Farmland Investments in Africa: What’s the Deal?

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Abstract

Large-scale foreign investments in African farmland are rising and may contribute to agricultural productivity growth and economic development. However, host countries sometimes have to wait longer for the economic benefits to arrive than initially expected. In this respect, the timing of project development is crucial and depends on the economic incentives provided to the investors. We therefore present a dynamic stochastic programming model that reflects the typical bargaining situation concerning large land deals in Africa and allows the effect of market- and country-specific risks and taxation to be assessed. The model shows that commodity price volatility increases the value of the land development option, but slows down the land development process. Furthermore, it shows that host country attempts to negotiate fixed commitments to the speed of project development may run counter to the structure of economic incentives at the project site. The applicability of the model is demonstrated for a recent 10,000-hectare cotton project in Ethiopia. Response surface estimations suggest that Ethiopia has negotiated a contract under which it will receive about half the expected total project value, as long as it levies the regular corporate tax rate.

KEYWORDS: FOREIGN DIRECT INVESTMENT, LAND LEASING, REAL OPTIONS, NASH BARGAINING.

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1 Introduction

Foreign direct investment (FDI) by multinational corporations or governments in the agricultural land of developing and least developed countries\(^1\) is an ongoing trend (e.g. Visser and Spoor 2011; Cotula et al. 2009) which appears to be driven by the rising and increasingly volatile prices of agricultural commodities (see e.g. von Braun and Meinzen-Dick 2009; Collier and Venables 2012; Deininger et al. 2011; Hall 2011a). These recent price trends reflect a combination of a rising global demand for food, biofuels and agricultural raw materials on the one hand (e.g. Chakravorty, Hubert, and Nøstbakken 2009) and many distorting national farm policies still in place on the other (Franco et al. 2010).

Ideally, this type of FDI would benefit not only investors but also the host countries, since it may lead to infrastructure development, technology transfer and job opportunities in rural areas (see e.g. von Braun and Meinzen-Dick 2009).

In Africa, such investment projects are typically established as long-term land leases and the host countries usually aim to benefit through negotiated investor commitments towards certain infrastructure projects, through taxation of the project, or through a combination of both (Hall 2011b). However, from the perspective of the African host countries, several such projects have proved to be disappointing (see e.g. Collier and Venables 2012), since investors either failed to take the acquired land under cultivation at all, or cultivation has been developing slowly compared with the expectations that were raised when the contract was signed. In addition, it is frequently observed that expected and contractually established benefits from project-related investments in the development of infrastructure do not seem to be delivered by the investors (Cotula et al. 2009).

From an economic perspective, there are several potential reasons why a large-scale FDI project in African farmland may only sluggishly deliver the expected benefits to the host country. While some authors blame exploitative or speculative intentions by investors (see e.g. Borras and Franco 2010), official reports of international organizations tend to attribute such failures to the institutional difficulties and governance problems encountered by foreign investors in the host countries (von Braun and Meinzen-Dick 2009; Cotula et al. 2009).

The objective of this article is to analyze whether, and under what policy settings, the interest of both parties to maximize the project value may lead to an outcome that also maximizes their respective payoff from the investment project.

The article aims to contribute to the discussion surrounding such large-scale land deals by developing a dynamic stochastic programming model. This model includes many aspects of the typical bargaining situation between host country and investor. In fact, as for many large-scale land deals in Africa, the model involves a foreign investor willing to invest in land development and corresponding agricultural activities, and a host country land owner potentially willing to provide access to land on the basis of a long-term leasing contract. For simplicity, we use the terms “host country” and “host country government” synonymously with “host country land owner”, even though our model could also reflect empirical cases in which land owner and government represent two distinct parties. In any case, we assume that the ‘land owner’ is that agent who has effective control over the land and therefore

\(^1\)Large acquisitions in sub-Saharan Africa concern projects with more than 1000 ha but include examples of a 452,500 ha biofuel project in Madagascar, a 150,000 ha livestock project in Ethiopia, and a 100,000 ha irrigation project in Mali.
enters the negotiations with the investor. Access to land is costly, however. The foreign investor must pay a fixed rent to the host country which is negotiated by the parties. In addition, taxes may be levied on the investor’s profits. Once the lease contract is signed, the investor has full control of the land development process. However, the investor must take land development decisions by accounting for 1) uncertainty about global market conditions for agricultural products, 2) the risk of adverse natural or political events in the host country, and 3) a fixed sunk capital cost for the activation of land as input for the production of agricultural goods. We solve the underlying land development problem by determining the optimal time trajectory for land conversion to agriculture and the value of the land development project.

Once the value of a hypothetical land development project has been assessed, we proceed by determining the optimal rental payment that the host country should require. This is done by considering two possible settings for the actual investment contract negotiations, namely a cooperative and a non-cooperative setting.

The article makes the following contributions: First, we analyze the typical situation of many large-scale farmland investments in Africa through a theoretical model that reflects the economic incentives for investor and host country government under risk and uncertainty. Insights from this analysis can debunk some of the myths around large-scale land deals by identifying the role that economic incentives play for land development under uncertain conditions, and this may also inform and guide a concerned public in African countries when critically assessing the negotiation strategies of their governments.

Second, it is possible to use the modeling framework presented for numerical assessments of the actual value of a specific investment project. Empirical specifications in this context necessarily involve a lot of incertitude and therefore we use stochastic Monte Carlo-type simulations with plausible ranges of key parameters rather than purely deterministic simulations. We demonstrate this approach by calibrating our model to a large-scale land contract signed between the government of Ethiopia and the Indian "Whitefield Cotton" company for an investment concerning 10,000 hectares of cotton in the Ethiopian district of Dasenech Nebremus. We use response surface design to evaluate the sensitivity of the model with respect to exogenous parameters that may vary in certain plausible ranges, and we infer the degree of bargaining power the respective parties were able to exert during negotiation of this contract. Any indication of very unequal bargaining power may provide a concerned public with valuable information about the way its government trades away domestic land resources.

Third, we contribute to the literature on FDI under uncertainty through introduction of a novel way to model the pricing of the investment option, and we introduce econometric response surface estimation as a convenient way to assess the response behavior of the dynamic stochastic programming model.

The next section reviews the related literature and introduces our model. We then explain how the project value is determined and what role the timing of the land conversion process plays in this respect. In Section 3 we derive implications for the optimal rental payment and the optimal profit taxation under alternative bargaining situations. In the last two section we introduce our empirical implementation, and discuss and draw conclusions on the implications of our analysis.
2 The Model

The model developed in this article should be viewed in the context of two specific strands of literature. The first strand includes scholars investigating the inter-temporal allocation of land under uncertainty (e.g. Capozza and Li 1994; Bulte et al. 2002; Schatzki 2003; Isik and Yang 2004; Song, Zhao, and Swinton 2011), while the second refers to research focusing on irreversible FDI under uncertainty (Pennings 2005; Yu, Chang, and Fan, 2007; Sarkar 2012; Di Corato 2013). A unifying aspect for these two lines of research is represented by the use of the option theory as the approach for the economic analysis (see Dixit and Pindyck 1994 for a complete treatment of the theory of optimal investment under uncertainty and irreversibility). This approach is based on the following considerations: Irreversible decisions under uncertainty may be later regretted, and therefore postponing such decisions in order to gather information about future net benefits may generate value, the so-called option value. Hence, for instance, when considering investment decisions, the investment should be undertaken as soon as future prospects are promising enough to cover the full cost of the underlying decision, i.e., sunk investment costs plus option value.

In the first strand of the literature, Capozza and Li (1994) pioneer the application of option theory to land development by studying the problem of land conversion in the presence of an option to adjust the land-capital ratio. In Bulte et al. (2002), a social planner must determine if and when forestland should be developed. The optimal conversion plan must be set taking into account the trade-off between potential profits accruing from agriculture and the uncertain benefits, in terms of environmental goods and services, provided by the forest if conserved. Schatzki (2003) and Isik and Yang (2004) investigate the decision to set aside land under the Conservation Reserve Program. Schatzki (2003) shows that hysteresis may characterize the decision of switching to permanent land uses and concludes that not accounting for it may influence the outcome of conservation policies. Isik and Yang (2004) study the decision to participate in the Conservation Reserve Program under uncertainty about agricultural profits and set-aside payments. They show that the probability of program participation may be importantly affected by option value considerations. Song, Zhao, and Swinton (2011) adopt a standard entry-exit model à la Dixit (1989) in order to study how land should be allocated when two mutually exclusive destinations are available, namely cultivation of food crops and energy crops. By allowing for the option to switch back and forth between food and energy crops, they show how regime reversibility impacts on land allocation decisions.

In the second line of research, Pennings (2005) studies the decision of a foreign monopolist who may either export or set up productive capacity in a host country in order to feed the local market. The decision must be taken under uncertainty about future demand and irreversible foreign investment. Domestic benefits are maximized by strongly subsidizing the initial foreign investment and by absorbing, through taxation, the benefits exceeding the gains from exporting. In addition, taking a global welfare perspective, in the absence of subsidies domestic welfare maximization induces underinvestment. Yu, Chang, and Fan (2007) consider the impact on the timing of FDI associated with an entry cost subsidy and a tax rate reduction. The two instruments are considered separately and then compared. Those authors conclude that in order to foster FDI, the host government should favor an entry cost subsidy, since this tool is less costly and more effective. In contrast, Sarkar (2012)
shows that it may be optimal for the government to combine both investment subsidy and tax reductions. This result is obtained by letting the government discount future net benefits at a rate different from that used by the foreign investor. In Di Corato (2013), a foreign investor contemplates the opportunity of investing in a project for the extraction of a natural resource in a developing country. FDI is compensated by a share of the profits accruing from the mining project. Residual profits are used to reward the host country for providing access to the resource.

The model presented in this article shares several features with this approach. In our model, for instance, land development is incremental as in Bulte et al. (2002). In addition, the conclusions regarding the role played by taxation on the investor initiative are in line with the literature on FDI. Furthermore, as in Penning (2005) and Di Corato (2013), government and foreign investor share the investment project’s value on the basis of an agreement reached in a cooperative frame.

However, the model that we present departs from the previous literature by considering the "entry fee" (or its reduction) differently. In most previous studies, the entry fee is usually represented by the initial investment cost and the analysis mainly focuses on the impact that a subsidy lowering this cost has on the speed at which foreign firms invest in the host country.

In contrast, our model assumes that 1) no subsidies are paid and 2) the entry fee is represented by the rental payment due to the host country. The foreign investor, by signing the lease contract, commits to this payment. The rental payment is not conditional on land development and must be paid in any case. Hence, in technical parlance, the initial transfer between the two parties does not affect the strike price of the option to develop land. It follows that, by setting the rental payment, the two parties are implicitly pricing the option to invest in land development. It is worth highlighting how this option is priced. First, the option is priced in a cooperative frame. This is important since it accounts for 1) the fact that both parties have as their final goal the initiation of the development project and 2) the difference in the bargaining power of the two parties. Second, the option price takes into account future tax revenue and the effect that taxation has on the exercise of the option to develop. Third, the option is priced using an interest rate, which is adjusted in order to account for host-country specific risk.

2.1 Basic set-up

Consider a risk-neutral host country (hereafter, HC) where a certain surface, \( L \), of land still in pristine condition, e.g. savannah, forestland, wetland, etc., is available. Assume that HC is financially constrained and cannot fund a project for the development of this land for the purpose of agricultural production, say marketable crops for food or bioenergy production. A risk-neutral foreign investor (hereafter, FI) is willing to invest in such a project if conveniently rewarded. Suppose that at a generic time period \( t \) the two parties can reach a bilateral agreement for the lease of \( L \) hectares of land. On the basis of this agreement, HC leases land to FI for a fixed and certain total rental payment, \( R \geq 0 \).\(^2\) FI then has the right

\(^2\)This amount may be thought of as the net present value (hereafter NPV) of a periodic rental payment, \( r \), per hectare paid over the entire contract duration. So, assuming that the contractual agreement has a term sufficiently long that can be approximated by infinity, we can set \( R = (r/\rho)L \) where \( \rho \) is the discount rate. This can be done at no loss in terms of generality for our results given the generally long duration of
to develop the land and devote it to agriculture. A corporate income tax, $s \in (0, 1)$, must be paid on each unit of profit accruing from the land once developed.

Denoting the hectares of land developed and under agriculture by $A_t$ and the extent of land still in its pristine state by $L_t$, at each $t \geq 0$ land is allocated as follows,

$$A_t + L_t = L, \text{ with } A_0 = 0$$

(1)

Assume that land under cultivation guarantees the following profit flow:\(^3\)

$$\pi(\theta_t, A_t) = \theta_t A_t^{1-\phi}/(1 - \phi)$$

(2)

where $0 < \phi < 1$ is a constant term representing the degree of decreasing returns to scale (DRTS) and $\theta_t$ is a random variable shifting profits, $\pi(\theta_t, A_t)$, over time. For simplicity, we assume that no benefit accrues to the landholder when land is undeveloped. This assumption comes at no cost in terms of the generality of our results. Note in fact that our model may be easily adjusted in order to account for a potential source of income from undeveloped land (e.g., carbon credits).

Let $\theta_t$ evolve according to the following diffusion:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dZ_t, \text{ with } \theta_0 = \theta$$

(3)

where $\mu$ and $\sigma$ are drift and volatility parameters and $dW_t$ is the increment of a Wiener process with $E[dZ_t] = 0$ and $E[dZ_t^2] = dt$.

Using (2) and (3), we can express profit dynamics as follows:

$$d\pi(\theta_t, A_t) = (\partial\pi(\theta_t, A_t)/\partial \theta_t)d\theta_t + (\partial\pi(\theta_t, A_t)/\partial A_t)dA_t$$

$$= [(\mu dt + \sigma dZ_t) + (1 - \phi)(dA_t/A_t)]\pi(\theta_t, A_t)$$

(3.1)

where the first term represents the marginal effect of changes in $\theta_t$ while the second captures the marginal effect due to additional land conversion.\(^4\)

Finally, we complete our set-up by including the following assumptions:

1. Land development is costly and irreversible. In particular, we assume that land development requires a sunk investment in capital costing $k$ per hectare.

2. Land development is undertaken in the presence of country-specific risk. In this respect, our definition of risk includes all socio-political factors (war, riots, crime, etc.) and natural events (as drought, floods, etc.) reducing the profitability of the land development project initiated by FI. We regulate the occurrence of such adverse events such projects in Africa.

\(^3\)Our profit function is consistent with a standard setting such as a price-taking farm whose production technologies show decreasing returns to scale (see Appendix). Note that it may also apply to the case of a monopolist using a constant returns to scale technology and facing a demand curve with $-1/\phi$ as constant elasticity and a multiplicative shocks $\theta_t$. In our model $\phi = 1/[c(\varepsilon - 1) + 1]$ where $c$ and $1 - c$ are the cost shares for each specific input factor of a Cobb-Douglas production function and $\varepsilon > 1$ indicates the degree of decreasing returns to scale, see Appendix A.1.

\(^4\)Note in fact that $\partial\pi(\theta_t, A_t)/\partial A_t = (1 - \phi)/\pi(\theta_t, A_t)/A_t = \theta_t A_t^{-\phi}$. 

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by a Poisson process (see e.g., Clarke 1997) with intensity $\lambda \in (0, \infty)$ and denote by $\omega \in (0, 1]$ the percentage of project value lost due to the adverse event. This means that at each generic $t$, for each $\$ of project’s value, a loss equal to $\omega$ may occur with probability $\lambda dt$.

### 2.2 Project value and optimal land conversion policy

In this section, we view FI as holding the option to develop land. We study the optimal land development policy to be followed once the contract is signed and determine the value attached to the land development project. As one can easily see, the opportunity of developing land does not depend on the rental payment once the contract has been signed, since $R$ must be paid irrespective of the development state of the land. However, the opportunity does depend on 1) the random fluctuating convenience of agriculture, and 2) the threat of adverse events that may partially or totally destroy the value of the development project.

Suppose that at the generic time period $t$ a surface $A_t \leq \bar{L}$ is developed while the remaining area, i.e., $L_t = \bar{L} - A_t$, is still undeveloped. Hence, assuming that $\pi(\theta_t, A_t)$ is such that the optimal strategy is to maintain the current land allocation, the value of the investment project for FI is given by the following Bellman equation:

$$
V^{FI}(\theta_t, A_t) = (1 - s)\pi(\theta_t, A_t)dt + (1 - \omega\lambda dt)\frac{E[V^{FI}(\theta_t + d\theta, A_t)]}{1 + \rho dt}
$$

where $\rho(\geq \mu)$ is the discount rate.\(^5\)

Solving Eq. (5), we show in the appendix that:

**Proposition 1** FI develops land ($dA_t > 0$) every time the process $\{\theta_t : t \geq 0\}$ reaches the barrier

$$
\theta^*(A_t) = \frac{\beta - 1}{1 - s}(\delta - \mu)A_t^\delta
$$

or, rearranged in terms of profit, whenever current profit, $\pi(A_t)$, reaches the critical threshold profit level

$$
\pi^*(A_t) = \frac{\beta - 1}{1 - s}(\delta - \mu)\frac{A_t}{1 - \phi}
$$

where $\delta = \rho + \omega\lambda$ and $\beta(> 1)$ is the positive root of the equation $\Phi(\beta) = (\sigma^2/2)\beta(\beta - 1) + \mu\beta - \delta = 0$.

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\(^5\)Note that $e^{-\lambda dt}$ is the probability that a drop in the project’s value, due to expropriation, does not occur over the next $dt$ whereas $(1 - e^{-\lambda dt})$ is the probability that a portion $\omega$ is seized. Hence, in expected terms, for each $\$ of project’s value we have:

$$
e^{-\lambda dt} \cdot 1 + (1 - e^{-\lambda dt})(1 - \omega) \\
\simeq (1 - \lambda dt) + \lambda dt(1 - \omega) = 1 - \omega \lambda dt
$$

\(^6\)Note that $\rho > \mu$ is needed in order to guarantee that the discounted stream of profits converges. In addition, note also that, in order to account for risk aversion, one may use CAPM and calculate a risk-adjusted discount rate.
Proof. See Appendix.

The critical profit threshold, \( \pi^*(A_t) \), is linearly increasing in \( A_t \). That is, the larger the surface under agriculture, the higher the agricultural profit inducing additional land conversion should be. This implies that the expected timing for the development of the next marginal unit of land increases as land is developed. This makes intuitive sense considering that agricultural profits are concave in the degree of (decreasing) returns to scale. Note also that \( \partial \pi^*(A_t)/\partial \phi > 0 \). That is, the lower the degree of (decreasing) returns to scale (\( \phi \to 0 \)), the earlier land development occurs in expected terms. As can be expected, the critical threshold in (6.2) is also increasing in \( s \), which means that the higher the corporate tax rate, \( s \), the slower the land development. A further element deterring conversion is represented by higher capital investment costs \( k \); since the critical profit threshold is rising in higher fixed costs that are associated with the investment, i.e., \( \partial \pi^*(A_t)/\partial k > 0 \).

Let us now discuss the corresponding effect of a change in the remaining parameters \( \sigma \), \( \mu \), and \( \delta \). In order to do so, we rearrange (6.2) as follows:

\[
(1 - s)\theta^*(A_t)A_t^{-\phi} = [(\sigma^2/2)\beta + \delta]k 
\]

(6.3)

The LHS of (6.3) shows the marginal net benefit from developing a hectare of land, while the RHS shows the corresponding marginal cost. Note that the marginal cost is represented by the rental cost of a unit of capital, \( \delta \), adjusted by adding the term, \( (\sigma^2/2)\beta \), to account for market uncertainty. The impact of expected profit growth, \( \mu \), and profit volatility, \( \sigma \), on the critical conversion threshold is in line with findings in the real options literature. In particular, we note that as future agricultural net returns become more volatile, the critical conversion threshold rises and land development is postponed, i.e., \( \partial \theta^*(A_t)/\partial \sigma^2 > 0 \). In contrast, the higher the expected profit growth rate, \( \mu \), the lower the critical threshold that triggers additional land conversion, i.e., \( \partial \theta^*(A_t)/\partial \mu < 0 \). Note also that \( \lim_{\sigma \to 0}[(\sigma^2/2)\beta + \delta] = \delta \), i.e., as market uncertainty vanishes, land conversion occurs whenever marginal profits cover the rental cost of capital, \( \delta k \). Finally, a higher discount induces delayed land conversion, i.e., \( \partial \theta^*(A_t)/\partial \delta > 0 \). This result deserves further comment on each specific component of the discount rate \( \delta \). A higher \( \rho \) implies a higher rental cost for the capital, \( \rho k \), while a higher \( \lambda \) and \( \omega \) imply a more likely loss in the project value and a larger loss due to adverse events, respectively. It is immediately apparent that all these considerations lead to a more prudent land development strategy for FI.

Now, let us determine the values of the land development project for both parties. In the Appendix we show that:

**Proposition 2** For any land allocation \( A \leq T \), the value functions of FI and HC are given, respectively, by:

\[
V^{FI}(\theta, A) = \frac{k}{\beta - 1} \int_A^T \left( \frac{\theta}{\theta^*(\xi)} \right)^\beta d\xi + (1 - s)\frac{\pi(\theta, A)}{\delta - \mu} 
\]

(7.1)

and

\[
V^{HC}(\theta, A) = \frac{\beta}{\beta - 1} \frac{s}{1 - s} k \int_A^T \left( \frac{\theta}{\theta^*(\xi)} \right)^\beta d\xi + s\frac{\pi(\theta, A)}{\delta - \mu} 
\]

(7.2)

\(^7\)Note that \( \partial \beta/\partial \delta > 0 \), \( \partial \beta/\partial \mu < 0 \) and \( \partial \beta/\partial \sigma^2 < 0 \). See section A.3 in the appendix.
where \( \theta^*(\xi) = \frac{\beta}{\beta - 1} \frac{k}{1-s} (\delta - \mu) \xi^\phi \).

**Proof.** See Appendix. ■

In (7.1) the first term represents the value of the option to develop the surface \( L - A \geq 0 \), while the second term represents the expected present value of the project if the current land allocation \( A \leq L \) is kept forever. A similar interpretation can be given to the terms in (7.2). However, it is worth highlighting that the main difference between the two parties is that only FI has control over the development process. In fact, while FI, on the basis of the contractual agreement, keeps under its own control the land development process, \( dA \), HC may attach to the surface potentially developable only the expected value of the potential earnings which can be obtained through the taxation of the profits. Note that the term \( (\theta/\theta^*(\xi))^\beta \) is a stochastic discount factor which discounts future potential earnings accruing from the future development of the surface \( L - A \).

Finally, let us conclude this section by studying the factors determining the dynamics of land development in the long run. Using (6.1) and denoting the long-run average growth rate of land development by \( E[d\ln A]/dt \), we can prove that:

**Proposition 3** For any land allocation \( A \leq L \) the expected long-run growth rate of land development is given by:

\[
\frac{1}{dt} E[d\ln A] \simeq \begin{cases} 
(\mu - \sigma^2/2)/\phi & \text{for } \mu > \sigma^2/2 \\
0 & \text{for } \mu \leq \sigma^2/2
\end{cases}
\]

**Proof.** See Appendix. ■

It is worth highlighting here that expected profit growth must be strong enough to have a positive long-run average development rate, i.e., \( \mu > \sigma^2/2 \). Otherwise, due to the deterring effect of profit volatility, the rate is null. In line with these considerations, note that the long-run development rate is increasing in \( \mu \) and decreasing \( \sigma^2 \). Note also that, as one could expect, land development speed is decreasing in the degree of (decreasing) returns to scale, \( \phi \). Finally, from (15), an immediate consideration is that the expected land development rate is independent of the rate of corporate taxes, \( s \). Note in fact that the change in the optimal developed land surface is random because \( \theta \) evolves randomly. In contrast, the corporate tax is constant over time and thus it does not affect the long-run optimal development path. This in turn implies that as concerns long-run dynamics, HC’s fiscal policy has a neutral impact.

3 **The optimal rental payment**

The value of the project for both parties depends on the timing of land development. This is in turn dictated by the optimal development trigger, \( \pi^*(A) \), which, as highlighted above, is set by the party having control over the development process, i.e., FI. However, it is important to stress the role that other two crucial aspects have on the development process: 1) the rental payment, \( R \), to be paid by FI in order to have access to the exploitation of land surface \( L \), and 2) the tax rate, \( s \), set by HC on FI’s profits.

First, concerning \( R \), as one can immediately see, the start of the land development project is conditional on the two parties reaching agreement on the terms of the contract. Once such
agreement is reached, the contract is signed and the project can start. In this respect, setting $R$ is crucial. The rental payment must in fact be set in order to satisfy a basic set of participation constraints. That is, at $t = \tilde{t}$ where $\tilde{t}$ is the time at which the contract agreement is reached, the following conditions must hold:

$$
\begin{align*}
W^{FI}(\theta, R) &= V^{FI}(\theta) - R \geq 0 \\
W^{HC}(\theta, R) &= V^{HC}(\theta) + R \geq 0
\end{align*}
$$

(9.1-9.2)

where $\theta_{\tilde{t}} = \tilde{\theta}$.

Note that by [9.1-9.2] we are simply requiring that for both parties the expected value attached to the project is non-negative.

Second, note that:

**Proposition 4** At $t = \tilde{t}$, given a certain tax rate, $s$, an agreement between FI and HC over $R$ always entails the immediate development of the following land surface:

$$
\tilde{A} = \left\{ \frac{(1 - s)}{[(\sigma^2/2)^{\beta} + \delta]k} \tilde{\theta} \right\}^{1/\phi}
$$

(9.3)

The interpretation of (9.3) is straightforward. By (6.1), the level of $\theta$ at $t = \tilde{t}$ is high enough to support some land development, $\tilde{A}$. Note in fact that for any $\theta > 0$, land should be developed up to the amount at which the control $\theta^*$ stops the conversion process, i.e., $\theta^* \geq \theta$. The magnitude of this amount of land depends, via (6.1), on, among other parameters, As shown by (9.3), the relationship between $\tilde{A}$ and $s$ is negative, i.e., $\partial\tilde{A}/\partial s < 0$. That is, the higher the corporate tax rate, the lower the land area that FI finds profitable to develop in the first place. Hence, in technical parlance, by viewing $\tilde{L}$ as a set of options to develop, HC is splitting it into a subset composed of $\tilde{A}$ options "in-the-money" and a subset composed of $\tilde{L} - \tilde{A}$ "out-of-the money". The first group of options must be exercised as soon as the contract is signed, while the remainder may be exercised later using (6.1). Changing perspective, HC, by fixing $s$, is implicitly 1) setting short-run goals concerning the development of the land surface $\tilde{L}$, and 2) setting the amount of land over which FI would exercise control. These considerations seem in line with what is observed in the reality, where HC are often willing to concede tax holidays to foreign investors.\(^8\)

### 3.1 Cooperative and non-cooperative solutions

Meeting the goal of fast and vast land development would, however, come at a cost in terms of tax revenues. As pointed out, this would in fact require a lower tax rate on profits accruing to FI. This loss may be balanced (or reduced) by setting a proper rental payment, $R$. Clearly, as stressed above, this is not a trivial issue, since $R$ must be set such that FI’s initiative is not deterred. This choice is the subject of this section, where, given a certain taxation regime, we study the definition of an optimal rental payment in two possible settings, namely a cooperative and non-cooperative setting.

\(^8\)In this respect, note that, depending on $\tilde{\theta}$ and $\tilde{L}$, it may be feasible to set $s$ such that $\tilde{A} = \tilde{L}$. 

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Cooperative solution - Assume that HC and FI are engaged in a cooperative cake-splitting game where 1) both parties are neutral to the risk of internal conflicts and 2) have bargaining power, $\psi$ and $1-\psi$ with $\psi \in (0, 1)$, to each of them, respectively. As is well known, we can solve the underlying game by applying the Nash bargaining solution concept (Nash 1950; Harsany 1977).

A feasible Nash bargaining solution, $R_1^* \geq 0$ solves the following maximization problem:\[\max_{R_1 \geq 0} \Omega_1 = \psi \ln[W^{FI}(\tilde{\theta}, R_1)] + (1 - \psi) \ln[W^{HC}(\tilde{\theta}, R_1)]\] (10)

In the Appendix, we show that:

Proposition 5 At $t = \tilde{t}$, when FI and HC jointly decide upon the optimal rental payment, $R_1^*$, in a Nash-bargaining frame, then the optimal payment is set as follows \[R_1^* = (1 - \psi)V^{FI}(\tilde{\theta}) - \psi V^{HC}(\tilde{\theta})\] (10.1)

The interpretation is straightforward. The optimal payment is set on the basis of the relative strength of the two parties. Note in fact that, as expected, $R_1^*$ is increasing in HC’s bargaining power and decreasing in FI’s strength. Note also that given a certain power allocation $(\psi, 1-\psi)$, a lower $R_1^*$ is paid as the expected value of tax revenues, $V^{HC}(\tilde{\theta})$, increases. Consistently, a higher payment is due when a higher expected value is attached to FI’s net revenues, $V^{FI}(\tilde{\theta})$. Substituting (10.1) into (9.1-9.2) yields:

\[W^{FI}(\tilde{\theta}, R_1^*) = \psi V(\tilde{\theta}), \quad W^{HC}(\tilde{\theta}, R_1^*) = (1 - \psi)V(\tilde{\theta})\] (10.2-10.3)

where \(V(\tilde{\theta}) = V^{FI}(\tilde{\theta}) + V^{HC}(\tilde{\theta})\).

That is, the two parties share the total value at stake, $V(\tilde{\theta})$, in shares which are given by their respective bargaining powers. It is worth highlighting that, by bargaining, the two parties are basically setting an optimal risk-sharing contract. Note in fact that HC’s revenues include a certain component represented by $R_1^*$ and a volatile component represented by tax revenues, $V^{HC}(\tilde{\theta})$. In this respect, one may also view the tax rate $s$ as HC’s share of FI’s volatile profits.

In addition, as can be easily shown, a sufficient condition for $dR_1^*/ds < 0$ is:\[s < \frac{1}{1 + \psi(\beta - 1)}\] (10.4)

This means that within this specific range of values for $s$, a lower rental payment should be paid if HC sets a higher tax rate. Note that, by lowering the rental payment, HC may be seen

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9Note that our frame may easily apply to the analysis of a Nash bargaining game where the two parties are characterized in terms of risk aversion. It would in fact suffice to set the Nash product equal to $(W^{FI})^p(W^{HC})^q$, where $0 < p \leq 1$ and $0 < q \leq 1$, measure the level of risk aversion for each of the parties involved.

10The objective function (10) is defined on the net gains from bargaining. Disagreement pay-offs are null, since without agreement the land development project is not activated.

11In the interval $s > \frac{1}{1 + \psi(\beta - 1)}$, the sign of the derivative depends on the amount of land developed as soon as the contract is signed, i.e., $A$. 

11
as implicitly subsidizing FI. This is done in order to provide better contractual conditions and encourage the signing of the lease contract. In fact, without an agreement land would not be developed. This initial transfer will be repaid later by higher taxes. It is important to stress that by doing this, HC assumes a more risky position. In fact, he will share with FI the uncertainty surrounding future profits and consequently the tax revenue. From FI’s perspective, sharing risks and paying a lower rental payment is beneficial and covers the cost of facing higher tax rates. In this respect, note in fact that as $\sigma \to \infty$ ($\beta \to 1$) condition (10.4) holds for any $0 \leq s < 1$. Basically, as uncertainty soars up, the advantage attached to risk sharing increases. In contrast, the impact of higher taxes is lower due to 1) land development occurring only when profits are very high and 2) the effect of discounting. In fact, the higher the uncertainty, the slower the land development process.

**Non-cooperative solution** - Assume that HC and FI are engaged in a two-stage game. In the first stage, HC sets the rental payment maximizing local benefits, i.e., $W^{HC}(\tilde{\theta}, R)$. If profitable, FI signs the leasing contract and contemplates land development in the second stage.

A feasible solution, $R^*_2 \geq 0$ solves the following maximization problem:

$$\max_{R_2 \geq 0} \Omega_2 = W^{HC}(\tilde{\theta}, R_2), \text{ s.t. } W^{FI}(\tilde{\theta}, R_2) \geq 0$$  \hspace{1cm} (11)

It is easy to show that:  

**Proposition 6** At $t = \tilde{t}$, when HC decide upon the optimal rental payment, $R^*_2$, in a non-cooperative setting, then the optimal payment is set as:

$$R^*_2 = V^{FI}(\tilde{\theta})$$  \hspace{1cm} (11.1)

**Proof.** Straightforward from Proposition 5. ■

The interpretation is as follows: The optimal payment to be set by HC is equal to the expected value attached by FI to the development project. In other words, HC can implicitly fully expropriate the benefits accruing from the initiative that FI may undertake. Note in fact that:

$$W^{FI}(\tilde{\theta}, R^*_2) = 0, \quad W^{HC}(\tilde{\theta}, R^*_2) = V(\tilde{\theta})$$  \hspace{1cm} (11.2-11.3)

However, it must be said that a non-cooperative outcome is extremely unlikely. In the real world, HC must compete with other countries in order to attract FDI. Thus, competition for capital leads, by increasing the bargaining power of foreign investors, to the development of negotiations where the two parties must play cooperatively.

### 3.2 Corporate taxation of profits

Let us conclude this section by checking the impact that corporate taxation has on the final payoffs.

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12 Note that the problem in (11) corresponds to the problem in (10) for the case where FI has no bargaining power, i.e., $\psi \to 0$. 

12
Cooperative solution - By taking the derivative of $V(\theta)$ with respect to $s$ we obtain:

$$\frac{\partial V(\theta)}{\partial s} = \frac{\partial V^{FI}(\theta)}{\partial s} + \frac{\partial V^{HC}(\theta)}{\partial s}$$

$$\quad = -\beta \frac{s}{(1-s)^2} k \int_{A}^{T} (\frac{\theta}{\theta'(A)})^\beta dA + \frac{\partial \widetilde{A}}{\partial s} k < 0 \quad (12)$$

This in turn implies that

$$\frac{\partial W^{FI}(\theta, R_1^*)}{\partial s} = \psi \frac{\partial V(\theta)}{\partial s} < 0, \quad \frac{\partial W^{HC}(\theta, R_1^*)}{\partial s} = (1-\psi) \frac{\partial V(\theta)}{\partial s} < 0 \quad (12.1-12.2)$$

That is, a complete tax exemption would maximize both total value and each party’s payoff. The effect of no taxation would be two-fold in that, firstly, taxes would not distort the definition of land development timing, and, secondly, the value of the land project would be maximized.

By setting $s = 0$ we would have:

$$W^{HC}(\theta, R_1^*) = R_1^* = (1-\psi)V(\theta) = (1-\psi)V^{FI}(\theta) \quad (13.1)$$

$$W^{FI}(\theta, R_1^*) = \psi V(\theta) = \psi V^{FI}(\theta) \quad (13.2)$$

That is, each party receives a portion of the value generated by the foreign initiative, $V^{FI}(\theta)$, which is proportional to its own bargaining power. Note, however, that in this case the agreement will not entail any risk-sharing between the two parties, since once HC has cashed the payment $R_1^*$ the whole uncertainty characterizing the project will only affect FI’s net benefits.

Non-cooperative solution - Since $\partial V(\theta)/\partial s < 0$, HC would be better off setting a zero tax rate. In fact, this would entail the opportunity of fully absorbing the value generated by a more valuable project.

Comparison between cooperative and non-cooperative settings shows that the host country would always prefer to lead the game and impose its will on the foreign investor. However, given that in reality host countries may compete with each other in order to attract FDI, the bargaining power of the foreign investor increases and the two parties may reach agreement only by negotiating in a cooperative framework.

4 Empirical implementation

Calibrating the modeling framework to an actual investment project is straightforward as long as some core data about size, duration, location-specific variable costs of the planned agricultural production and fixed costs of establishing the farm and development of the land are known. For new projects, our framework can serve as a rule-of-thumb planning tool that allows both investors and host country negotiators to compute the expected value of an investment by explicitly taking market uncertainty and risk in the host country into account.

Furthermore, contracts that already exist can be assessed by our framework if negotiated annual rental payments and profit tax rates are known. With this information, one can for
instance assess the host country’s share of the expected total value of the investment, or one can determine the distribution of bargaining power between host country and foreign investor. Both can provide transparency and can support local interest groups and the concerned public in the host country who may not be directly involved in the negotiations.

We therefore demonstrate the applicability of our model by analyzing an existing large-scale investment project in Ethiopia:

As a visible outcome of growing critical public awareness about large-scale land deals, individual governments have yielded to public pressure in a few instances and now publish the contractual details of some recently signed large-scale land deals (Ethiopian Land Portal 2012). We demonstrate that our model closely reflects the conditions stated in some such publicly available contracts. Specifically, we calibrate the model to a land lease contract that has been signed between the Government of Ethiopia and the Indian company "Whitefield Cotton" (Ethiopian Land Portal 2012). The contract covers 10,000 hectares for cotton production. The agreement between Whitefield Cotton and the Ethiopian government was signed on August 1, 2010 and the contract duration is 25 years. According to the contract, the annual rent amounts to 158 Birr/ha. Furthermore, the contract requires 25% of the land to be developed in year 1 and 100% by year 4. Both parties can terminate the contract within 6 months unless grand majeure forces (e.g. draught, civil conflict, etc.) are the reason. However, the contract does not contain any information on potential refunding of the investor in the event of grand majeure forces. This most likely means that the investor bears the full risk of such events.

The total net present value for Ethiopia, after taking the negotiated 3-year grace period into account, amounts to 15426.8 thousand Birr (TBirr) for the whole farm, which is equivalent to 2.9 TBirr/ha (own computations based on Ethiopian Land Portal 2012).

With this information, we can calibrate the contractual part of the model. Furthermore, in order to determine the profitability of the cotton production process, we use output and input price data for cotton production around the time when the contract was signed. Table 1 presents all parameters that are exogenous to the model, some of which can only be considered within plausible ranges due to incertitude or lack of precise information available. Therefore, no attempt is made to present individual parameters at an overly ambitious level of accuracy, and instead the incertitude attached to these parameters is explicitly incorporated into the numerical implementation of the model by allowing them to vary stochastically within the specified ranges according to uniform distributions. Some of the specified ranges (e.g. corporate tax rate) are wider than the official Ethiopian corporate tax rate of 35% would require. This is in order to ensure that we assess the response behavior of the model well around relevant values and within plausible parameter ranges, rather than too narrowly for individual values that may need to be adjusted in reality. For instance, the investor may perceive certain transaction costs (e.g. inefficient bureaucracy, corruption) related to the export of agricultural products and the import of production inputs as effectively additional taxation that would come on top of the corporate income taxes (Hall 2011b).

The cotton price in Table 1 is set according to the world market price (from www.cotlook.com) around the time when the contract was negotiated (assumed 2010), re-expressed per hectare under the assumption of the standard cotton yield of 3 metric tons/hectare that can be expected for such a project in Ethiopia (Ethiopian Embassy 2013). The specific cotton price for the starting period of a project simulation is drawn from a uniform distribution and
then simulated to move according to a Geometric Brownian Motion with drift and volatility parameters, as specified in Table 1. The latter two parameters are also allowed to vary over slightly wider ranges than can e.g. be observed for cotton over the past decade, but are in line with observed volatility estimates for financial instruments based on agricultural raw material indices since 2008 (OECD 2011). This decision about a rather wide parameter range for drift and volatility is made in order to reflect the often cited claim that rising volatility has triggered the recent interest in large-scale land acquisitions. In other words, we anticipate that investors may focus disproportionately on recent short-term time windows with higher than long-term average price volatility, and that price volatility may be measured not only according to real commodity prices, but also according to price movements for financial derivatives that are based on agricultural raw materials (OECD 2011).

Table 1: Exogenous Parameters Used for Simulation of the Whitefield Cotton Project

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value or Range</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>Project size</td>
<td>10,000 ha</td>
<td>from Whitefield’s contract</td>
</tr>
<tr>
<td>$t$</td>
<td>Duration</td>
<td>25 years</td>
<td>from Whitefield’s contract</td>
</tr>
<tr>
<td>$k$</td>
<td>Cost of developing 1ha</td>
<td>13.48 TBirr</td>
<td>plowing 2.1TBirr/ha and fixed cost to set up farm</td>
</tr>
<tr>
<td>$w$</td>
<td>Total Average Cost / ha</td>
<td>6.472 TBirr</td>
<td>for an assumed yield of 3000kg/ha</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>cotton price volatility</td>
<td>[0.05;0.5]</td>
<td>randomly drawn</td>
</tr>
<tr>
<td>$\mu$</td>
<td>cotton price drift</td>
<td>[0.005;0.04]</td>
<td>randomly drawn</td>
</tr>
<tr>
<td>$p_t$</td>
<td>starting price cotton</td>
<td>[11,14]</td>
<td>Average world cotton price (2010 in TBirr/ha)</td>
</tr>
<tr>
<td>$t$</td>
<td>Degree of DRTS</td>
<td>[2.00]</td>
<td>higher $t \rightarrow$ CRTS</td>
</tr>
<tr>
<td>$c$</td>
<td>Cobb-Douglas</td>
<td>0.25</td>
<td>Factor elasticity for non-land inputs</td>
</tr>
<tr>
<td>$\rho$</td>
<td>risk-free interest rate</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>loss (probability)</td>
<td>[0.04;0.08]</td>
<td>Poisson process; lower bound: one event in 25 years</td>
</tr>
<tr>
<td>$\omega$</td>
<td>loss (share)</td>
<td>[0.5;1]</td>
<td>share of investment lost due to political event</td>
</tr>
<tr>
<td>$s$</td>
<td>Corporate income tax</td>
<td>[0.0;0.5]</td>
<td>Ethiopian tax office</td>
</tr>
</tbody>
</table>

For potential agricultural investors, the Ethiopian government provides illustrative standard gross margin calculations for the production of the most common cash crops. These standard gross margins refer to typical large-scale investment projects and the data are distributed through the diplomatic body of various foreign Ethiopian embassies (Ethiopian Embassy 2013). Even though these gross margin computations may in some instances appear simplistic, we assume that they provide a fundamentally realistic approximation of the ex ante cost estimations that investors make about a prospective cotton project in reality.

For the purpose of calibration, these cost figures are re-expressed in terms of per hectare cost (Table 1). For instance, the cost of developing 1 ha is estimated as the cost of surveying, clearing and leveling farm land and main canal drainage, plus building access through farm roads including hydraulic structure constructions. This total cost is estimated to be about Birr 4.2 million for a hypothetical 2000 ha farm, which is 2.1 Tbirr/ha (Ethiopian Embassy 2013).

The specification of socio-political or natural risk of losses that may occur seldom, but can nevertheless pose a major threat to the investment, is difficult. One option would be to consider one of the various country-specific risk indicators provided by the World Bank and
other organizations. This information could be combined with location-specific data about the past occurrence of extreme weather events such as droughts or floods. However, the sometimes dynamically changing socio-economic and political conditions in some African countries suggest that historical data about these risks could be a poor predictor of the future. Therefore, investors may just have to form an educated guess about the magnitude of these risk sources.

As Table 1 shows, here we allow both the probability of a catastrophic event and the share of the investment lost due to this event to vary over a relatively wide range in order to assess the sensitivity of the model with respect to such shocks, and in order to reflect the high degree of uncertainty that has to be attributed to these risk sources.

Furthermore, the model imposes for cotton production a Cobb-Douglas technology with DRTS. Production inputs are land and all other inputs required to farm this land. The approximate factor elasticities are obtained by computing the share of variable cost of farming 1 ha of cotton in total cost per hectare. The degree of DRTS is captured by a separate parameter that needs to be specified exogenously in order to allow the factor elasticity on land to be derived as a composite to the factor elasticity for all other inputs. As the DRTS parameter approaches infinity, the production technology approaches the behavior of constant returns to scale.

4.1 Simulation Experiment 1: Response Surface

Below we illustrate the effect of each of the exogenously chosen parameters on the project value to the foreign investor at the moment of signing the investment deal. For this purpose, 500 investment projects were generated under parameter settings that were simultaneously and randomly chosen from the ranges specified in Table 1 ("Monte Carlo simulations"). Based on these data, an econometric response surface is estimated. Specifically, the econometric response surface is an Ordinary Least Squares (OLS) regression model that takes the following form:

\[ y_i = \alpha_0 + \alpha'X_i + \epsilon_i \]  

where \( X_i \) is a vector containing the elements \( \sigma_i, \mu_i, p_i, \lambda_i, \omega_i, s_i \) for every investment project \( i = 1, \ldots, 500 \) and \( \epsilon_i \) is a term capturing random disturbances that are assumed to follow a standard normal distribution. The coefficients \( \alpha_0 \) and \( \alpha \) represent, respectively, a constant term and a vector of regression coefficients to be estimated on the elements of \( X_i \).

The dependent variable in this regression model is expressed as the logarithm of the project value to FI. Furthermore, we opted to approximate the nonlinear functional relationships within the model by expressing the explanatory variables in \( X_i \) as their logarithmic values. Additionally, we found that the number of hectares that the investor is willing to develop immediately after signing the contract (\( \bar{A} \) in equ. 9.3) is a very good predictor for the dependent variable; however, including this predictor as a regressor in the response surface is not possible because of its endogenous relationship with the dependent variable.

For the case of the Whitefield Cotton contract, Figure 1 illustrates the occurrence of initial

\footnote{We are aware of factor elasticity estimates for land in the literature. However, all such estimates that we know of refer to conditions in developed countries and therefore we opted not to apply such estimates here.}
land development $\tilde{A}$ for $n=500$ simulated projects, given the parameter settings specified in Table 1: In more than 300 out of 500 cases, initial land development, $\tilde{A}$, covers between 9000 ha and the maximum of 10,000 ha. In about 150 out of the 500 cases, $\tilde{A}$ falls into the category of up to 1000 ha. Interestingly, the results in Figure 1 also show that initial activation of land takes in less than 10% of these 500 simulated project cases values between 1000 and 9000 ha.

Figure 1: Simulated values of the land immediately developed after signing the contract ($\tilde{A}$)

Figure 1 indicates that initial land development according to our model takes the overall characteristic of an 'everything or nothing' strategy. The government of Ethiopia may therefore need to reconsider its current practice of contractually requiring initial land development during the first four years (Ethiopian Land Portal 2012). Fixing the land development path in this artificial way may address the Ethiopian desire to avoid projects under which foreign investors acquire land without actually getting any development started. However, our model suggests that the economic driving forces of land development on the side of the investor are likely to be very strong, making land development either profitable or not, and a contract that tries to regulate this may interfere severely with the investor’s perceived risk situation. In other words, if the investor finds the project convenient overall, land development will in most cases be conducted as soon as possible. However, there is also the possibility that the investor initially does not find land development on a large scale profitable. According to the model, such a situation indicates that the combination of uncertainties at the time after
signing suggests holding the option to develop the land later instead.

Table 2 shows econometric response surface estimates based on the Ordinary Least Squares Estimator and the experimental set-up for the exogenous model parameters according to Table 1.

| Estimate | Std. Error | Pr(>|t|) | Estimate | Std. Error | Pr(>|t|) |
|----------|------------|---------|----------|------------|---------|
| (Intercept) | 0.3228 | 0.3008 | 0.2838 | -4.3111 | 1.2064 | 0.0004 |
| ln[μ] | 0.3673 | 0.0179 | <2e-16 | 0.3968 | 0.0721 | 5.89E-08 |
| ln[σ] | -0.0269 | 0.0082 | 0.0012 | 0.0846 | 0.0331 | 0.0108 |
| ln[β] | 4.218 | 0.1006 | <2e-16 | 2.5643 | 0.4035 | 4.74E-10 |
| ln[σ₁] | -1.4816 | 0.0489 | <2e-16 | -0.5844 | 0.1924 | 0.0025 |
| ln[σ₂] | -0.6388 | 0.0345 | <2e-16 | -0.4371 | 0.1384 | 0.0017 |
| ln[σ₃] | -0.5691 | 0.0116 | <2e-16 | -0.4336 | 0.0464 | <2e-16 |
| ln[σ₄] | 0.5039 | 0.0359 | <2e-16 | 0.5588 | 0.1441 | 0.0001 |

Table 2 contains results from two different response surface estimations, both using the same data and explanatory variables. The first set of columns in Table 2 refers, as described, to the log of the project value to the foreign investor at the moment of signing. Almost all regressors are significant at the 5% level or better, and the coefficient of determination suggests a satisfactory fit to the data. One advantage of the log-log transformation is that estimated regression coefficients can be directly interpreted as the corresponding partial elasticities, with a 1% change in the regressor inducing a corresponding percentage change in the dependent variable. The estimated coefficient of the intercept has to be interpreted as the log of the mean model response when all other regressors take zero values. A closer inspection of the estimated coefficients in Table 2 reveals that the partial elasticities of the market price of cotton, the drift of this market price and the DRTS technology parameter each have a positive effect on the project value from the viewpoint of the foreign investor. In line with expectations from the theoretical properties of the model, the estimated project value will ceteris paribus be higher if the natural conditions of the investment project allow for milder rather than stronger degrees of DRTS. In other words, the closer the production technology in reality to constant returns to scale, the higher the expected project value.

In contrast, the estimated elasticities confirm that the share of the investment lost due to a potential adverse event and due to the corresponding probability of this event occurring decrease the project value. The same holds for the introduction of corporate profit taxes. It is interesting to note that a 2.8% corporate tax can roughly offset the value gains from a 1% increase in the market price of cotton at the time of signing the contract. The sign of the estimated coefficient on cotton price volatility (log of sigma) appears negative and significant. This contradicts reports in the literature (see introduction) that rising price volatility would, among other factors, actually attract global land deals. Due to the interplay of various factors in our model, however, the positive role of market price volatility on project value is dominated by the negative role that price volatility has on land development and initial land
conversion (compare Figure 1). In the second set of regression results in Table 2 we therefore present a response surface regression with the dependent variable given by the project value to FI at the moment of signing (=dependent variable from first set of regression results) but now divided by $\bar{A}$. This dependent variable can be interpreted as the per hectare project value to the foreign investor (Whitefield Cotton) that it would immediately develop. In this regression, the estimated coefficient on cotton price volatility is positive and significant, which confirms the conventional insight that volatility is driving the project value in a positive way and initial land conversion in a negative way; apparently, the negative effect dominates the positive one for the case of the Whitefield Cotton contract. All other estimated coefficients maintain their previously estimated sign. The substantially lower coefficient of determination in the second set of regressions is due to outliers generated by the distribution of $\bar{A}$ (Figure 1): Most observations are divided by the maximum land available for development (10,000 ha), while $\bar{A}$ varies widely below this value for only a small number of observations. However, given the overall robustness of the estimated coefficients when comparing the first against the second set of regression results in Table 2, we conclude from this scenario that the method of econometric response surface estimation with double-log specification can be a parsimonious way to assess the effect of individual exogenous parameters on the aggregated model response, both in absolute terms and relative to other exogenous parameters.

4.2 Simulation Experiment 2: Estimating Ethiopia’s Bargaining Power

The aim of the second response surface simulation scenario is to assess if the Ethiopian government may have exercised a bargaining power that can be considered in line with the public interest of Ethiopia. Since bargaining power enters the model as a parameter in the range [0,1], intuition may suggest that a bargaining share of 0.5 reflects a balanced negotiating power under which both parties meet ‘on eye level’. Major imbalances in this bargaining share instead may reflect either that one party has signed the contract without insisting on getting a near to fair share of the expected total project value, or that this party factors in additional benefits from the contract that are not directly observable. Such additional benefits may reflect the Ethiopian government’s hope that a project such as Whitefield Cotton will generate further benefits through forward and backward linkages within the local economy, and that the investor will provide e.g. infrastructure available for public use.

However, such unobserved benefits could potentially also reflect the attempt by some host country negotiators to acquire individual shares in this investment project (i.e., corruption) without necessarily passing them on to the public.

In order to determine ex post Ethiopia’s bargaining power in the case of the Whitefield Cotton contract, it is therefore necessary to assess all benefits that the host country is definitely going to receive. In this respect, the Whitefield Cotton contract states only the rental payment over the 25-year contract period, which amounts to a total net present value of 15426.8 TBirr for Ethiopia after taking the negotiated 3-year grace period into account.

However, a second potential source of revenue, not further specified in the contract, is the taxation of profits once the farm has been established. Since no income tax is mentioned in the Whitefield Cotton contract, we initially assume that no income taxes are levied. However,
domestic businesses in Ethiopia certainly face corporate income taxes that progress according
to level of profit. Expected profits from the Whitefield Cotton project would usually fall
into the highest tax rate of 35%. However, the Ethiopian government frequently grants
tax holidays of up to seven years, e.g. for start-up firms. We therefore compare three
different scenarios of 500 simulated projects each. All three scenarios use exactly the same
specifications for the exogenous parameters as in the previous response surface experiment
(Table 1). However, the first of the three bargaining share scenarios fixes the corporate
income tax at zero, the second at 24% (which is equivalent to 35% under a 7-year tax
holiday) and the third scenarios taxes at 35%. The last two scenarios represent the most
generous possible and the maximum possible taxation scenario, respectively, as long as the
official taxation rules for domestic firms are also applied to the Whitefield Cotton project.

Table 3 shows the corresponding inferred median and mean bargaining share that Ethiopia
has exercised in the Whitefield Cotton contract, given uncertainty in the parameters as in
Table 1.

<table>
<thead>
<tr>
<th>Corporate Tax</th>
<th>n</th>
<th>Median</th>
<th>Mean</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>500</td>
<td>0.027</td>
<td>0.033</td>
<td>Whitefield contract does not mention taxes</td>
</tr>
<tr>
<td>24%</td>
<td>500</td>
<td>0.270</td>
<td>0.257</td>
<td>35% adjusted for initial 7-year tax holiday</td>
</tr>
<tr>
<td>35%</td>
<td>500</td>
<td>0.383</td>
<td>0.430</td>
<td>Relevant Ethiopian corporate income tax</td>
</tr>
</tbody>
</table>

On assuming zero corporate tax, Table 4 reveals that this would correspond to a rather low
bargaining share of Ethiopia, with a mean around 3%. However, based on kernel density
estimates Figure 2 illustrates that this first scenario, despite its rather low mean, can still
reflect bargaining shares of around 10-15% in few instances. For the second and third
scenarios, Figure 2 demonstrates that the distribution of simulated bargaining shares is
much wider than for the first scenario, making a 'fair' (0.5) or near to fair bargaining share
certainly realistic. However, Figure 2 also indicates that, in rare events of bargaining shares
exceeding 50%, Ethiopia may actually have negotiated with Whitefield Cotton a deal that
would be especially favorable for Ethiopia.

Figure 2: Kernel density estimates of Ethiopia’s inferred bargaining share

In summary, the results from the simulations indicate that for the case of the Whitefield Cotton contract: The Ethiopian public can be assured that Ethiopia may very well have negotiated with the investor ‘on eye level’ only under the assumption that Ethiopia applies a 35% corporate tax on the Whitefield Cotton project. However, in order to attract the investor initially, Ethiopia may have granted a tax holiday of up to seven years, despite the fact that our simulation results suggest that under assumed price drift and volatility combinations for cotton as in the year 2010, when the contract was signed, such a tax holiday may have been unnecessary. Of course this does not automatically imply that the implementation of this land lease project may not be unfair to other interest groups (although for the case of Whitefield Cotton we do not have such information); the model only states that the bargaining power exercised can broadly be justified as being in line with the interests of Ethiopian society as a whole.

Furthermore, it should be stressed that none of the additional benefits in terms of infrastructure or job creation that the host country may hope to receive has been explicitly stated in the Whitefield Cotton contract as definite deliverables. Additional benefits that may come in terms of infrastructure provision and job creation will be provided in the interests of the foreign investor anyway, if at all, as part of the profit-seeking operation of cotton.
production. Therefore, Ethiopia does not need to ‘pay’ for the provision of these additional benefits from the investment.

5 Conclusions

The theoretical model developed in this article reflects the typical bargaining situation between a foreign investor and a host country for many currently ongoing or recently signed large-scale land deals in Africa. The literature provides ample evidence that there is currently a high degree of uncertainty about whether a certain project will prove to be beneficial or rather disappointing for the host country (e.g., Deininger et al. 2011). Such disappointment is often linked to unfulfilled expectations about the speed of development of the project and related commitments to infrastructure for the host country.

Our model highlights that the speed at which land development under such an investment project takes place is driven by the investor’s incentive to maximize profits under uncertainty. We solve the underlying cooperative game between host country and investor and determine the optimal rental payment. It is shown that 1) the parties share the total value generated by the land development project on the basis of their relative bargaining strength, 2) the optimal rental payment should be such that, once added to tax revenues, the value accruing to the host country is equal to its share of the total value, and 3) the land lease contract is equivalent to a risk-sharing contract between the parties. In this respect, we show that the host country’s payoff includes a riskless component represented by the rental payment and a volatile component represented by taxes on the uncertain profits earned by the foreign investor. This implies that, for instance, by setting higher taxes and a lower rental payment, the host country assumes a more risky position. In contrast, the foreign investor could reduce the risk of the project by obtaining a reduction in the fixed rental payment; such a reduction would function as an implicit subsidy.

Our results from Proposition 1 show that the larger the surface under agriculture, the higher the agricultural profit required to induce additional land development. This implies that the expected timing for the development of the next marginal unit of land increases as more land is developed. This in turn means that even if global food prices remain at high levels, the probability of further land conversion will decline.

However, the level of the rental payment has no impact on the land development policy. In fact, by signing the contract, the foreign investor commits to this payment irrespective of the destination given to leased land during the contract duration. In contrast, the level of taxation levied by the host country on future corporate profits may negatively affect the extent of land developed in the short-run. However, when studying the long-run dynamics, we find that the long-run rate of land development is not affected by tax considerations.

Proposition 2 presents a straightforward relationship between the expected long-run growth rate of land development and price trend, price volatility and returns to scale. This relationship can be empirically tested. Proposition 3 shows that as future agricultural profits become more volatile, the foreign investor postpones land conversion. In contrast, a higher expected profit growth rate triggers faster land conversion. The model shows that commodity price volatility increases the value of the land development option, but slows down the land development process. This result is in line with findings in the real options literature.
As expected, socio-political and natural risk and associated losses slow down development. A similar effect is associated with profit taxation.

Hence, the sluggish land development sometimes observed in large-scale investment projects in Africa may appear to the outsider to be speculative “land grabbing”, but according to our model there are underlying economic reasons that can partly be influenced by the government of the host country. Furthermore, the model shows that by keeping political risk and transaction costs for investors low, for instance through good governance, host country governments can increase the total and thus their own payoff from an investment project. While our model is unable to directly capture issues of bad governance surrounding such investment projects, it nevertheless highlights the profit-maximizing incentive of both negotiating parties under uncertainty and country-specific risk. This risk may very well include riots and unrest triggered by possible violations of human rights during project implementation.

In other words, if project development proceeds more sluggishly than expected by the host country, this is an indication that market price conditions for agricultural outputs and inputs, but also host country-specific risks, have proved to be more challenging for the investor than assumed when the contract was signed.

Nevertheless, we do not deny that the occasionally reported cases (Hall 2011b) of violations of human rights in the host country can occur in relation to the introduction of large-scale land deals, for instance if the host country is eager to offer land for investment ‘free from local people’. Under such conditions of unclear property rights, it is even more important that a critical and concerned public requests transparency about the negotiations and insists on an appropriate distribution of the total project value.

Our model cannot provide a solution to governance problems in the host country, but contributes a robust and easily adaptable framework that all stakeholder groups involved can use as a basis for discussion. We calibrated this model to one specific land contract in Ethiopia for which we were able to determine plausible ranges within which important exogenous model parameters are likely to occur. By using Monte Carlo simulations and letting these parameters simultaneously vary from corresponding random distributions, we were able to assess the model response surface econometrically and estimate the corresponding bargaining share that the Ethiopian government was likely able to raise in the case of this specific contract. Such response surface estimates, once established for a specific contract, may enable host country administrators to conduct rule-of-thumb predictions about the change in the project value if e.g. negotiations over potential tax exemptions take place.

The findings from the simulations indicate that the foreign investor will most likely seek to develop all land immediately, but in about one third of simulated cases it will instead postpone almost the entire land development. Therefore attempts by the host country to fix a specific land development path in the negotiated contract are unnecessary.

However, our simulations also confirm that, as long as it taxes corporate profits from this investment project according to the rules that apply to domestic Ethiopian firms, the Ethiopian government has on average exercised a near to fair bargaining share. Otherwise, our results can inform stakeholders who were not involved in the negotiations about the approximate size of the share of the total project value.

However, the role of cotton price volatility is more ambiguous than often claimed in the literature. Our model shows that the positive contribution to project value is outweighed by other factors that rather slow down land development.
This article by no means intends to play down the seriousness of the human rights infringements reported for some large-scale land deals. However, our analytical framework seeks to deconstruct such deals in developing and less developed countries from the perspective of economic theory and fundamental economic incentives that are likely at play in reality. In this context, we find little justification to argue about ‘good’ or ‘evil’ investors, as some authors distinguish them (e.g., Collier and Venables 2012). Furthermore, the malfunctioning of public institutions within weak states cannot be expected to improve very soon and appeals for a ‘good investment code of conduct’ alone will not have much power unless they are supported by very strong economic incentives (see von Braun and Meinzen-Dick 2009).

In addition, despite attempts to empower local people during negotiations about large-scale land deals, it is perhaps realistic to assume that people who could potentially be adversely affected by large-scale land deals initially have only very limited bargaining power at the time when a contract is signed. For this reason, there is no third party directly represented in our model.

With respect to future research, we suggest case studies to clarify the extent to which a higher degree of land development also increases the forward and backward linkages with the local economy for the practice of actually observed land deals. Furthermore, high global food prices may also directly increase the likelihood of political unrest in low-income countries, which suggests that high profitability of farmland investment projects might be causally related to a high risk of expropriation. If such a relationship between profit and political risk receives empirical support, the results could be used directly in the calibration of our model. For now, the model suggests that, as long as a ruthless strategy against the local society creates tensions that raise the level of policy risk, the expected net present value of the land development project will decline. Thus, a core finding of the model presented here is that an economic rationale for long-term sustainable economic growth based on large-scale foreign direct investment in African farmland may very well exist. In reality, however, this rationale may need to be supported through planning tools such as that presented here.
A Appendix

A.1 Profit function: a price-taker farmer

Suppose the farmer produces a certain agricultural product combining land, $A_t$, with a variable input factor (or several other input factors), $B_t$, e.g., labor, according to the following Cobb-Douglas production function:

$$Q_t = (A_t^{c} B_t^{1-c})^{1-1/(1/c)}$$  \hspace{1cm} (A.1.1)

where $c$ and $1-c$ are the cost shares for each specific input factor and $i > 1$ indicates the degree of decreasing returns to scale.

In the following, in order to simplify the notation we introduce $a = c[1 - (1/i)]$ and $b = (1-c)[1 - (1/i)]$. Let $p_t$ and $w$ be the output price and the unit price for $B_t$, respectively. The operating profit at each $t$ is then given by $\pi(p_t, B_t, A_t) = p_t Q_t - wB_t$. Since apart from land other inputs can be costlessly and instantaneously adjusted to maximize $\pi(p_t, B_t, A_t)$ over time, then the instantaneous short-run profit is given by:

$$\pi(\theta_t, A_t) = \theta_t A_t^{1-\phi} / (1 - \phi)$$

where $\phi = 1 - \gamma_a / (1 - \gamma_b)$ and $\theta_t = G p_t^{1/(1-\gamma_b)}$ with $G = \gamma_a (\gamma_b / w)^{\gamma_b / (1-\gamma_b)}$.

Now, assume that the price $p_t$ is stochastic and fluctuates according to the following geometric Brownian motion:

$$dp_t = \mu p_t dt + \sigma p_t dW_t$$

where $\mu$ and $\sigma$ are expected growth rate and volatility of $p_t$, and $dW_t$ is the increment of a Wiener process with $E[dZ_t] = 0$ and $E[dZ_t^2] = dt$.

It can immediately be shown that $\theta_t$ follows the geometric Brownian motion:

$$d\theta_t = [G p_t^{\gamma_b/(1-\gamma_b)} / (1 - \gamma_b)](\mu p_t dt + \sigma p_t dW_t) +$$

$$+ (\sigma^2/2) \gamma_b G p_t^{\gamma_b/(1-\gamma_b) - 1} [p_t / (1 - \gamma_b)]^2 dt$$

$$= \{ \mu / (1 - \gamma_b) + \gamma_b / 2 [\sigma / (1 - \gamma_b)]^2 \} G p_t^{1/(1-\gamma_b)} dt +$$

$$+ \sigma / (1 - \gamma_b) G p_t^{1/(1-\gamma_b)} dW_t = \mu \theta_t dt + \sigma \theta_t dW_t$$  \hspace{1cm} (A.1.3)

where $\mu = \mu / (1 - \gamma_b) + \gamma_b [\sigma / (1 - \gamma_b)]^2 / 2$ and $\sigma = \sigma / (1 - \gamma_b)$.

A.2 Proof of Proposition 1

Assume $V^{FI}(\theta, A)$ to be twice continuously differentiable. By using Ito’s lemma, the Bellman equation in (5) can be rearranged as follows:\footnote{We drop the time index for notational convenience.}

$$(1/2) \sigma^2 \theta^2 V^{FI}_{\theta \theta}(\theta, A) + \mu \theta V^{FI}_{\theta}(\theta, A) - \delta V^{FI}(\theta, A) = -(1 - s) \pi(\theta, A),$$

for $0 \leq A \leq \overline{A}$  \hspace{1cm} (A.2.1)
where $\delta = \rho + \omega \lambda$.

Differentiating (A.2.1) with respect to $A$ we obtain:

$$(1/2)\sigma^2 \theta^2 v_{\theta \theta}^{FI}(\theta, A) + \mu \theta v_{\theta}^{FI}(\theta, A) - \delta v^{FI}(\theta, A) = -(1 - s) \theta A^{-\phi}$$  \hspace{1cm} (A.2.2)

where $v^{FI}(\theta, A) = V_A^{FI}(\theta, A)$ is the expected marginal net benefit from the conversion of a marginal unit of land.

The closed form solution for the differential equation in (A.2.2) is given by:

$$v^{FI}(\theta, A) = f(A, \theta) + \sum_{i=1}^{2} h_i(A) \theta^{\beta_i}$$ \hspace{1cm} (A.2.3)

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\Phi(\beta) = (\sigma^2/2) \beta(\beta - 1) + \mu \beta - \delta = 0$, $h_1(A)$ and $h_2(A)$ are two constants to be determined and

$$f(A, \theta) = (1 - s)(\theta/(\delta - \mu)]A^{-\phi}$$

The second term in (A.2.3) is the solution to the homogeneous part of (A.2.2) and represents the value of the option to develop an additional unit of land. However, since the value of such an option should vanish as $\theta$ tends to 0 then we must set $h_2(A) = 0$.

The boundary conditions for (A.2.3) are:

$$v^{FI}(\theta^*, A) = k, \ v_{\theta}^{FI}(\theta^*, A) = 0, \ h_1(L) = 0$$ \hspace{1cm} (A.2.4-A.2.6)

Conditions (A.2.4) and (A.2.5) are the value-matching and smooth-pasting conditions for the FI’s optimal development policy. Optimality requires that marginal benefit and marginal cost are tangent at the threshold $\theta^*$ (see Dixit and Pindyck 1994, p. 364).

Condition (A.2.6) simply imposes that $A \leq L$. Substituting (A.2.3) into [A.2.4-A.2.5] yields:

$$\begin{cases}
  h_1(A) \theta^*(A)^\beta + (1 - s)(\theta^*(A)/(\delta - \mu)]A^{-\phi} = k \\
  h_1(A) \beta \theta^*(A)^{\beta-1} + (1 - s)[1/(\delta - \mu)]A^{-\phi} = 0
\end{cases}$$

Solving the system for $\theta^*(A)$ and $h_1(A)$ we obtain:

$$\begin{align*}
  \theta^*(A) & = \frac{\beta}{\beta - 1} \frac{k}{1 - s} \frac{(\delta - \mu) A^\phi}{A}, \hspace{1cm} (A.2.7a) \\
  h_1(A) & = -\frac{1 - s}{\beta} \frac{A^{-\phi}}{\delta - \mu} \theta^*(A)^{1-\beta} < 0 \hspace{1cm} (A.2.7b)
\end{align*}$$

Note that (A.2.7a) can be easily rearranged in terms of $\pi^*(A)$. That is:

$$\pi^*(A) = \frac{\beta}{\beta - 1} \frac{k}{1 - s} \frac{(\delta - \mu) A}{1 - \phi}$$ \hspace{1cm} (A.2.8)

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15This is determined by using the method of undetermined coefficients. Note that it corresponds to the net discounted value attached to the conversion of a marginal unit of land.

16Note that $\beta = \beta_1$, hereafter.
A.2.1 Comparative statics

In this section we analyze the impact of each parameter on the critical threshold for land development. It can immediately be shown that
\[\pi^*(A) = [(\sigma^2/2)\beta + \delta]k A/(1 - \phi),\]  
(A.2.9)
it is easy to show that
\[\partial \pi^*(A)/\partial \mu = (\sigma^2/2)k A/(1 - \phi) = -\frac{(\sigma^2/2)\beta}{\sigma^2(\beta - 1/2) + \mu}k A/(1 - \phi) < 0.\]  
(A.2.10)

Taking the derivative with respect to \(\sigma^2\) we obtain:
\[\frac{\partial \pi^*(A)}{\partial \sigma^2} = \frac{k}{2}(\beta + \sigma^2 \frac{\partial \beta}{\partial \sigma^2}) A/(1 - \phi) = \frac{k}{2}\{\beta - \frac{(\sigma^2/2)\beta(\beta - 1)}{\sigma^2(\beta - 1/2) + \mu}\} A/(1 - \phi) \]
(A.2.11)

Finally, the derivative with respect to \(\delta\) is
\[\frac{\partial \pi^*(A)}{\partial \delta} = [(\sigma^2/2)\frac{\partial \beta}{\partial \delta} + 1]k A/(1 - \phi) = \frac{\sigma^2 \beta + \mu}{\sigma^2(\beta - 1/2) + \mu}k A/(1 - \phi) > 0\]  
(A.2.12)

Note that since \(\delta = \rho + \omega \lambda\) then \(\partial \pi^*(A)/\partial \rho > 0, \partial \pi^*(A)/\partial \omega > 0\) and \(\partial \pi^*(A)/\partial \lambda > 0\).

A.3 Proof of Proposition 2

Inserting (A.2.7b) into (A.2.3), we obtain the expected marginal net value from the conversion of a marginal unit of land. That is:
\[v^{FI}(\theta^*(A), A) = f(A, \theta^*(A)) + h_1(A)\]  
(A.3.1)

By integrating it over \(0 \leq A \leq \bar{L}\) we obtain the value function for FI:\(^{17}\)
\[V^{FI}(\theta, A) = \int_A^\bar{L} v^{FI}(\theta^*(\xi), \xi) d\xi = (1 - s)\left[\int_A^\bar{L} \frac{\theta^*(\xi) - \phi(\theta^*(\xi))^{\beta}}{\beta^{\delta - \mu}(1 - \phi)} d\xi + \frac{\theta}{\delta - \mu} A^{1-\phi}\right] = \frac{k}{\beta - 1} \int_A^\bar{L} \left(\frac{\theta}{\theta^*(\xi)}\right)^{\beta}d\xi + (1 - s)\frac{\theta}{\delta - \mu} A^{1-\phi}\]  
(A.3.2)

Let us now compute the value function for HC. Once the contract is signed, HC implicitly transfers the option to develop the land surface \(\bar{L}\) to the counterpart. This implies that the

\(^{17}\)Note that to guarantee the integral convergence we need to impose \(1 - \phi \beta > 0\).
strategy for the exercise of this option is fixed by FI and taken for granted by HC which receives, as payment, a share of the profits accruing as land is developed over time. Given a certain land allocation \( A \leq \mathcal{L} \), the value of the development project, \( V^{HC}(\theta, A) \), for HC is given by the solution of the following differential equation:

\[
(1/2)\sigma^2 \theta^2 \nu_{\theta}^{HC}(\theta, A) + \mu \theta \nu_{\theta}^{HC}(\theta, A) - \delta \nu^{HC}(\theta, A) = -s \theta A^{1-\phi}/(1-\phi) \quad (A.3.3)
\]

Differentiating (A.3.3) with respect to \( A \) we obtain

\[
(1/2)\sigma^2 \theta^2 \nu_{\theta \theta}^{HC}(\theta, A) + \mu \theta \nu_{\theta \theta}^{HC}(\theta, A) - \delta \nu^{HC}(\theta, A) = -s \theta A^{-\phi} \quad (A.3.4)
\]

where \( \nu^{HC}(\theta, A) = V_{A}^{HC}(\theta, A) \) is the expected marginal net benefit from the conversion of a marginal unit of land.

The general solution to equation (A.3.4) is given by:

\[
\nu^{HC}(\theta, A) = x(A) \theta^\beta + s \frac{\theta}{\delta - \mu} A^{-\phi} \quad (A.3.4a)
\]

where \( x(A) \) is a constant to be determined.

Since HC takes as given the land development strategy, we can determine \( \nu^{HC}(\theta, A) \) by simply imposing the a value-matching condition \( \nu^{HC}(\theta^*(A), A) = 0 \). This yields:

\[
x(A) = -s \frac{\theta^*(A)^{1-\beta}}{\delta - \mu} A^{-\phi} \quad (A.3.5)
\]

Finally, substituting (A.3.5) into (A.3.4a) and integrating \( \nu^{HC}(\theta, A) \) over \( 0 \leq A \leq \mathcal{L} \) we have:

\[
V^{HC}(\theta, A) = \int_{A}^{\mathcal{L}} \theta^*(\xi) \frac{\theta}{\delta - \mu} \xi^{-\phi} + \frac{\theta}{\delta - \mu} A^{1-\phi} \frac{d\xi}{\theta^*(\xi)} = \frac{\beta}{\beta - 1} s \int_{A}^{\mathcal{L}} \frac{\theta}{\delta - \mu} \xi^{-\phi} + s \frac{\theta}{\delta - \mu} A^{1-\phi} \quad (A.3.6)
\]

### A.4 Proof of Proposition 4

Relation (6.1) can be equivalently rearranged as follows:

\[
\eta_t = (1-s)\theta A^{-\phi}, \quad \text{for } \eta \leq \tilde{\eta} = [(\sigma^2/2)\beta + \delta]k \quad (A.4.1)
\]

where the process \( \{\eta_t : t \geq 0\} \) and \( \tilde{\eta} \) represent the expected marginal profit and the marginal cost attached to the conversion of an additional hectare of land, respectively. In technical parlance \( \{\eta_t : t \geq 0\} \) is a regulated process having \( \tilde{\eta} \) as upper reflecting barrier (see Harrison 1985, chp. 2). Note that 1) \( \eta \) moves as \( \theta \) fluctuates and 2) \( A \) will increase to prevent the process from passing the barrier, \( \tilde{\eta} \), whenever \( \eta \) reaches the barrier \( \tilde{\eta} \).

Taking the logarithm of (A.4.1) we get:

\[
\ln \eta = \ln (1-s) + \ln \theta - \phi \ln A \quad (A.4.2)
\]
which, on the basis of a straight-forward application of Ito’s lemma, follows the same Brownian motion that \( \ln \theta \) does. That is

\[
d\ln \theta = (\mu - \sigma^2/2)dt + \sigma dZ
\]

Following Dixit (1993, pp. 58-68) the long-run density function for \( \ln \eta \) fluctuating between a lower reflecting barrier, \( t \rightarrow -\infty \), and an upper reflecting barrier, \( \ln \tilde{\eta} \), is given by the following truncated exponential distribution:

\[
f(\ln \eta) = \begin{cases} 
(2\mu/\sigma^2 - 1)e^{-(2\mu/\sigma^2 - 1)\ln(\tilde{\eta}/\eta)} & \mu > \sigma^2/2, \\
0 & \mu \leq \sigma^2/2.
\end{cases}
\]  

(A.4.3)

for \(-\infty < \ln \eta < \ln \tilde{\eta}\).

It follows that the long-run average of \( \ln \eta \) is given by

\[
E[\ln \eta] = \int_{-\infty}^{\ln \tilde{\eta}} \ln \eta f(\ln \eta) d\ln \eta = \ln \tilde{\eta} - \sigma^2/(2\mu - \sigma^2)
\]  

(A.4.4)

Rearranging (A.4.2) and taking the expected value on both sides, we obtain:

\[
E[\ln A] \approx [\ln (1 - s) + E[\ln \theta] - E[\ln \eta]]/\phi
= [\ln (1 - s) + \ln \theta_0 + (\mu - \sigma^2/2)t - E[\ln \eta]]/\phi
\]  

(A.4.5)

Note that since \( E[\ln \eta] \) is independent on \( t \), differentiating (A.4.5) with respect to \( t \) gives the following expected long-run rate of land development:

\[
E[d\ln A]/dt = (\mu - \sigma^2/2)/\phi
\]  

(A.4.6)

### A.5 Proof of Proposition 5

The first-order conditions for the maximization problem in (10) are\(^{18}\)

\[
\partial \Omega_1/\partial R|_{R=R^*_1} = -\frac{\psi}{W^{FI}(\tilde{\theta}, R^*_1)} + \frac{1 - \psi}{W^{HC}(\tilde{\theta}, R^*_1)} = 0
\]  

(A.5.1)

From (A.5.1) we obtain

\[
R^*_1 = (1 - \psi)V^{FI}(\tilde{\theta}) - \psi V^{HC}(\tilde{\theta})
\]  

(A.5.2)

or, differently put,

\[
W^{HC}(\tilde{\theta}, R^*_1) = V^{HC}(\tilde{\theta}) + R^*_1 = (1 - \psi)V(\tilde{\theta}) > 0
\]  

(A.5.3)

\(^{18}\)As one can easily check, the second-order condition holds.
References


