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Crime and Education in a Model of Information Transmission

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Abstract

We model the decisions of young individuals to stay in school or drop-out and engage in criminal activities. We build on the literature on human capital and crime engagement and use the framework of Banerjee (1993) that assumes that the information needed to engage in crime arrives in the form of a rumor and that individuals update their beliefs about the profitability of crime relative to education. These assumptions allow us to study the effect of social interactions on crime. We first show that a society with fully rational students is less vulnerable to crime than an otherwise identical society with boundedly rational students. We also investigate the spillovers from the actions of talented students to less talented students and show that policies that decrease the cost of education for talented students may increase the vulnerability of less talented students to crime. This is always the case when the heterogeneity of students with respect to talent is sufficiently small.

Keywords: human capital, the economics of rumors, social interactions, urban economics.

JEL codes: D82, D83, I28
1 Introduction

Many developing countries and poor areas in developed countries are plagued by high crime rates and low levels of education. Young people seem to be particularly vulnerable to crime engagement. Oftentimes, once crime has started it spreads in an epidemiological way through a community. We here suggest a theory of juvenile crime that is motivated by the idea that the further people are from receiving a return on educational investments they have made, and the more likely they are to be surrounded by other young criminals, the more they will be willing to engage in crime. It allows us to investigate the effect that educational policies have on the diffusion of crime among young people.

Our theory is motivated by the fact that crime is a social phenomenon. Following Becker (1968), economic theory sees crime as an occupational choice or investment opportunity. A person compares the streams of payoffs from crime versus other occupations or investments in human capital such as going to school to obtain a good job later. Particularly interesting is Lochner (2004) who builds a dynamic model of education and crime engagement and explains the decreasing age-petty crime pattern. The more individuals have invested in education, the larger the opportunity cost of crime. Hence, older people who have accumulated more human capital or are closer to graduation, will be less prone to engaging in crime.

There is evidence supporting the ideas of Becker and Lochner (see Levitt, 1998, Mocan and Rees, 2005). But there is also evidence showing that crime engagement decisions are not completely described by the traditional neo-classical model and that a model with that aim should introduce new features. In particular, social interactions are important determinants of crime engagement. Ludwig et al. (2001) and Kling et al. (2005) show that neighborhood’s wealth has an incidence in youth crimes. Particularly important to our paper is the evidence found by Case and Katz (1991) who show that in low-income Boston neighborhoods the behavior of peers appears to affect youth behaviors in a manner suggestive of contagion models. Another important piece of evidence is provided by Luallen (2006) who shows that
reducing school incapacitation increases crime rates among youngsters.

Taken together, the previous literature shows that the causal link between crime and low levels of human capital is quite complex. However, there seems to be agreement that fostering education is a good way to fight crime.

We investigate the interaction between educational policies and juvenile crime. We assume that everybody is rational, but that information on the opportunity to become a criminal is not readily available. Rather it is transmitted through an information diffusion process in society: people who have become criminals meet students and students learn about the possibility to become a criminal rather than going to school. Our assumption is in line with the evidence cited above. We investigate the nature of the information transmission process between criminals and students and carry out an investigation on the policies that reduce the cost of education such as scholarships, meals or transport subsidies, better teachers and materials.

We consider social interactions using a model of a rumor process à la Banerjee (1993). People are rational, they are young and go to school. Going to school costs some effort or money. Some of the students are more talented, thus they have lower costs, while others are less talented, and have higher costs of going to school. Talent (or ability) is private information. There is aggregate uncertainty: crime may pay or not and, because of differences in the opportunity costs of crime engagement, the payoff of engaging in crime depends on whether you are talented or not. Information on crime is not common knowledge but travels as a rumor. Upon hearing the rumor, a student updates the likelihood of crime being profitable and decides whether to stay in school or become a criminal. The time that passes before a given student meets a criminal for the first time provides crucial information about the probability that crime is profitable. This is so because the speed of the rumor transmission depends on the number of criminals, which in turn depends on profitability of crime.

We show that there is a point in time after which talented students will not be tempted anymore to become criminals. There is also a point in time for the less talented, but it occurs later. Hence, the less talented are more
vulnerable to crime engagement. Indeed if students are fully rational and take into account the time passed before they hear the crime rumor, they will be less likely to engage in crime. This holds for both the talented and the less talented. Hence, social interaction need not increase crime, provided that people understand the diffusion process. This can be seen as a rationale for information campaigns about crime.

The second result is arguably more important for policy considerations. We show that social interactions play a role in fully rational students’ decisions. The behavior of the talented students affects the behavior of the less talented ones, but not the other way round. Consider a policy reducing the cost of schooling for talented students (for instance, a meritocratic scholarship program). This policy directly reduces the vulnerability to crime of talented students. To understand the effect on less talented students, the way the rumor about crime spreads at any time afterwards is crucial. Individuals update their beliefs of the profitability of crime by taking into account the time that passes until they meet a criminal for the first time. Older rumors are a signal that crime is less profitable; this is the effect that appears in Banerjee (1993). However, there is a second effect that is caused by the reduction in the number of talented students that become criminals. We can show that this effect makes less talented students believe that crime is more profitable. Consequently, a policy reducing the cost of education of talented students may increase or decrease the vulnerability of less talented students depending on the strength of each of the two effects. Moreover, when the heterogeneity of students is sufficiently small, such a policy always increases crime among less talented students.

The paper is related to a broader literature on information diffusion, such as Banerjee (1992) and Scharfstein and Stein (1990), who develop models of herd behavior. In those models information goes through a process of word-of-mouth learning and they are thought to explain financial runs, behavior facing new products, etc. In the context of social economics, Jackson and Yariv (2008) have recently reviewed the literature on the influence of social networks on diffusion processes in different realms, such as disease contagion, technology adoption, vote decisions, etc. Previously, Akerlof (1997)
developed a model that shows how social position may affect decisions such as education attainment or childbearing. Economic models of social interactions and crime were firstly developed by Sah (1991) and Glaeser et al. (1996). The former develops a model in which the decision of a person to commit crime reduces the probability of other offenders to be arrested. The latter develop a model in which the individuals decision about crime depends on their neighbors’ decisions about criminal activities. The model helps them to explain the cross-city variance of crime rates.

The rest of the paper is divided as follows: in the next section, we present our model and a benchmark. In the benchmark the probabilities associated to the profitability of crime are exogenous. In our model this is a result of assuming that students are boundedly rational - they do not understand the rumor process. Alternatively, the same results are obtained assuming that the information set available to students is more limited. We maintain the bounded / full rationality terminology across the whole paper. In the third section, we present our main results when we consider a society conformed by fully rational students. In the fourth section, we analyze the social interactions among student types and how this may affect education policies. In the last section we conclude with some final remarks.

2 Model Setting

We consider population of students given by the interval $[0, 1]$ with equal life length $T$. We denote $s$ the length of schooling of a student. After graduation, students earn an income of $W$ in each moment of the rest of their lives. Education is costly; the instantaneous cost of education (in terms of effort, tuition etc) is $e$. There are two types of students: a proportion $q$ of the students have high costs, $\pi$, and a proportion $1 - q$ of the population have low costs, $\sigma$. Leaving problems of access to credit markets aside (a topic that is beyond the scope of this paper), notice we can refer for simplicity to high-cost students as “less talented” and low-cost students as “talented”.

To simplify the model, we assume that the discount rate is equal to zero and that:
Assumption 0  At $t = 0$ the entire population is attending school.

Education is a riskless project.$^1$ Its value depends on the moment of life of a person. At any moment in time $t < s$ the instantaneous continuation value of education is:

$$R(t) = \frac{(T - s)W - (s - t)e}{T - t}$$  \hspace{1cm} (1)

We have then $R(t)$ and $R(t)$ for $\tau$ and $\varepsilon$, respectively. The idea of $R(t)$ is that students must study a proportion $s/T$ of life in order to obtain a degree and to earn $W$ in each moment of the rest of their lives. Students hence first have to invest the cost of education to obtain its benefits afterwards. Clearly, the value of education increases in $t$. The sunk-cost nature of education will be a crucial feature in our model. We will simply refer to $R(t)$ as the value of education.

Assume for a first benchmark that becoming a criminal were a riskless project with instantaneous returns $a_0$. Then if $a_0 < R(0)$ there is no crime. If $R(0) < a_0 < R(0)$, the less talented (high-cost) students commit crime during their entire life and talented (low-cost) students commit no crime and the total number of criminals is $q$. And, if $R(0) < a_0$, all students commit crime during all life, in this case the total number of criminals is 1.

It makes much more sense, however, to consider that being a criminal is a risky project. Consider that its returns are $a$ with probability $p$, $b$ with probability $r$ and $d$ with probability $1 - p - r$, where $a > b > 0 > d$.\(^2\)

We make the following assumptions about the interaction between education, crime and different types of students:

Assumption 1  $(T - s)W - s\bar{e} > 0$

Assumption 2  $pa + rb + (1 - p - r)d < 0$

$^1$One can argue that education may also be a risky project. However, the existence of institutions like minimum wages, that are common in both developed and developing countries, make the education project less risky than the crime project. Moreover, in those contexts in which education is riskier than crime, rumors about criminal projects may be more pervasive. Hansen and Machin (2001) present empirical evidence showing that the establishment of minimum wage actually causes a decrease in crime rates.

$^2$At the cost of further complication but without much benefit in terms of economic insights one could assume that each of the states $a$, $b$ and $d$ were lotteries themselves, with $a$ the best lottery and $d$ the worst.
Assumption 3 \( W \geq a \)

Assumption 4 \( R(0) < b < R(0) < a \)

Assumption 1 says that education pays for the less talented students and hence also for the talented one. Assumption 2 is a somewhat stronger assumption: it says that the \textit{ex ante} expected value of crime is negative such that without further information neither type of student would engage in crime. It allows us to focus attention on crime due to social interaction. Assumptions 3 and 4 are about the crime-education decisions during schooling time. Assumption 3 says that the riskless reward of education is larger or equal to the largest payoff of crime. As a consequence, nobody becomes a criminal after \( s \), when all educational efforts are sunk. Assumption 4 says that, at \( t = 0 \), crime is profitable for the talented type only if the true state of the world is \( a \). Crime is profitable for the less talented type if the true state of the world is either \( a \) or \( b \).

The previous assumptions deserve further discussion. Assumptions 1 and 2 together say that, ex-ante, education pays more than crime. This is in line with previous evidence on gang earnings showing that risks of criminal activities more than offset its wage premium with respect to legal earnings (Levitt and Venkatesh, 2000). Assumption 3 gathers the findings of Lochner and Moretti (2004), that high school graduation significantly reduces engagement in crime.\(^3\)

At \( t = 0 \), all the population is attending school. A proportion \( x \) of the population learns the true state of the world, which is either \( a \), \( b \) or \( d \). These students then choose whether to drop out of school (and commit crime) or to attend school (and exert effort).

Assumption 5 \textit{If the student commits crime once, he stays a criminal for the rest of life, that is there is no way back to school once it has been interrupted.}\(^6\)

From \( t = 0 \) on, each student meets another agent in each instant. The agent may be either a criminal or a student. The student learns whether

\(^{3}\text{This is so because wages after graduation are much higher than wages with no graduation.}\)
the agent is a criminal or not, but he does not learn the true state of the world or the agent’s type (talented or less talented as a student). We will let \( m \) denote the event in which the student effectively meets an individual who had previously engaged in crime. When a student meets a criminal for the first time, he can choose whether to commit crime or not. This reflects the idea that crime is an occupational choice that becomes available only through social interaction. In order to adopt crime, one needs to have contact with other people who are criminals, because there are no formal channels through which one can take this type of career.\(^4\) Upon meeting a criminal, a student is then confronted with the choice of staying in school or engaging in a very different type of career.

Assume now, for a second benchmark, that updating is boundedly rational in the following sense:

**Assumption 6** A student who meets a criminal at time \( t \) updates his belief about the state of the world, but he does not take into account the point of time \( t = 0 \) at which the crime diffusion process has started.

Assumption 6 is meant to capture people’s limited knowledge or understanding of the diffusion process of crime. Assumption 6 implies that the only information used by students to update their beliefs about the profitability of crime when they meet a criminal is the distribution of types of students and the \textit{ex-ante} distribution of the profitability of crime. Students who have never met a criminal act as if nobody had committed crime, and a student who meets a criminal at any time \( \tau > t \) assigns the same informational value to meeting a criminal as a student who meets the criminal at time \( t \).

Formally, suppose the true state of the world is either \( a \) or \( b \) (we do not have to consider the true state of the world \( d \) because then nobody commits crime). When a student meet a criminal he learns that crime may be profitable for him and updates the probability of each state of the world.

\(^4\)We exclude that one can become a criminal without having any contact with other criminals, as we are interested in crime, education and social interaction through information diffusion and not in the isolated decision of an individual to commit crime which has been thoroughly studied by Becker and other scholars building on his work.
Let $\pi = \frac{\pi}{p+r}$ be the updated probability of state $a$. Students will commit crime if and only if

$$EC_{br} \equiv \frac{\pi}{\pi + q(1-\pi)} a + (1 - \frac{\pi}{\pi + q(1-\pi)}) b > R(t).$$

**Result 1** Under boundedly rational updating, there is a time at which a student with cost of education $e$ will not be tempted to adopt crime, but will stay in school. This time is given by the function $\tau_{br}(e)$ which follows:

$$\tau_{br}(e) = \frac{T \cdot EC_{br} + se - (T - s)W}{EC_{br} + e}.$$ 

Let $\overline{\tau}_{br} = \tau_{br}(\overline{e})$ and $\overline{\tau}_{br} = \tau_{br}(\overline{e})$, we have that $\overline{\tau}_{br} < \tau_{br}$.

This benchmark brings across the direct intuition from any extension of a Becker-type model of crime. The lower the opportunity costs of crime, the more likely people will adopt it. In our model, the opportunity cost modeled explicitly is not the risk of being detected and punished (this is contained in reduced form in the parameters $a, b, d$). Rather, we look at the process through which education is acquired. Over time, education becomes relatively less costly, because effort has already been sunk. Hence the opportunity costs of crime increase over time, independently whether one is talented or not. Further, the opportunity costs for the talented are higher, which explains why their temptation to engage in crime ends earlier.

### 3 Fully Rational Students

We maintain Assumptions 1 to 5 and now assume that students take the time dimension into account when updating:

**Assumption 7** Students know the distribution of types and the date ($t = 0$) in which the rumor started.

Under Assumption 7, the process of information transmission about crime becomes a rumor process in the sense of Banerjee (1993). Criminals become a source of the rumor on crime and, thus, the probability of hearing the rumor (meeting a criminal) increases with the number of criminals. Anybody’s decision whether or not to engage in crime thus creates
information externalities. Under our assumptions, especially Assumptions 1 and 2, nobody will invest in crime unless somebody learns that someone else has already committed crime. Rumor begins if the true state of the world is either $a$ or $b$. If $a$, a proportion $x$ of people will become criminals at $t = 0$. If $b$, a proportion $qx$ will do so. If the true state is $d$, nobody will.

In our analysis there are three critical points in time for understanding the decision of individuals regarding their education vs. crime decisions; these are $\tau^*, \tau$, and $\tau$. The latter two are the moments in time when talented and less talented students respectively stop engaging in crime; $\tau^*$ is the moment in time at which less talented students would stop engaging in crime if the probability of state $a$ were zero. From equation (1) we obtain $\tau^*$ equating $R(\tau^*)$ to $b$; From Assumption 4, by continuity of $R(t)$ we know that $\tau^* > 0$.

We now turn to determining $\tau$ and $\tau$ which requires some additional notation. A talented student at any $t$ or a less talented student at any $t > \tau^*$ who meets a criminal will commit crime if:

$$EC(t) \equiv p(t)a + (1 - p(t))b > R(t).$$

where $p(t)$ is the probability of the true state being $a$ given that the student meets a criminal for the first time at time $t$. It is estimated using Bayes’ rule:

$$p(t) = \frac{\pi}{\pi + \frac{Prob[m|b,t]}{Prob[m|a,t]}(1 - \pi)}$$

with $\pi = \frac{p}{p + r}$. $Prob[m|s,t]$ is the probability that in state $s \in \{a,b\}$ a student meets a criminal for the first time at $t$. The ratio of $Prob[m|b,t]$ and $Prob[m|a,t]$ determine $p(t)$; this ratio will be crucial for our analysis and we hence define it formally.

**Definition 1**

$$z(t) \equiv \frac{Prob[m|b,t]}{Prob[m|a,t]}.$$  

We also define, $z^*(t)$, the net gain of engaging in crime when it is profitable, relative to the net loss when it is not profitable:

**Definition 2**

$$z^*(t) \equiv \frac{\pi(a - R(t))}{(1 - \pi)(R(t) - b)}.$$
Because $z^* (t)$ depends on the type of student which affects $R(t)$:

$$z^* (t) \equiv \frac{\pi (a - R(t))}{(1 - \pi) (R(t) - b)} \text{ and } \bar{z}^* (t) \equiv \frac{\pi (a - \bar{R}(t))}{(1 - \pi) (\bar{R}(t) - b)}.$$  

Notice also that $\bar{z}^* (t)$ is defined in the interval $(\tau^*, T]$ while $z^* (t)$ is defined in $[0, T]$.

It can then readily be shown that inequality (2) holds for talented students if $z(t) < z^* (t)$ and for less talented students if $z(t) < \bar{z}^* (t)$. This analysis is summarized in the following result:

**Result 2** Under fully rational updating, the behavior of students is as follows:

1. A less talented student who meets a criminal for the first time at $t \leq \tau^*$ engages in crime.

2. A less talented student who meets a criminal for the first time at $t > \tau^*$, engages in crime if and only if $z(t) \leq \bar{z}^* (t)$.

3. A talented student who meets a criminal for the first time, engages in crime if and only if $z(t) \leq z^* (t)$.

We have shown that the crucial element in the decision to become a criminal or not when hearing the rumor is the relative probability of meeting a criminal in each of the two states of the world. Notice that the rumor on crime only begins if the condition in Equation (2) holds at $t = 0$, that is, both types must be vulnerable to crime at $t = 0$. The updating of fully rational students uses the age of the rumor to calculate the number of criminals in each state of the world. The decision rule stated in Result 2 identifies a critical level for $z(t)$ after which talented and less talented students keep on going to school; the critical level depends on the costs that each individual faces to complete school.

From Result 2, it becomes clear that both types of students are vulnerable to crime. Less talented students are more vulnerable than talented students since they are likely to become criminals during a longer period of life. To understand the dynamics of the diffusion process and, in the next
step, the effect of different policies, we need to investigate the properties of \( z(t), \hat{z}^*(t) \) and \( \mathcal{E}^*(t) \). Result 3 states the properties of \( \hat{z}^*(t) \) and \( \mathcal{E}^*(t) \).

**Result 3** The functions \( \hat{z}^*(t) \) and \( \mathcal{E}^*(t) \) are both monotonically decreasing in \( t \) and convex.

Proofs are in Appendix ???. According to Result 3 the profitability of crime decreases with time for both types, in particular because the education cost is continuously sunk at each moment of time. The result holds in the respective domain of each function; that is, for \( \hat{z}^*(t) \) in \( t \in [0, T] \), and for \( \mathcal{E}^*(t) \) in \( t \in (\tau^*, T] \).

The analysis of \( z(t) \) is more challenging and it requires the use of additional notation and definitions. The following is borrowed from Banerjee (1993).

**Definition 3** For \( i = a, b \), we define:

1. \( N(i, t) \equiv \) the proportion of the population that has committed crime until time \( t \) in state \( i \).

2. \( P(i, t) \equiv \) the proportion of the population that has not met a criminal until time \( t \) in state \( i \).

Using Definition 3 and the fact that

\[
\text{Prob}[m|b, t] = N(b, t)P(b, t)
\]

and

\[
\text{Prob}[m|a, t] = N(a, t)P(a, t),
\]

we can then write:

\[
z(t) = \frac{N(b, t)P(b, t)}{N(a, t)P(a, t)}.
\]

Notice that \( P(a, 0) = 1 - x, P(b, 0) = 1 - x, N(a, 0) = x, N(b, 0) = xq \), and that \( z(0) = q \).

**Definition 4** We distinguish between Regime 1, where \( z(t) \leq \hat{z}^*(t) \) and Regime 2, where \( z(t) > \hat{z}^*(t) \).
We now show that there is a moment in time, which we call $\tau$, in which $z(\tau) = z^*(\tau)$; i.e. a moment in time in which the system switches from Regime 1 to Regime 2.

Notice that the process must start off in Regime 1 (i.e. the condition in Equation 2 must hold at $t = 0$), otherwise, there will be no uncertainty about crime. If the process began in Regime 2, everybody who meets a criminal will know the criminal is a less talented student. Formally, the process must start off when $z(t) \leq z^*(t)$, which at $t = 0$ boils down to

$$q \leq \frac{\pi (a - R(0))}{(1 - \pi)(R(0) - b)}.$$

In Regime 1, the dynamics of $N(i,t)$ and $P(i,t)$ are given by

$$\frac{dP(i,t)}{dt} = -N(i,t)P(i,t),$$

$$\frac{dN(i,t)}{dt} = N(i,t)P(i,t).$$

In Regime 2, the dynamics of $N(i,t)$ and $P(i,t)$ are given by

$$\frac{dP(i,t)}{dt} = -N(i,t)P(i,t),$$

$$\frac{dN(i,t)}{dt} = qN(i,t)P(i,t).$$

The intuition for the difference is of course that in Regime 2 only less talented students (a proportion $q$ of the total population) may become criminals at $t > \tau$.

Furthermore, for an economy that has always been in Regime 1 holds:

$$P(a,t) = 1 - N(a,t),$$

$$P(b,t) = 1 - x(1 - q) - N(b,t).$$

For an economy that has made its first transition to Regime 2 at moment $\tau$ holds:

$$q[P(a,\tau) - P(a,t)] = N(a,t) - N(a,\tau),$$

$$q[P(b,\tau) - P(b,t)] = N(b,t) - N(b,\tau).$$

Equations (8) and (9) evaluated in $t = 0$ together with equations (10) and (11) evaluated in $t = \tau$ provide the initial conditions for the differential
equations (4)-(7), respectively. These differential equations differs depending on the true state of the world. In both, Regime 1 and Regime 2, we have that $P(a, t) < P(b, t)$ and $N(a, t) > N(b, t)$. This means that the proportion of individuals that engage in crime is higher in state $a$ than in state $b$; consequently, in a given $t$, the number of individuals that have not heard the rumor is smaller in state $a$ than in state $b$. With these things established the following Result can readily be shown (see the formal proof in Banerjee, 1993).

**Lemma 1 (The Banerjee effect)** The ratio $z(t)$ increases monotonically in $t$ and is unbounded.

The Lemma states that the older the rumor (i.e. the larger $t$), the stronger the believe of students that the true state is $b$. The later a student meets a criminal for the first time, he believes that it is more likely that the benefits of crime are low.

From equations (4)-(11) one can see that since the dynamics in both regimes differ, $z(t)$ will have different forms in each regime. For $t < \tau$, in Regime 1, $z(t)$ is defined by (4), (5), (8) and (9), we will let $z^r_1(t)$ represent this part of $z(t)$. For $t \geq \tau$, in Regime 2, $z(t)$ is defined by (6), (7), (10) and (11), we will let $z^r_2(t)$ represent this part of the function. $z^r_1$ depends on $x$ and $q$; $z^r_2$ depends on $x$, $q$ and the parameters that determine $z^*$ (since $\tau$ is determined by the equality $z^* = z^r_1$). Indeed $\tau$ defines the initial conditions for (and determines the actual path followed by) $z^r_2$. We can then define formally the function $z(t)$ as follows:

$$z(t) = \begin{cases} 
    z^r_1(t), & \text{if } t \leq \tau; \\
    z^r_2(t), & \text{if } t > \tau.
\end{cases} \quad (12)$$

Explicit expressions for $z^r_1(t)$ and $z^r_2(t)$ can be easily obtained using (3), (4)-(11). For our analysis we will need the explicit expression for $z^r_2$ which can be expressed in terms of $P(i, \tau)$, $N(i, \tau)$ and $(t - \tau)$, as follows

$$z^{r_2} = \frac{dN(b, t)}{dN(a, t)} = \frac{qN(b, t) P(b, t)}{qN(a, t) P(a, t)} = \frac{dN(b, \tau)}{dN(a, \tau)} \frac{g(P(b, \tau), t - \tau)}{f(P(a, \tau), t - \tau)} \quad (13)$$

The specific forms $z^{r_2}$ and $z^{r_1}$ appear in the Appendix 1 where it will be clear that $g$ and $f$ are two specific functions.
The following Lemma shows that $\tau$ exists.

**Lemma 2** Provided that $q \leq \frac{\pi(a - R(0))}{(1-\pi)L(0) - b}$, there will be an instant $\tau$ at which there will be a transition from Regime 1 to Regime 2.

We have so far established that the beliefs on the true state of the world converge to $b$ (Lemma 1), that the value of education relative to crime is increasing over time (Result 3), and that $\tau$ exists (Lemma 2). It is important to note that the rumor on crime goes beyond $\tau$. Consider a student who meets a criminal in $t > \tau$. Although he knows that talented students are not vulnerable anymore, he also knows there are criminals who had been talented students, but met a criminal before $\tau$. At $\tau$ non talented students are still vulnerable to crime.

Following a similar reasoning we can show how the less talented students behave. Indeed, from Results 3 and 1 we have that $z(t) - \tau(t)$ is monotonically increasing. Therefore there exists a $\tau$ such that $z(\tau) = \tau(\tau)$. Since an increasing amount of education cost is sunk over time, there is a moment in time in which no student becomes a criminal anymore. After $\tau$, education is more valuable than crime for both types. The total number of criminals thus reaches its maximum at $\tau$. After $\tau$, some students will still meet criminals (who can be of either type) but they will not engage in crime. As we have argued before, the time at which talented students stop engaging in crime, $\tau$, is strictly shorter than the time at which less talented students do so, $\tau'$. In a nutshell, less talented students are more vulnerable to crime than talented students since their cost of schooling is larger ($s\tau > s\tau'$). Figure 1 depicts the solutions we presented above.

A comparison with the cutoff values for boundedly rational students derived in the preceeding section establishes the following proposition.

**Proposition 1** A society with fully rational students will be less vulnerable to crime then an otherwise identical society with boundedly rational students.

Since boundedly rational students do not fully understand the diffusion process, they believe that $z(t) = q$ for all $t$. The updating process of unboundedly rational students makes them believe that $z(t)$ is increasing in $t$. 

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and everywhere above $q$. Since $z^*(t)$ and $\tau^*(t)$ are both decreasing in $t$, all this implies that $\tau < \tau_{br}$ and $\tau < \tau_{br}$.

There is one more characteristic of $z(t)$ that we must consider, this is stated in the following result.

**Lemma 3** At $t = \tau$, there is a downward kink in $z(t)$, that is, for $t$ very near to $\tau$, the slope of $z(t)$ is larger for $t \leq \tau$ than for $t > \tau$.

There is a kink at $\tau$ because beyond this point, talented students who have not yet met a criminal will never engage in crime. At the kink the speed of the rumor decreases, which has important consequences for the policy effects we present below.

### 4 Policy: the effect of changing the costs of education

In the previous section we have investigated how crime spreads in a society and how it affects education. We here show that policies reducing the cost of education may have surprising effects. Examples of such policies are...
reductions of tuition fees, food for school programmes, improvements in school infrastructure or teachers. These measures can be given depending on performance of a student, which makes them contingent on their type (talented vs less talented).

Consider policies such that $e$, $\tau$ or both are reduced. These reductions have a direct effect on the vulnerability to crime of the targeted type of student, but there is also an indirect effect through the transmission of information about crime profitability among students of different types. The direct effect of reducing $e$ ($\tau$) shifts $\tau$ ($\tau$) to the left. That is, the point in time at which no more students of a given type will engage in crime occurs earlier when their cost of attending and succeeding at school decreases. Put differently, students become less vulnerable to crime.

The indirect interaction, i.e. effects between different types, are much more subtle, but they only go from talented students to less talented students. To understand this statement notice first that both the talented and the less talented students carry out the same type of comparison between costs and benefits of engaging in crime. More precisely, they both have the same uncertain benefit of engaging in crime and they assign the same probability distribution to the states $a, b, d$. Also, wages upon graduation are assumed to be the same for both types. The opportunity costs of engaging in crime, however, are type-dependent: talented students have lower costs of going to school than less talented ones, which explains why $\tau$ is to the
left of $\tau$.

For any $t$ smaller than $\tau$, any change in the parameters affects equally the expected benefits both types of students assign to crime. This implies that among criminals, the proportion of talented and less talented types is constant, reflecting the respective proportions in the entire population. There are interaction effects here, but they do not depend on the types. This changes at $t = \tau$. Here, no further talented student engages in crime, while less talented students who meet a criminal continue to do so, implying that the proportion of less talented types among criminals increases. Hence, when one wants to understand the interaction effects between different types, it suffices to investigate how a shift in $\tau$ will affect the behavior of less talented students. This is stated in the following proposition.

**Proposition 2** Effects of a decrease in $e$ (the costs of education for talented students): (i) $\tau$ shifts to the left i.e. talented students become less vulnerable to crime; (ii) the effect of a reduction in $e$ on $\tau$ is ambiguous, in particular a reduction of $e$ may result in an increase of $\tau$ i.e., less talented students may become more vulnerable to crime.

To understand the second part of the proposition recall that students vulnerability to crime depends on $z(t)$ in particular, if $z(t)$ increases because of an intervention, students become less vulnerable, and if $z(t)$ decreases they become more vulnerable.

The introduction of a subsidy scheme for more talented students will cause an instantaneous reduction of the number of criminals in both state of the world, because the subsidy makes the talented students less vulnerable to crime. If a student meets a criminal at a given $t$ he will have stronger beliefs about the true state of the world being $a$ if the subsidy scheme is in place. To formally see this effect consider two situations, one without the subsidy and one with the subsidy; let $\tau'$ and $\tau''$ be the two moments in time in which talented and less talented students stop engaging in crime without the subsidy and $\tau'''$ and $\tau''''$ with the subsidy. We know from Lemma 3 that at the moment in time in which talented students are not vulnerable to crime any more, there is a kink in $z(t)$ this implies that at $\tau'''$ the slope to
the right of \( z(t) \) with and without a subsidy is different. Without a subsidy the right derivative of function \( z(t) \) at \( \tau'' \) is equal to

\[
z (\tau'') \left[ 2 \left( P \left( b, \tau'' \right) - P \left( a, \tau'' \right) \right) + x \left( 1 - q \right) \right].
\]

When there is a subsidy, the right derivative of \( z(t) \) at \( \tau'' \) is

\[
z (\tau'') \left[ (1 + q) \left( P \left( b, \tau'' \right) - P \left( a, \tau'' \right) \right) + x \left( 1 - q \right) \right]
\]

which is smaller than the previous one. This means that the introduction of a subsidy scheme makes state \( a \) more likely at \( \tau'' \).

From \( \tau'' \) onwards this effect coexists with the Banerjee effect (Lemma 1), which makes the beliefs about \( b \) being the true state of the world increase with time. The two effect hence go in opposite directions.

Another way to formally see these two effects is by looking at the derivative of \( z(t) \) in Regime 2, that is after the time talented students cease to be vulnerable to crime, with respect to \( \tau \). Deriving the function \( z^r_2 \) in Equation (13) we obtain

\[
\frac{\partial z^r_2}{\partial t} = \left[ \frac{\partial^{2 N}(b,t)}{\partial t^2} - \frac{\partial^{2 N}(a,t)}{\partial t^2} \right] \frac{\partial g}{\partial P(b,t)} \frac{\partial P(b,t)}{\partial t} - \frac{\partial g}{\partial P(b,t)} \frac{\partial P(b,t)}{\partial t} - \frac{\partial f}{\partial P(a,t)} \frac{\partial P(a,t)}{\partial t} - \frac{\partial f}{\partial P(a,t)} \frac{\partial P(a,t)}{\partial t} \right].
\]

The first term in on the right-hand side corresponds to the subsidy effect that increases the belief of state \( a \) being the true state of the world. The second term gathers the interaction of the first effect with the Banerjee effect in each state of the world. Notice that the size of the interaction depends on \( t \). Less talented students will become more (less) vulnerable to crime if at \( t = \tau' \) the first effect is stronger (weaker) than the second effect.

Two different systems that differ in \( \varepsilon \) only differ in their dynamics after the lower \( \tau \). The differences are consequence of changing the initial conditions for \( z^r_2 \) and on changes in the dynamics of \( z(t) \) after \( \tau \). Precisely, what one wants to know is whether, for \( t > \tau \), \( z'' \) is to the left or to the right of \( z' \) or whether they cross. If they cross one also wants to know if \( z'' \) crosses \( z' \)
from above or from below. If $z''$ is to the right (left) of $z'$ this would mean that reducing the cost of talented students increases (reduces) the vulnerability of less talented students. In other words if $z''$ is to the right (left) of $z'$ a reduction in $e$ would bring the undesirable effect of increasing $\tau$; in the other case reducing $e$ would have a positive externality since it would also reduce $\tau$. A full comparison of $z'(t)$ and $z''(t)$ is in general not possible. However, the following example shows that for two sets of parameters in which the only difference is the value of $a$, a change in $e$ of the same size induces changes in $\tau$ of different signs.

**Example 1** Consider the following values for the parameters of our model: $x = 0.1, q = 0.6, \tau = 10, W = 69, a = 55, b = 15, \pi = 0.6$ and $T = 34$. The effort of the less talented students is $\bar{e} = 153$ and the effort of the talented students is $\bar{e} = 100$. With this information, all less talented students that hear the rumor about crime before $\tau \approx 5.44$ choose to become criminals. Similarly, all talented students that hear the rumor before $\tau \approx 4.07$ will do so. Let us consider a policy that reduces the costly effort of education for talented students. It reduces $e$ to 35. This policy reduces the vulnerability to crime of talented students to 1.58 and makes less talented students more vulnerable to crime increasing $\tau$ to 5.51. This example is depicted in Figure 3.

Now, consider an alternative situation in which crime pays more, such that $a = 64$. In this case the initial $\tau$ is 5.39; the initial $\tau$ is 5.96. When the policy that reduces $e$ from 100 to 35 is implemented, the vulnerability of talented students decreases to 2.68 but less talented students become less vulnerable to crime; $\tau$ decreases from 5.96 to 5.93. The example is in figure 4.

The next proposition identifies a sufficient condition under which there is no ambiguity.

**Proposition 3** Comparative statics with respect to student heterogeneity $(\bar{e} - e)$. For sufficiently low levels of student heterogeneity, a decrease in the cost of education of talented students makes less talented students more vulnerable to crime.
Figure 3: Example of policy that reduces vulnerability of talented students but increases vulnerability of less talented students

Figure 4: Example of policy that reduces vulnerability of talented and less talented students
Proof. Consider a situation in which \( \tau - \varepsilon \) is small. Consequently \( \tau - \tau \) is also small. Consider two levels of cost of education for talented students \( \varepsilon' \) and \( \varepsilon'' \), such that \( \varepsilon' > \varepsilon'' \). The corresponding moments in time in which talented students stop engaging in crime are \( \tau' \) and \( \tau'' \), and they satisfy \( \tau' > \tau'' \). We also have two functions for \( z(t) \); let these functions be \( z'(t) \) and \( z''(t) \). These two functions are exactly the same for any \( t \leq \tau'' \) and differ for \( t > \tau'' \). Consider a \( t \) such that \( \tau' > t > \tau'' \). Since \( \tau' > t > \tau'' \), \( t \) belongs to Regime 1 when \( \varepsilon = \varepsilon' \) and to Regime 2 in the second case. From Lemma 3, we have that there is downward kink at \( \tau \). Therefore, since functions \( z(t) \), \( z^*(t) \) and \( z^*(t) \) are continuous, for \( t \) near enough to \( \tau'' \) we have that \( z'(t) > z''(t) \) for \( t > \tau' \). Since \( \tau' \) is near \( \tau' \), \( \tau'' \) is near \( \tau'' \) and \( \tau(t) \) is downward sloping we then have that \( \tau' < \tau'' \). ■

5 Concluding remarks

We have suggested a theory of education and crime of young individuals. Information needed to engage in crime is not available to everybody; to become a criminal an individual has to hear information that affects the profitability of crime from someone who is already a criminal. Crime thus spreads in an epidemic fashion, as in the literature on rumors. We have shown that social interactions among fully rational students reduce crime engagement. Second, we have studied how informational externalities between different types of individuals affect crime engagement. The informational externality is such that policies aiming to decrease the costs of education of talented students have effects also on the education success and on crime engagement of less talented students. In particular, they may increase the vulnerability of less talented students to crime, which is always the case when heterogeneity of students with respect to talent is sufficiently low.
Appendix

A The form of $z^r_1$ and of $z^r_2$

The forms $z^r_1$ is

$$z^r_1 = \frac{dN(b,t)}{dN(a,t)}$$

where

$$\frac{dN(b,t)}{dt} = \frac{(1-x)(1-x(1-q))^2 xqe^{(1-x(1-q))t}}{(1-x+xqe^{(1-x(1-q))t})^2}$$

and

$$\frac{dN(a,t)}{dt} = \frac{x(1-x)e^t}{(1-x+xe^t)^2}.$$ 

The form of $z^r_2$ is

$$z^r_2 = \frac{dN(b,\tau)}{dN(a,\tau)} g(P(b,\tau), t-\tau)$$

where

$$\frac{dN(b,\tau)}{dt} = \frac{q(1-x)(1-x(1-q))^2 xqe^{(1-x(1-q))\tau}}{(1-x+xqe^{(1-x(1-q))\tau})^2},$$

$$\frac{dN(a,\tau)}{dt} = \frac{qx(1-x)e^\tau}{(1-x+xe^\tau)^2},$$

$$g(P(b,\tau), t-\tau) = \frac{(1-x(1-q)-(1-q)P(b,\tau))^2 e^{(1-x(1-q)\tau-1-q)P(b,\tau)(t-\tau)}}{[qP(b,\tau) + (1-x(1-q)-P(b,\tau))e^{(1-x(1-q)\tau-1-q)P(b,\tau)(t-\tau)}]^2},$$

and

$$f(P(a,\tau), t-\tau) = \frac{(1-(1-q)P(a,\tau))^2 e^{(1-(1-q)P(a,\tau)(t-\tau)}}{[qP(a,\tau) + (1-P(a,\tau))e^{(1-(1-q)P(a,\tau)(t-\tau)}]^2}.$$ 

B Proofs

1. Proof of Result 3:

Since $z^*(t)$ is continuous and differentiable then to show the result it is enough to to analyze the signs of the first two derivatives of $z^*(t)$.

We first need to know that

$$R'(t) = \frac{R(t) + e}{1-t} > 0,$$
\[ R''(t) = 2 \frac{R'(t)}{(1-t)} > 0. \]

Taking the first derivative of \( z^*(t) \), we obtain that
\[ z''(t) = -\frac{R'(t)}{(R(t)-b)} \left( \frac{\pi}{1-\pi} + z^*(t) \right) < 0. \]

Taking the second derivative
\[ z'''(t) = \left( \frac{\pi}{1-\pi} + z^*(t) \right) \left[ 2 \left( \frac{R'(t)}{(R(t)-b)} \right)^2 - \frac{R''(t)}{(R(t)-b)} \right] > 0. \]

2. Proof of Lemma 2:

Since \( q \leq \frac{\pi(a-R(0))}{(1-\pi)(R(0)-b)} \), the rumor on crime starts off. Let us first consider the case of strict inequality. Once a talented student meets a criminal, he updates beliefs on the state of the world and takes decisions following the rule in Result 2. Since, \( z(0) < z^*(0) \), the crime is profitable for talented students. Those students meeting criminals at \( t = 0 \) will become criminals. This will be the behavior of talented students for all \( t > 0 \) provided that \( z(t) < z^*(t) \). From Result 3, \( z^*(t) \) is monotonically decreasing with time and from Result 1, \( z(t) \) is monotonically increasing with time. Therefore, the difference \( z(t) - z^*(t) \) monotonically increases with time. At \( t = 0 \), it is negative, then it becomes zero and finally it becomes positive. Let \( \tau \) be the moment in time at which \( z(\tau) - z^*(\tau) = 0 \). For all \( t > \tau \) the talented student that meets a criminal will stay in school. At \( \tau \) there will be a transition from Regime 1 and Regime 2. Now let us consider the case of strict equality. In this case the talented students that know the true state of the world are indifferent between staying at school and becoming criminals. Those talented students that hear the rumor at \( t = 0 \) are also indifferent between school and crime. However, since \( z(t) - z^*(t) \) is monotonically increasing, all talented students that hear the rumor at all \( t > 0 \) will stay at school. In this case, \( \tau = 0 \).

3. Proof of Proposition 1:
The idea of the proof is to compare the moment in time at which both types of students stop to become criminals under two models: one with time-independent updating ($\tau_{br}$ and $\overline{\tau}_{br}$), the other with time-dependent updating ($\tau$ and $\overline{\tau}$). We need to show that $\tau_{br} > \tau$ and $\overline{\tau}_{br} > \overline{\tau}$.

On the one hand, with time-independent updating we have that

$$
\tau_{br} = \frac{T \cdot E_{br} + se - (T - s)W}{E_{br} + \bar{\epsilon}} \quad \text{and} \quad \overline{\tau}_{br} = \frac{T \cdot E_{br} + \bar{s}\bar{\epsilon} - (T - s)W}{E_{br} + \bar{\epsilon}}.
$$

On the other hand, with time-dependent updating we have that

$$
\tau = \frac{T \cdot E(t) + s\epsilon - (T - s)W}{E(t) + \epsilon} \quad \text{and} \quad \overline{\tau} = \frac{T \cdot E(t) + \bar{s}\bar{\epsilon} - (T - s)W}{E(t) + \bar{\epsilon}}.
$$

We also have that at $t = 0$, $z(0) = q$. Therefore $E_{br}(0) = E(0)$.

Finally,

$$
\frac{\partial \tau}{\partial z(t)} = \frac{\partial \tau}{\partial E(t)} \cdot \frac{\partial E(t)}{\partial z(t)},
$$

$$
\frac{\partial \tau}{\partial E(t)} = \frac{(T - s)(\epsilon + W)}{(E(t) + \epsilon)^2} > 0
$$

and

$$
\frac{\partial E(t)}{\partial z(t)} < 0.
$$

Then

$$
\frac{\partial \tau}{\partial z(t)} < 0.
$$

Since

$$
\frac{T \cdot E_{br} + se - (T - s)W}{E_{br} + \epsilon} = \frac{T \cdot E(0) + s\epsilon - (T - s)W}{E(0) + \epsilon}
$$

and from Lemma 1, $z(t)$ is always increasing, then, $\tau_{br} > \tau$ for all $t > 0$.

Following the same procedure we obtain that $\overline{\tau}_{br} > \overline{\tau}$.

4. Proof of Lemma 3:

To show that there is a downward kink at $\tau$, we have to show that

$$
\lim_{t \to \tau^-} z'(t) > \lim_{t \to \tau^+} z'(t).
$$
Indeed,
\[
\lim_{t \to \tau^-} z'(t) = \lim_{t \to \tau^-} [z(t) [2 (P(b,t) - P(a,t)) + x (1 - q)]]
= z(\tau) [2 (P(b,\tau) - P(a,\tau)) + x (1 - q)]
\]

and
\[
\lim_{t \to \tau^+} z'(t) = \lim_{t \to \tau^+} [z(t) [2q (P(b,t) - P(a,t))
+ (1 - q) (P(b,\tau) - P(a,\tau)) + x (1 - q)]]
= z(\tau) [(1 + q) (P(b,\tau) - P(a,\tau)) + x (1 - q)]
\]

Since 1, \(P(b,\tau) > P(a,\tau)\) and 2 > 1 + q, then there is a downward kink in \(\tau\).
References


