make excessive demands and the possibility of various fixed-shared types creates a distribution of initial demands, both of which lead to the occurrence of the war of attrition. On the other hand, we conjecture that the logic behind Proposition 3 remains valid in the models with various irrational types and, when the probabilities of irrational types are small, the equilibrium payoffs of the players are close to the ones derived in this paper.

In any case, we ought to pay more attention to bargaining power based on reputation since the possibility of stubborn negotiators can substantially change the bargaining outcomes.

Appendix 1: Nash Program

We provide the proof of Proposition 4 here.

Even with risk-averse players, we can prove the same statements as Lemma 0. However, both a_i and T_i take different forms. In the same way as in Section 3, player j is indifferent between accepting and waiting in the war of attrition. Hence, for a small $\triangle$, we have

$$\begin{aligned} &\text{accepting:} && V_j = u_j(1-x_i), \\ &\text{waiting:} && V_j = a_i\triangle u_j(x_j) + (1-a_i\triangle)e^{-r_j\triangle}V_j. \end{aligned}$$

Solving the system of equations and applying L'Hospital's theorem by taking $\triangle$ to zero, we get

$$a_i = \frac{r_j u_j(1-x_i)}{u_j(x_j) - u_j(1-x_i)}.$$

From this, we can compute the potential exhaustion time for player i.

$$T_i = \frac{1}{a_i}\log\frac{1}{z_i} = \frac{u_j(x_j) - u_j(1-x_i)}{r_j u_j(1-x_i)}\log\frac{1}{z_i}.$$

Now compute the just compatible demand pair $\langle x_1^{**}, x_2^{**}\rangle$ that reflects natural bargaining powers. It is defined by the limit of demands when $x_1 + x_2 \downarrow 1$

such that $T_1/T_2 = 1$. Note that

$$\begin{aligned}\frac{T_i}{T_j} &= \frac{\log z_i(u_j(x_j)-u_j(1-x_i))}{r_ju_j(1-x_i)}\frac{r_iu_i(1-x_j)}{\log z_j(u_i(x_i)-u_i(1-x_j))}\\ &= \frac{\log z_i}{r_ju_j(1-x_i)}\frac{u_j(x_j)-u_j(1-x_i)}{x_i+x_j-1}\frac{r_iu_i(1-x_j)}{\log z_j}\frac{x_i+x_j-1}{u_i(x_i)-u_i(1-x_j)}.\end{aligned}$$

By taking the limit of the left hand side as $x_1 + x_2 \downarrow 1$ and by equating it to one, we find that $\langle x_1^{**}, x_2^{**}\rangle$ satisfies,

$$\frac{\log z_i u_j'(x_j^{**})}{r_j u_j(x_j^{**})}\frac{r_i u_i(x_i^{**})}{\log z_j u_i'(x_i^{**})} = 1.$$

Observe that $\langle x_1^{**}, x_2^{**}\rangle$ is the solution to the following maximization problem that describes an asymmetric Nash bargaining solution where player i's weight is given by $1/(-r_i \log z_i)$ for $i = 1, 2$.

$$\begin{aligned}\max_{x_1,x_2} \quad & u_1(x_1)^{1/(-r_1\log z_1)}u_2(x_2)^{1/(-r_2\log z_2)}\\ \text{s.t. } & x_1+x_2=1\\ & x_1 \in [0,1].\end{aligned}$$

We can show that

$$\frac{\partial T_i/T_j}{\partial x_i} = \frac{T_i}{T_j}\left(\frac{u_j(x_j)}{u_j(1-x_i)}\frac{u_j'(1-x_i)}{u_j(x_j)-u_j(1-x_i)} - \frac{u_i'(x_i)}{u_i(x_i)-u_i(1-x_j)}\right).$$

Since $x_j > 1 - x_i$, $\frac{u_j(x_j)}{u_j(1-x_i)} > 1$. By the concavity of u_j and u_i, we have $u_j'(1-x_i)(x_i+x_j-1) \geq u_j(x_j)-u_j(1-x_i)$ and $u_i'(x_i)(x_i+x_j-1) \leq u_i(x_i)-u_i(1-x_j)$. Combining these, we get $(\partial(T_i/T_j))/\partial x_i > 0$. Even when the players are risk-averse, if one of them increases her own demand, she increases her potential exhaustion time relative to that of her opponent. Hence, the same argument as in the proof of Proposition 1 can be used to prove the first claim of the proposition.

For the convergence result, we consider player i's strategy for initially demanding $x_i^{**} - \epsilon'$.

When player j demands no more than $x_j^{**} + \epsilon'$, then the demands are compatible and player i obtains a payoff no less than $u_i(x_i^{**} - \epsilon')$.

Now suppose that player j initially demands $x_j^{**} + \xi + \epsilon'$ for some $\xi > 0$. Note that player j is the one who makes the initial mass acceptance here. Let P^{ma} be the probability of player j's initial mass acceptance. The same analysis as in Section 3 can show that

$$P^{ma} = 1 - z_j^{(1-(T_i/T_j))}.$$

Evaluate T_i/T_j at $x_i = x_i^{**} - \epsilon'$ and $x_j = x_j^{**} + \epsilon'$ and denote it RT_j^i. Since T_i/T_j is decreasing in x_j and is increasing in x_i, we obtain $T_i/T_j < RT_j^i < 1$.

Observe that $1 - RT_j^i$ is independent of the absolute sizes of z_1 and z_2 when, as supposed, the ratio $\log z_i / \log z_j$ is kept constant. Hence, $1 - z_j^{1-RT_j^i}$ converges to one as z_j (or z) tends to zero. (Note that the convergence is uniform with respect to ξ.) Since $P^{ma} > 1 - z_j^{1-RT_j^i}$, P^{ma} also converges to one uniformly. This implies that there exists a positive $\underline{z}$ such that, for any $z < \underline{z}$, player i obtains more than $u_i(x_i^{**} - \epsilon') - \epsilon'$.

Hence, player i obtains more than $u_i(x_i^{**} - \epsilon') - \epsilon'$ no matter what player j does. By choosing ϵ' appropriately, this implies that for any $\epsilon > 0$ there exists a positive $\underline{z}$ such that, for any $z < \underline{z}$, player i obtains more than $u_i(x_i^{**}) - \epsilon$. By symmetry, we can show that the equivalent statement is true for player j. Since the sum of the players' shares must be no more than one, this will prove the statement about convergence.

Appendix 2: Inborn Stubbornness

This Appendix proves Proposition 5 in two parts. In the proof, the flexible (stubborn) type of player i is referred to as flexible (stubborn) player i.

The following proofs often need to evaluate the players' payoffs when the stubborn and flexible types make identical demands. In that case, the situation given initial demands is identical to the one studied in Section 3 and we can use the same analyses. When the initial demands are not compatible and when player j makes the initial mass acceptance, the payoffs of player i are as follows.

$$\begin{aligned} &\text{Flexible:} \quad EP_{if} = (1 - x_j) + P^{ma}(x_i + x_j - 1), \text{ or} \\ &\text{Stubborn:} \quad EP_{is} = (1 - x_j) + P^{ma}(x_i + x_j - 1) - z_j e^{-r_i T_i}(1 - x_j), \end{aligned}$$

where "Flexible" means the expected payoff of the flexible type and "Stubborn" means that of the stubborn type. For stubborn player i, the impasse occurs when her opponent is also stubborn, which happens with probability z_j. Thus, the payoff of stubborn player i is lower than that of flexible player i by $z_j e^{-r_i T_i}(1 - x_j)$. On the other hand, player j, who makes the initial mass acceptance, expects

$$\begin{aligned} &\text{Flexible:} \quad EP_{jf} = (1 - x_i), \text{ or} \\ &\text{Stubborn:} \quad EP_{js} = (1 - x_i) - z_j e^{-r_i T_i}(1 - x_i). \end{aligned}$$

The Proof of the First Part of Proposition 5

Suppose that player i is expected to demand x_i^* initially and that she uses the prior belief z_j after any initial demand of player j, including deviant ones.

Either type of player j has no incentive to demand less than x_j^* since $\langle x_1^*, x_2^* \rangle$ is just compatible.

We now examine player j's deviation to demand more than x_j^*. Since player i believes that the deviation is carried out by both types, flexible player j has to make the initial mass acceptance. Then, as shown above, neither type of player j increases his payoff.

This proves that the above is an equilibrium and, moreover, that there is no successful deviation. Therefore, the outcome in the proposition is supported in a semiperfect sequential equilibrium.

The Proof of the Second Part of Proposition 5

Let x_{is} be the initial demand of stubborn player i and let x_{if} be that of flexible player i. The next lemma is an extension of the first part of Lemma 0.

Lemma A.1. *Either if $x_{jf} \neq x_{js}$ or if $x_{if} \neq x_{is}$, then flexible player i, faced with offer x_{js}, immediately accept the offer in the unique continuation equilibrium.*

Proof: If $x_{jf} \neq x_{js}$ and if flexible player i is faced with offer x_{js}, then she has no better alternative than immediate acceptance.

If $x_{jf} = x_{js}$ and $x_{if} \neq x_{is}$, then Lemma 0 has shown that flexible player i immediately accepts player j's offer in the unique continuation equilibrium.

Combining these, we obtain the desired statement. Q.E.D.

The proof proceeds in three parts, depending on the compatibility of the initial demands by the stubborn types.

(i) The stubborn types of both players initially make incompatible demands: $x_{1s}+x_{2s} > 1$. Supposing that $x_{is} = x_{if}$ and $x_{js} = x_{jf}$, we show a contradiction as follows. Without loss of generality, suppose that player i does not make initial mass acceptance. Then, the above has shown that $EP_{js} < 1 - x_{is}$. Since stubborn player j can obtain the payoff of $1 - x_{is}$ by initially making the compatible demand, this is a contradiction.

Next, supposing that $x_{is} \neq x_{if}$, we derive a contradiction. (x_{js} may or may not be equal to x_{jf}.)

First, we compute the expected payoffs of player i. By Lemma A.1, x_{is} is immediately accepted by flexible player j. Since the demands of stubborn types are incompatible, the stubborn types get zero when they are matched against each other. Thus stubborn player i expects $(1 - z_j)x_{is}$. Flexible player i has the option of mimicking stubborn player i by demanding x_{is} and of accepting the demand of stubborn player j when her demand is not accepted. Since the former happens with probability $1 - z_j$, her payoff from this strategy is no less than $(1 - z_j)x_{is} + z_j(1 - x_{js})$. Flexible player i expects to obtain at least this much.

Now consider the expected payoffs of player j. By Lemma A.1, x_{js} is immediately accepted by flexible player i. Thus, we can use a similar argument to that above and show that stubborn player j expects $(1 - z_i)x_{js}$ while flexible player j expects at least $(1 - z_i)x_{js} + z_i(1 - x_{is})$.

From these, the *ex ante* expectation of the sum of the payoffs is at least

$$\begin{aligned} & z_i(1-z_j)x_{is} + (1-z_i)((1-z_j)x_{is} + z_j(1-x_{js})) \\ & +z_j(1-z_i)x_{js} + (1-z_j)((1-z_i)x_{js} + z_i(1-x_{is})) \\ = & \quad (1-z_i)(1-z_j)(x_{is}+x_{js}-1) + 1 - z_1 z_2 \\ > & \quad 1 - z_1 z_2. \end{aligned}$$

Since stubborn types cannot accept each other's offer, the benefit from agreement is lost with probability $z_i z_j$. This implies that the expected sum of their payoffs cannot be more than $1 - z_i z_j$. This is a contradiction.

Combining the above arguments, we can conclude that $x_{is} + x_{js} > 1$ does not occur in equilibrium.

(ii) The stubborn types of both players initially make strictly compatible demands: $x_{1s} + x_{2s} < 1$. When both are stubborn, the initial demands are compatible and stubborn player i obtains a payoff that is more than x_{is} and less than $1 - x_{js}$. When flexible player j demands x_{js}, stubborn player i expects to obtain the same payoff as above. If he does not demand x_{js} initially, by Lemma A.1, he immediately accepts and stubborn player i obtains x_{is}. Hence, in any case, the payoff of stubborn player i is strictly less than $1 - x_{js}$ and is at least as much as x_{is}.

Now, consider player i's deviation to demand $1 - x_{js}$. If the deviation is believed to be carried out by both types, the above argument still holds and acceptance by player j is immediate for sure. Then, both types of player i receive at least $1 - x_{js}$.

If this deviation is not successful, then flexible player i needs to have obtained more than $1 - x_{js}$ in the original equilibrium. Since the stubborn types expect to obtain at least what they demand as argued above, this implies that flexible player i expects to obtain more than $1 - x_{js}$ from flexible player j. Since the same is true for player j and $(1 - x_{js}) + (1 - x_{is}) > 1$, this is impossible. Therefore, this case does not happen in any semiperfect sequential equilibrium.

(iii) The stubborn types of both players initially make just compatible demands: $x_{1s} + x_{2s} = 1$. Consider player i's strategy for initially demanding x_{is}. When player j's initial demand is x_{js}, the demands are compatible and player i receives the payoff of x_{is}. When flexible player j makes an incompatible demand, by Lemma A.1, the unique continuation equilibrium is that flexible player j accepts player i's offer immediately. Hence, by demanding x_{is}, player i can secure herself at least the payoff of x_{is}. The same logic applies for player j, who should get at least x_{js}. Since the sum of the expected payoffs cannot be more than one and since $x_{1s} + x_{2s} = 1$, this implies that the settlement has to be immediate at $\langle x_{1s}, x_{2s} \rangle$.

Next, we show the convergence.

Suppose that player j obtains $x_j^* + \epsilon$ and player i obtains $x_i^* - \epsilon$ for some $\epsilon > 0$. Since demanding x_i^* should not be a successful deviation for player i, the payoff from the deviation cannot be higher than $x_i^* - \epsilon$. (If flexible player i is expected to demand x_i^* originally, we consider a demand slightly bigger than

x_i^* for the deviation.) If both types deviate to a new demand x_i^*, then we can apply the same argument as in the proof of Proposition 3 and show that

$$EP_{if} > EP_{is} \geq x_i^* - \left(z_j + \frac{1}{e(-\log z_j)} \right) x_i^*.$$

Hence, for this deviation not to be successful, it is necessary that

$$\epsilon < \left(z_j + \frac{1}{e(-\log z_j)} \right) x_i^*.$$

This gives the lower bound for player i's payoff. Since the sum of the players' expected payoffs is no greater than one, we can obtain the upper bound by subtracting player j's lower bound from one.

This concludes the proof of Proposition 5.

References

Abreu, D., and Gul, F. (1997). "Bargaining and Reputation," mimeo. (subsequently published in 2000 at *Econometrica* **68**, 85–117.)

Binmore, K., and Dasgupta, P. (1987). *The Economics of Bargaining*, Oxford, UK: Blackwell.

Binmore, K., Rubinstein, A., and Wolinsky, A. (1986). "The Nash Bargaining Solution in Economic Modeling," *Rand J. Economics* **17**, 176–188.

Compte, O., and Jehiel, P. (1996). "On Stubbornness in Negotiations," mimeo.

Crawford, V. (1982). "A Theory of Disagreement in Bargaining," *Econometrica* **50**, 607–637.

Fershtman, C., and Seidmann, D. J. (1993). "Deadline Effects and Inefficient Delay in Bargaining with Endogenous Commitment," *J. Econ. Theory* **60**, 306–321.

Fudenberg, D., and Levine, D. K. (1989). "Reputation and Equilibrium Selection in Games with a Patient Player," *Econometrica* **57**, 759–778.

Grossman, S., and Perry, M. (1986). "Perfect Sequential Equilibrium," *J. Econ. Theory* **39**, 97-119.

Kornhauser, L., Rubinstein, A., and Wilson, C. (1989). "Reputation and Patience in the 'War of Attrition'," *Economica* **56**, 15–24.

Kreps, D., Milgrom, P., Roberts, J., and Wilson, R. (1982). "Rational Cooperation in the Finitely Repeated Prisoners' Dilemma," *J. Econ. Theory* **27**, 245–252.

Kreps, D., and Wilson, R. (1982): "Reputation and Imperfect Information," *J. Econ. Theory* **27**, 253 – 279.

Milgrom, P., and Roberts, J. (1982). "Predation, Reputation, and Entry Deterrence," *J. Econ. Theory* **27**, 280–312.

Muthoo, A. (1996). “A Bargaining Model Based on the Commitment Tactic,” *J. Econ. Theory* **69**, 134–152.

Nash, J. F. (1953) “Two-Person Cooperative Games,” *Econometrica* **21**, 128–140.

Osborne, M., and Rubinstein, A. (1990). *Bargaining and Markets*, San Diego, CA: Academic Press.

Schelling, T. (1960). *The Strategy of Conflict*, Cambridge, MA: Harvard University Press.

Schmidt, K. M. (1993a). “Reputation and Equilibrium Characterization in Finitely Repeated Games,” *Econometrica* **61**, 325–351.

Schmidt, K. M. (1993b). “Commitment through Incomplete Information in a Simple Repeated Bargaining Game,” *J. Econ. Theory* **60**, 114–139.

Watson, J. (1993). “A “Reputation” Refinement without Equilibrium,” *Econometrica* **61**, 199–205.