

PREFACE TO “HIERARCHIES OF BELIEFS AND INTERACTIVE EPISTEMOLOGY IN DYNAMIC GAMES”

Marciano Siniscalchi

The epistemic approach to the analysis of games utilizes two basic ingredients. The first is a formal framework, or language, that allows the modeler to provide a precise, exhaustive description of what players can *do* and *think about each other*. The second is a set of assumptions, stated in the formal language one chooses to adopt, about the players’ *decision rules* (e.g. sequential rationality) and *mutual beliefs* (e.g., “Player 1 believes that Player 2 believes that 1 is sequentially rational”).

These two ingredients are then “combined” to produce *behavioral implications*: that is, one proves that, within the framework of choice, a given set of assumptions is satisfied (if and) only if players choose certain actions or strategies.

This approach is attractive because it allows the modeler to justify predictions, or characterize solution concepts, in terms of clearly defined assumptions about behavior and beliefs.

Furthermore, it promises to bridge the gap between decision theory and game theory. Savage [4] and Anscombe and Aumann [1] provided axiomatic characterizations of expected-utility maximization. In much the same spirit, Aumann and Brandenburger [2] provide an axiomatic characterization of Nash equilibrium, Tan and Werlang [5] characterize normal-form rationalizability, etc.

I remember Robert Wilson making this observation on several occasions. His insistence on the importance of establishing proper foundations for game theory, as well as his remarks on the subtle aspects of backward and forward induction, prompted me to investigate strategic rationality in extensive games from the point of view of interactive epistemology.

The following paper is the first in a series I coauthored with Pierpaolo Battigalli, then at Princeton University. Its aim is to provide the first, basic ingredient for the epistemic analysis of extensive games—a framework that allows one to formulate assumptions about the players’ mutual *conditional* beliefs at each history or information set. We show that, loosely speaking, the model we propose is rich enough to accommodate *any* combination of (coherent) assumptions, and indicate some applications.

Given our emphasis on mutual conditional beliefs, it would be impossible not to mention Kreps and Wilson's [3] influential paper on sequential equilibrium as a major source of inspiration. While this applies to most of the literature on extensive games since the early 80s, the following quotation from Kreps and Wilson's paper reveals that the connection with our own work is quite specific:

...making explicit the construction of beliefs off the equilibrium path enables discussion of which beliefs are "plausible" and which are not... And such comparisons can often help one to choose among sequential/perfect equilibria. ([3], p.864)

It is a pleasure to contribute the following paper to this collection in honor of Robert Wilson. I do so with heartfelt gratitude for his advice and guidance.

References

- [1] Anscombe, F. and R. J. Aumann (1963): "A Definition of Subjective Probability," *Annals of Mathematical Statistics*, 34, 199-205.
- [2] Aumann, R. J. and A. Brandenburger (1995): "Epistemic Conditions for Nash Equilibrium," *Econometrica*, 63, 1161-1180.
- [3] Kreps, D. and R. Wilson (1982): "Sequential Equilibria," *Econometrica*, 50, 863-893.
- [4] Savage, L.J. (1954), *The Foundations of Statistics*. Wiley, New York.
- [5] T. Tan and S. Werlang (1988): "The Bayesian foundation of solution concepts of games," *Journal of Economic Theory*, 45 , 370-391.