Interpreting the Oil Risk Premium: do Oil Price Shocks Matter?

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Summary

This paper provides an analysis of the link between the global market for crude oil and oil futures risk premium at the aggregate level. It offers empirical evidence on whether the compensation for risk required by the speculators depends on the type of the structural shock of interest. Understanding the response of the risk premium to unexpected changes in the price of oil can be useful to address some research questions, among which: what is the relationship between crude oil risk premium and unexpected rise in the price of oil? On average, what should speculators expect to receive as a compensation for the risk they are taking on? This work is based on a Structural Vector Autoregressive (SVAR) model of the crude oil market. Two main results emerge. First, the impulse response analysis provides evidence of a negative relationship between the risk premium and the changes in the price of oil triggered by shocks to economic fundamentals. Second, this analysis shows that the historical decline of the risk premium can be modelled as a part of endogenous effect of the oil market driven shocks.

Keywords: Crude Oil Risk Premium, Bayesian SVAR Model, Oil Price Speculation

JEL Classification: Q40, Q41, Q43, E32
Abstract

This paper provides an analysis of the link between the global market for crude oil and oil futures risk premium at the aggregate level. It offers empirical evidence on whether the compensation for risk required by the speculators depends on the type of the structural shock of interest. Understanding the response of the risk premium to unexpected changes in the price of oil can be useful to address some research questions, among which: what is the relationship between crude oil risk premium and unexpected rise in the price of oil? On average, what should speculators expect to receive as a compensation for the risk they are taking on? This work is based on a Structural Vector Autoregressive (SVAR) model of the crude oil market. Two main results emerge. First, the impulse response analysis provides evidence of a negative relationship between the risk premium and the changes in the price of oil triggered by shocks to economic fundamentals. Second, this analysis shows that the historical decline of the risk premium can be modelled as a part of endogenous effect of the oil market driven shocks.

Keywords: Crude oil risk premium, Bayesian SVAR model, Oil price speculation

JEL Codes: Q40, Q41, Q43, E32
1 Introduction

The international market for crude oil is exposed to price risk. The latter can affect the economic performance of a large number of oil companies. Therefore commercial firms hedge against oil price volatility by taking part of the oil futures markets. The risk premium arises because hedger offers to speculator (the counterparty side to the derivative contract) a monetary reward for non-diversifiable risk in the crude oil markets.

This paper emphasises the importance of the risk premium for two main reasons. First, it represents the opportunity cost that is accrued to commercial firms for hedging purposes. Second, it is an attractive investment return for oil speculators. This is motivated by the inflow of capital into crude oil futures markets from commodity index traders, also known as index funds. The latter are economic agents who wish to gain exposure to the oil futures price without holding the commodity in the physical market.

On a practical level institutional investors sell financial instruments in the over-the-counter (OTC) markets to commodity index traders. Therefore money managers who provide suitable instruments that replicate returns of commodity price indices hedge themselves by entering long in the oil futures markets.

The following strategy can have impacts on the crude oil risk premium as discussed in Hamilton and Wu (2014). The authors show empirical evidence of a structural change in the average and the volatility of the risk premium in the crude oil futures contracts as a significant effect of the inflow of money from index traders.

In this paper we investigate the interaction between unexpected changes in the economic fundamentals of the global market for crude oil and oil risk premium at the aggregate level.

The methodology is based on a Bayesian structural vector autoregressive (BSVAR) model as discussed in Baumeister and Hamilton (2015b). Relative to the existent literature on oil risk premium this work provides three main contributions.

First, it offers an empirical evidence on whether compensation for risk required by oil speculators depends on the type of structural shock in question. We document a negative relationship between the impact responses of the price of oil and the risk premium to
shocks of the economic fundamentals in the global oil market. This finding is consistent with theoretical framework based on the hedging pressure theory, the limits to the arbitrage theory and further considerations that will be presented and argued in this analysis. Moreover understanding the response of oil risk premium to unexpected changes in the price of oil is useful for some class of investors, such as speculators who usually take long positions in the oil futures markets.

This analysis addresses some research questions, among which: What is the relationship between crude oil risk premium and unexpected rise in the price of oil? On average, what should speculators expect to receive as a compensation for the risk they are taking on? Second, this work provides a specific investigation for the risk premium in the crude oil market as opposed to most of the empirical analysis based on a "portfolio approach". Understanding the economic factors driving the overall rate of return from a financial commodity portfolio (or index) can be misleading on several aspects.

First of all, the commodity index cannot be a good proxy for the performance of a single asset class as referred to crude oil. For example, the Standard and Poor’s-Goldman Sachs Commodity Index (SP-GSCI) represents the main benchmark for investment in the commodity markets but the share of crude oil futures contracts is only a fraction (about 40%) of its whole composition. For other indices like the Dow Jones-UBS Commodity Index (DJ-UBSCI) the total energy weight amounts to 30%. Moreover the weighting scheme of a commodity index might change over time.

Third, the choice of the econometric framework allows us to deal with reverse causality and consider the endogeneity of the crude oil risk premium with respect to macroeconomic and global oil market variables. This methodology is widely used in the empirical works for modelling the global price of crude oil, see Kilian (2009); Kilian and Murphy (2014); Kilian and Lee (2014).

The rest of the paper is set out as follows. Section 2 presents the literature review. Section 3 describes data and it offers stylised facts on the crude oil futures market. Section 4 discusses the methodology. Empirical results are presented in section 5. Finally, section 6 offers some conclusions.
2 Literature Review

According to the theory of normal backwardation proposed by Keynes (1930); Hicks (1939) and Kaldor (1939), on average the aggregate short hedging demand for futures outweighs the long hedging demand. As a result, to entice speculator to take a long side of the contract the crude oil futures price should be set below the expected future spot price.

For example the empirical analysis discussed in Bessembinder (1992); Bessembinder and Chan (1992) and De Roon and Veld (2000) find out that on average positive excess returns from holding futures contracts are correlated when hedgers are net short. Consistent with these findings Hong and Yogo (2012) show that the hedging pressure is an important determinant of crude oil risk premium.

In contrast, other studies such as Chang (1985) and Rouwenhorst and Tang (2012) do not provide robust results in linking the risk premium to position of speculators and hedgers. Gorton et al. (2013) and Alquist and Gervais (2013) highlight that changes in the net positions of oil traders do not predict oil prices movements. Conversely, the authors show that changes in oil prices help to predict changes in traders’ positions on the oil futures market.

Recently, index speculators have been exposed to commodity indices within a context of a portfolio diversification as discussed in Cheng and Xiong (2014).

Gorton and Rouwenhorst (2006) show that commodity futures returns derived from an equally-weighted index are low correlated with stocks and bonds but positively correlated with changes in inflation.

Hamilton and Wu (2014) propose a model describing the relationship between hedging demand from commercial producers, financial investors and the arbitrageurs. The equilibrium requires that the expected returns of futures prices depend on the arbitrageurs’ net exposure to non-diversifiable risk in the crude oil market.

The authors show that after 2005, the index-fund traders have considerably reduced the average level of the crude oil risk premium.

Studies by Irwin and Sanders (2012); Brunetti et al. (2013); Sanders and Irwin (2014); Brunetti and Reiffen (2014) investigate the role of speculation by exploiting the relation-
ship between commodity index positions and the path of prices in energy futures markets. In these works the authors conduct traditional time-series statistical test with mixed results to provide evidence of predictive link between commodity index investment and changes in energy futures prices. The empirical design behind this literature suffers from some limitations. First, these studies refer to a wide basket of commodities rather than the single market of crude oil. Second, these works treat position from the commodity index traders as exogenous to changes in futures prices leading to downward-biased estimates. Third, Granger-causality test says nothing about the causal relationship between futures prices and index speculators.

Another view consists of a link between the risk premium and the benefit derived of holding oil stocks. This economic view is typically based on the theory of storage. As discussed in Gorton et al. (2013) and Erb and Campbell (2006), the convenience yield can be interpreted as a risk premium linked to the level of inventories which might be able to explain the term structure of the crude oil futures curve. As a result, higher levels of oil stocks might cause a reduction of the risk premium because the risk of stock-outs falls. Empirical studies conducted by Alquist and Kilian (2010) and Valenti (2017) highlight that the oil futures-spot spread can be used as a proxy for the convenience yield but expressed with an opposite sign. The first investigation shows that the futures-spot spread is highly correlated with the cumulative effect of the real price of oil to precautionary demand shocks. The latter are identified from the oil market VAR model as discussed in Kilian (2009).

Valenti (2017) adopts a revised version of the model proposed by Kilian and Murphy (2014) and he shows that most of the fluctuations in the oil futures-spot spread can be explained by shocks to aggregate demand. Alternative methodologies based on volatility models confirm that the rises of the crude oil risk premium are associated with higher price volatility in the underlying asset, see Moosa and Al-Loughani (1994) and Considine and Larson (2001).

Pindyck (2001) argues that holding a commodity alone entails risk because the spot price of crude oil might covary positively with the global economy. Therefore the holders of a commodity will be rewarded for that risk in term of oil spot prices greater than relative
current futures prices.

Finally, the crude oil futures risk premium might be also affected by macroeconomic factors. For example, Coimbra and Esteves (2004) find positive correlation between oil futures forecast errors and market expectation errors on economic activity at the world level. Pagano and Pisani (2009) highlight the importance of the US business-cycle indicators to end up with precise estimates of oil futures prices adjusted for the risk premium. Analogously, Alquist et al. (2014) and Heath (2016) show that unspanned macroeconomic factors help to explain the behaviour of the crude oil risk premium.
3 Data and stylised facts on the crude oil futures market

The data we use in this work are monthly and cover the period 1983:4-2016:4. In this analysis two types of variables are employed: the global oil market variables and the oil risk premium predictors.

The former consist of data on production and price of the international market for crude oil available from the website of the Energy Information Administration (EIA). The global oil production is expressed in percent changes while the physical price is the refiner acquisition cost of crude oil imports. In order to capture the global demand for industrial commodities we consider the real economic activity index proposed by Kilian (2009).

The oil risk premium represents the predictable pay-off of an oil futures contract held to maturity. As opposed to the previous variables crude oil risk premium is not observable and it must be estimated from the data. Therefore to derive the monthly realized excess return we use three months futures contracts price as the end-of-month value and close daily spot price traded on WTI market.

As regards the set of risk premium predictors we include both macroeconomic and financial data. Table 1 reports a summary of the explanatory variables used for the estimation of the risk premium.

We use changes of the US consumer price index to derive a monthly measure for annual inflation rate (\(inf\)). Some empirical studies find out that the expected (\(ei\)) and unexpected (\(ui\)) component of the inflation rate is positively correlated with prospective returns of a commodity futures investment. This is consistent with the view that investors use crude oil futures contracts to hedge against inflation risks.

Following Casassus and Collin Dufresne (2006) we consider a proxy for the slope of the yield curve in order to capture the relationship between the US government bond and crude oil market. We refer to the change in the term structure yield curve (\(cts\)) which is defined as the difference between the 10-Year Treasury constant maturity rate and the Treasury Bill of maturity 3-months.

Variation in liquidity plays an important role in explaining the factor structure of the
Table 1: List of predictors for the estimation of the risk premium

<table>
<thead>
<tr>
<th>Id</th>
<th>Predictors</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>inf</td>
<td>Annual CPI inflation rate</td>
</tr>
<tr>
<td>2</td>
<td>ci</td>
<td>Expected US inflation</td>
</tr>
<tr>
<td>3</td>
<td>ui</td>
<td>Unexpected US inflation</td>
</tr>
<tr>
<td>4</td>
<td>cts</td>
<td>Change in the term structure yield curve</td>
</tr>
<tr>
<td>5</td>
<td>cdp</td>
<td>Change in the default premium</td>
</tr>
<tr>
<td>6</td>
<td>jbp</td>
<td>Junk bond premium</td>
</tr>
<tr>
<td>7</td>
<td>cli</td>
<td>Composite leading indicator</td>
</tr>
<tr>
<td>8</td>
<td>cip</td>
<td>Annual changes of U.S. industrial production index</td>
</tr>
</tbody>
</table>

global business cycle and it can be correlated with the risk premium. Therefore the analysis takes into account other two indicators. The first is the change in default premium (cdp) defined as the difference between Moody’s Baa corporate bond yield and 10-year treasury constant maturity rate. The second indicator is called junk bond spread (jbs) and is measured as the difference between Baa and Aaa corporate bond yields rated by Moody.

Moreover empirical results of Pagano and Pisani (2009) suggest that the risk premium in the commodity markets can be strongly affected by fluctuations in the business cycle of the global economy. Therefore this analysis includes the composite leading indicator (cli) and the yearly changes in the US industrial production index (cip).

### 3.1 The estimation of the crude oil risk premium

The oil risk premium represents the average returns that long investors expect to receive as a reward for non-diversifiable risk in the crude oil futures market. The risk premium is not observable but it can be estimated from the data. In this analysis we follow two different methodologies.

The first approach relies on a multivariate linear regression model. The second method is based on a Gaussian affine term-structure model in which time-varying crude oil risk premium depends on three latent factors. The first two factors are identified as the level and the slope of the term structure futures curve while the last can be though as a proxy for measurement error.

Regarding the first methodology we define the realized log-returns of a crude oil futures investment as 

\[ er_{t+3} = \ln \left( \frac{S_{t+3}}{F_{t,3}} \right) \]

where \( F_{t,3} \) denotes the price of futures contract at the end
of the day of month \( t \) (with maturity 3-months) and \( S_{t+3} \) is the corresponding realized daily spot price at the next 3-months from period \( t \).

The oil risk premium regression can be expressed as follow:

\[
er_{t+3} = \alpha + \sum_{i=1}^{K} \beta_i z_{it} + \nu_{t+3} \quad \text{for } t = 1, 2, \ldots, T
\]  

(1)

where \( z_{it} \) represents the \( i \) explanatory variable at time \( t \), \( \alpha \) and \( \beta_i \) represent the 3-months specific regression coefficients which can be consistently estimated by ordinary least squares and \( K \) is the total number of explanatory variables. Finally \( \nu_{t,t+3} \) is the mean zero error term of the risk premium regression model. Fitted values of \( \hat{er}_{t+3} \) represent consistent estimates of time-varying risk premium.

In this analysis we include four different measures of crude oil risk premium in order to assess the robustness of the empirical results. We estimate the first-three risk premium measures as follow:

\[
 rp_{t+3}^{(1)} \equiv \hat{er}_{t+3} = \hat{\alpha} + \hat{\beta}_1 \text{inf}_t + \hat{\beta}_6 \text{bpt}_t + \hat{\beta}_7 \text{cli}_t \quad \text{for } t = 1, 2, \ldots, T
\]  

(2)

\[
 rp_{t+3}^{(2)} \equiv \hat{er}_{t+3} = \hat{\alpha} + \hat{\beta}_3 \text{uit}_t + \hat{\beta}_4 \text{cts}_t + \hat{\beta}_7 \text{cli}_t \quad \text{for } t = 1, 2, \ldots, T
\]  

(3)

\[
 rp_{t+3}^{(3)} \equiv \hat{er}_{t+3} = \hat{\alpha} + \hat{\beta}_2 \text{ei}_t + \hat{\beta}_5 \text{cdp}_t + \hat{\beta}_8 \text{cip}_t \quad \text{for } t = 1, 2, \ldots, T
\]  

(4)

Although the risk premium regression analysis is widely accepted in the academic literature some concerns might arise about the selection criteria of the exogenous variables. Therefore we provide an alternative estimate of risk premium based on oil futures prices, only. This approach stems from an affine factor structure model as developed by Hamilton and Wu (2014).

The authors propose a model of the time-varying risk premium that imposes an affine factor structure which is common for oil futures prices and the economic fundamentals of the global market for crude oil.
The risk premium is identified by the difference between the observed futures prices and the rational expectation of future spot price. The latter depends on the risk price parameter which is thought as an affine function of the latent variables.

To estimate an affine term structure model we postulate the existence of three factors. The first-two factors represent level and slope of the nearest three futures contracts while the third factor is usually interpreted as measurement error.

Data are collected such that the maturity of futures contracts changes with each observation. Thus, implementation of the term structure model only requires oil futures prices collected in an unbalanced dataset in which the panel structure is given by different maturities and the monthly time-series dimension is made by the futures price on the last day of each week.

Following Hamilton and Wu (2012) the set of parameters included in the affine term structure model can be derived by applying the method of minimum-chi-square estimation (MCSE) to the unrestricted reduced form estimates. In this way it is possible to infer the crude oil risk premium as difference between the oil futures price based on the structural parameters under risk-neutrality assumptions and the observed oil futures price that characterize the real world dynamic. [1]

Figure 1 plots four alternative estimates of the risk premium based on the methodologies and the specifications that have been previously discussed. Two basic futures emerge. First, significant similarities can be seen between the pairs of risk premium estimates. In particular, the first and the second measures of risk premium have high positive correlation (0.93) over the entire sample. This becomes stronger (0.96) from January 2000 to April 2016. Moreover the risk premium estimate implied by the affine term structure model, \(rp_{t+3}^{(4)}\), is positively correlated with \(rp_{t+3}^{(3)}\). In particular their correlation ranges from 0.32 (January 1984 - April 2016) to 0.46 (January 2000 - April 2016).

Second, the last two measures of crude oil risk premium document a systematic downward shift of their average level.

Hamilton and Wu (2012) show that the MCSE minimizes a quadratic form in the difference between the reduced-form parameters implied by the structural model and the ols estimates derived from the reduced-form model. The quadratic form corresponds to the information matrix and the MCSE is asymptotically equivalent to full-information maximum likelihood estimator.
Note: Figure 1 plots the risk premium estimates across different specifications and methodologies over the period January 1990 - April 2016.

Following Baumeister and Kilian (2016) the accuracy of risk premium estimate is reflected by the mean squared prediction error (MSPE) ratio between the rational expectation of future spot price and the random-walk process.

Rational expectations of future spot price equals the futures prices adjusted for the crude oil risk premium. A case of MSPE ratio below one indicates an improvement in the accuracy of the random-walk process.

Table 2 reports the predictive accuracy of risk-adjusted futures price and the statistical significance of the MSPE reduction based on the test of Clark and West (2007).

Table 2: Predictive accuracy of risk-adjusted futures price.

<table>
<thead>
<tr>
<th>Risk premium</th>
<th>Mean squared prediction ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RP_1$</td>
<td>0.92</td>
<td>0.03</td>
</tr>
<tr>
<td>$RP_2$</td>
<td>0.95</td>
<td>0.03</td>
</tr>
<tr>
<td>$RP_3$</td>
<td>0.88</td>
<td>0.05</td>
</tr>
<tr>
<td>$RP_4$</td>
<td>0.82</td>
<td>0.02</td>
</tr>
</tbody>
</table>

3.2 Stylised facts on the crude oil futures market

Broadly speaking, participants in the oil futures market can be classified into three categories: hedgers, speculators and arbitrageurs.
The hedgers have economic interests in the physical market and they hedge against price risks by holding opposite positions in the spot and futures markets at the same time. For example, an oil producer can lock in the price of crude oil production by selling a certain amount of futures contracts in anticipation of a later spot market sale. In contrast, an oil consumer can hedge against rising crude oil prices by buying a given number of futures contracts in anticipation of an actual physical market purchase. Although hedging activities represent the simplest way to manage price risks they could also affect the total revenues accruing to both consumers and producers.

The oil speculators are not interested in making (or taking) delivery of the commodity in the physical market but they buy (or sell) paper barrels to make profits as an opportunity for a capital gain in anticipation of price changes or as component of a diversified portfolio.

For example, the Commodity Pool Operators (CPO’s) are investment vehicles that collect capital from a large number of investors, through a public or private offering, in order to facilitate investment opportunities in a portfolio of commodity futures. The CPO’s usually delegate the Commodity Trading Advisors (CTA’s) who are professional money managers able to engage futures transactions in the derivative markets. Analogously the hedge funds invest on behalf of rich people in conjunction with other investment products like stocks, currencies and bonds.

As a result the participation of financial institutions such as banks, hedge, mutual and pension funds, money managers can add liquidity to crude oil futures market serving as a counterparty for the hedgers.

The arbitrageurs are the third class of actors who attempt to eliminate any markets’ price discrepancies.

All categories above mentioned are easier to separate in principle than in practice but their definitions reported in this analysis are consistent with those proposed by the Commodity Futures Trading Commission (CFTC).

The following regulatory agency breaks down the number of outstanding short and long futures contracts for crude oil on the basis of two macro categories: the ”commercial” and ”non-commercial” firms.
The former include physical participants such as producers, merchants, processors and end-users that have a direct interest in physical oil production, consumption and trade. The latter are mainly made by financial participants like money managers and hedge funds that are interested in trading futures contracts for investment purposes. In this context commercial firms are considered hedgers while non-commercial firms are treated as speculative traders. At first sight it might be questionable to assume that commercial firms are only hedgers. For example a producer (or consumer) can hedge only a fraction of its physical underlying taking implicitly a speculative position, see Fattouh et al. (2013). However, since in this analysis the risk premium is defined as a monetary reward accrued to speculators for their non-diversifiable risk in the commodity market, it is reasonable to refer to a non-commercial firm as a speculative trader.

The hedging pressure theory states that the risk premium arises from the interaction between hedgers and speculators and it becomes higher (in absolute value) when the hedging demand increases.

The open interest represents the number of derivative contracts held by both commercial and non-commercial firms at the end of a trading day. This is a proxy for the flow of money injected into the futures market. This implies that the open interest can be used to define an aggregate measure of hedging (or speculative) pressure.

Therefore we define the net-hedging demand as a ratio between the net and gross positions of the futures contracts referred to commercial firms. Analogously, the same logic applied to non-commercial firms yields to a measure of net-speculative demand. This last can be also interpreted as proxy for net-hedging supply.

Interestingly, the CFTC considers financial institutions called swap dealers as commercial firms. Although these entities represent investment banks and commodity brokers/dealers, they act as intermediaries for producers and consumers suggesting that their positions on futures market should reflect hedging purposes. However, it may not always be obvious to understand whether swap dealers operate for commercial firms or not. For this reason we do not include the positions of swap dealers on the derivative market in the definition of hedging pressure measure.
Figure 2: Hedging pressure indicators.

Note: Figure 2 plots the WTI spot prices combined with the hedging pressure indicators over the period January 2006 - April 2016. Green histograms refer to the net-hedging demand from commercial firms. Negative values indicate that hedgers are net-short. Pink histograms refer to the net-speculative demand from money managers. Positive values indicate that speculators are net-long.

Figure 2 plots the monthly WTI spot price combined with the net positions held by both the hedgers and the speculators.

It is important to note that an investor might hold both long and short positions in the futures market. In particular for every contract that one trader is long there is another trader who is short such that the outstanding value of long and short futures contract is exactly offsetting.

Figure 2 shows that on average the hedgers are net-short and speculators are net-long. Interestingly, hedgers seem to follow price trend: they increase their net-short positions when the spot price falls and move from short to long positions when spot price rises. On the other hand, speculators seem to change their positions in the futures market with the object to replicate the spot price of oil, providing market liquidity.
4 Econometric method

The methodology is based on a Bayesian structural vector autoregressive (BSVAR) model inspired by Baumeister and Hamilton (2015b). In this section we provide an economic explanation for each of the structural equations and the corresponding informative prior distributions. Further details of the identification strategy and the Bayesian algorithm as proposed by Baumeister and Hamilton (2015a) are reported in Appendix A.

The SVAR model is the following:

\[ Ay_t = c + \sum_{j=1}^{24} B_j y_{t-j} + v_t \]  

(5)

where \( A \) is the matrix of instantaneous structural parameters and \( c \) is the vector of constant terms. The vector of endogenous variables is \( y_t \) and it includes the percent change in global crude oil production (\( q_t \)), the global real economic activity (\( rea_t \)), the real price of crude oil (\( p_t \)) and the crude oil risk premium (\( rp_t \)). The structural representation considered in (5) is based on a system of four equations:

\[ q_t = a_{q,p} p_t + \tilde{b}_1 x_{t-1} + v_{1t} \]  

(6)

\[ rea_t = a_{rea,q} q_t + a_{rea,p} p_t + \tilde{b}_2 x_{t-1} + v_{2t} \]  

(7)

\[ p_t = a_{p,q} q_t + a_{p,rea} rea_t + \tilde{b}_3 x_{t-1} + v_{3t} \]  

(8)

\[ rp_t = a_{rp,q} q_t + a_{rp,rea} rea_t + a_{rp,p} p_t + \tilde{b}_4 x_{t-1} + v_{4t} \]  

(9)

where \( \tilde{b}_1, \tilde{b}_2, \tilde{b}_3 \) and \( \tilde{b}_4 \) are row vectors of structural lagged coefficients related to the first-four equations and \( x_{t-1} \) is a column vector including a constant and the past variables.

The oil supply equation is given by (6). It is a function of only one contemporaneous structural parameter \( a_{q,p} \) which represents the short-run price elasticity of oil supply.

The real economic activity modelled in equation (7) is instantaneously affected by the global oil production and the real price of crude oil due to \( a_{rea,q} \) and \( a_{rea,p} \), respectively.

---

\[ A_{i} \] contains all structural lagged coefficients of the \( i \)th equation belongs to the first row of \( B_j \), for \( j = 1, \ldots, 24 \). In other words, \( A_{i} \) has a dimension \( (n \times m + 1) \) where \( n \) and \( m \) are the numbers of endogenous variables and lags, respectively.
The inverse demand function of the global market for crude oil is defined in equation 8 where $a_{p,q}$ represents the reciprocal of short-run price elasticity of oil demand.

Following [Baumeister and Hamilton (2015)](https://example.com), the structural coefficient $a_{p,rea}$ is the ratio between the income elasticity (derived from the real economic activity index) and the short run price elasticity.

The risk premium estimate modelled in equation 9 is contemporaneously affected by all endogenous variables that are considered in this analysis.

Finally $v_t = (v_{1t}, v_{2t}, v_{3t}, v_{4t})'$ denotes a vector of structural innovations with the following variance covariance structure: $E_t(v_t v_t') = D$ and $E_t(v_t v_s') = 0$ if $t \neq s$. The fact that $D$ is a diagonal matrix implies that the structural shocks can be economically interpreted in terms of shifts in demand and supply.

In particular, the first shock ($v_{1t}$), oil supply shock, is the unexpected changes in the global oil production. The second shock ($v_{2t}$), aggregate demand shock, reflects a rise in the demand for crude oil and other industrial commodities driven by fluctuations in the global business cycle. The third shock ($v_{3t}$), precautionary demand shock, is related to a unanticipated change in the demand for crude oil for future consumption.

Finally, the fourth shock ($v_{4t}$) is called risk premium shock and it is designed to capture unexpected changes in the risk premium which is not driven by the first-three structural shocks. For example, it might reflect an increase in the price of risk and/or capital constraints, difficulty to achieve a diversified investment portfolio, and the existence of profitable opportunities from other markets.

The matrix summarizing the simultaneous relations among the endogenous variables can be denoted as follow:

$$
A = \begin{pmatrix}
1 & 0 & -a_{q,p} & 0 \\
-a_{rea,q} & 1 & -a_{rea,p} & 0 \\
-a_{p,q} & -a_{p,rea} & 1 & 0 \\
-a_{rp,q} & -a_{rp,rea} & -a_{rp,p} & 1
\end{pmatrix}
$$

The first row of $A$ characterizes the oil supply equation, as reported in 6. We postulate that the global oil production does not respond to any changes in the economic activity and crude oil risk premium, within the same period. Therefore an instantaneous increase
in the global crude oil production is only caused by a rise in the price of oil reflected by \( a_{q,p} \), for which we assign student \( t(c_{q,p}, \sigma_{q,p}, \nu_{q,p}) \) positive truncated distribution, with mode at \( c_{q,p} = 0.0258 \), scale parameter \( \sigma_{q,p} = 0.2 \) and degrees of freedom \( \nu_{q,p} = 3 \).

The magnitude of the prior mode is very small in absolute value which is consistent with the assumption of a price inelastic supply curve, which is motivated by the existence of large production cost of the oil sector, see Pindyck (1994, 2001) and Kilian and Murphy (2014). The second row of \( A \) includes the structural parameters of the real economic activity equation. We remain agnostic about the prior beliefs of \( a_{rea,q} \) and \( a_{rea,p} \). We postulate that changes in crude oil risk premium do not instantaneously affect the real economic activity. For both coefficients we put relative uninformative prior student \( t \) distribution with mode at \( c_{rea,q} = 0 \), scale parameter \( \sigma_{rea,q} = 0.2 \) and \( \nu_{rea,q} = 3 \) degrees of freedom. As regards the structural parameter \( a_{rea,p} \) we truncate the prior distribution to be negative consistent with the view that an increase in the real price of oil causes a reduction of the economic activity, see for example Kilian (2009); Kilian and Murphy (2014).

In matrix \( 10 \) the structural coefficient \( a_{p,q} \) represents the reciprocal of short run price elasticity of oil demand. Therefore we put a student \( t(c_{p,q}, \sigma_{p,q}, \nu_{p,q}) \) prior distribution with mode at \( c_{p,q} = -3 \), scale parameter \( \sigma_{31} = 0.1 \), \( \nu_{31} = 3 \) degrees of freedom and truncated to be negative. We expect that the short price elasticity of oil demand is lower than its long-run estimate as discussed in Hausman and Newey (1995).

As regards the income elasticity of oil demand there are several estimates that range from 0.4 to 1, see Gately and Huntington (2002) and Csereklyei and Stern (2016).

In this analysis we postulate that the income elasticity (derived from the real economic activity index) is greater than the short run price demand elasticity. Thus their ratio is greater than one and we assign student \( t(c_{32}, \sigma_{32}, \nu_{32}) \) prior distribution with mode at \( c_{32} = 1.4 \), scale parameter \( \sigma_{31} = 0.2 \), \( \nu_{32} = 3 \) degrees of freedom and truncated to be positive.

Given the forward-looking nature of the risk premium we impose an exclusion restriction on the structural coefficient \( a_{p,rp} \).

Finally for the parameters of the risk premium equation we assign completely uninformative priors \( t \) student distribution, with location parameter \( c_i = 0 \), scale \( \sigma_i = 100 \) and
degrees of freedom $\nu = 3$. The prior density of matrix $A$ is given by the product of all student $t$ densities of each structural parameter subject to the sign restrictions previously discussed.
5 Empirical results

In this section we proceed to the analysis of the dynamic responses of the endogenous variables to each structural shock. The shocks have been normalized such that they imply an increase in the real price of oil.

Figure 3 plots the median impulse responses of oil production, real economic activity and price of oil to the three oil supply and oil demand shocks together with the corresponding pointwise 68% percentiles of the posterior distribution. All empirical results are reported for the specification including the crude oil risk premium estimated by adopting the affine term structure model as discussed in Hamilton and Wu (2014).

The impulse response estimates imply that an unexpected oil supply disruption causes a contemporaneous drop of the global crude oil production. This shock is associated with an increase in the price of oil and a decline in the real economic activity, on impact.

An unanticipated positive aggregate demand shock causes an instantaneous increase in the real economic activity, in the global oil production and in the real price of crude oil. Finally a positive precautionary demand shock causes a contemporaneous increase in the global oil production accompanied by an hump-shaped response of the real price of oil. The impact response of the real economic activity to a positive precautionary demand shock is negative.

Figure 3 suggests that the dynamic responses of the endogenous variables to each structural shock are consistent with the empirical results of Kilian (2009); Kilian and Murphy (2014); Kilian and Lee (2014) and Baumeister and Hamilton (2015b).

Figure 4 shows empirical evidence that an unexpected increase in the real price of oil triggered by an oil supply disruption and positive aggregate and precautionary demand shocks is associated with a contemporaneous decline in the oil futures risk premium. This result reflects several economic features. First, the expected speculative gains (hence, the crude oil risk premium) decrease as current oil prices increase.

This is consistent with the fact that higher oil prices require that speculators allocate more capital to purchase the same amount of contracts, causing the marginal value of the

3Notice that, the sign and the path of the impulse response functions are robust across different measures of crude oil risk premium.
Figure 3: Median impulse responses of global oil production, real economic activity and price of oil to oil market shocks

Note: Figure 3 plots the Bayesian posterior median responses to one-standard deviation structural shocks. Black lines indicate the impulse response estimates based on structural models satisfying the identification structure. Dashed lines indicate the 68% credible region.
investment to decrease. Suppose that the oil futures curve is upward sloping and the price to purchase a futures contract for a given maturity is $60. Now, imagine that the spot price of crude oil is $30, the storage cost at the net of the convenience yield is, say, 3% of the nominal futures price and the risk-free interest rate is 5%. Thus, oil traders would borrow $30, buy oil in the physical market and store it at the cost of $2 until the delivery date. As a result the total cost for the investor is $33.5 and the expected profit will be $26.5.

Now we assume that a shock causes an increase in the spot price of oil up to $100. The same shock also causes an upward-shift of the futures curve so that futures price is $130. In this case the total cost for the investor will be higher than before, reaching $108 while the expected long returns will be equal to $21.7, amounting to a 35% reduction.

Second, it is not surprising to find out that when the term structure of the oil future curve is in contango a very large number of speculators increases their long position in these contracts because they expect that the price of oil will be higher in the future. As a result the increase in the speculative purchase of futures contract and hence, the competition among oil speculators, might cause a decline of the average prospective return in the crude oil futures market.

Moreover, it is important to note that the decline of the risk premium might be reinforced by a reduction in the short-hedging demand of commercial firms. Although every hedging strategy implies an off-setting gain between spot and financial markets, it is well known that higher levels of oil prices might lead to a reduction of the incentive to hedge against price drops. This is motivated by a higher return than one hedgers would receive if they did not hedge.

This conjecture is consistent with the view that crude oil risk premium is higher when net-short hedging demand is higher as discussed in Acharya and Ramadorai (2013). This implies that during a high level of oil prices an increase in the hedging supply from speculators and/or a reduction in the hedging demand from commercial firms might cause a drop of the crude oil risk premium.

Third, the growing interests in commodity futures contract as an asset class for portfolio investment have attracted attention of many arbitrageurs causing the arbitrage profits to
increase and the risk premium earned by oil speculators to decline, as discussed in Duffie (2010) and Etula (2013).

The average excess return of a crude oil futures investment consists of a spot return and a roll return. The spot return is simply the appreciation (or depreciation) of the futures contract held to maturity. The roll-return (or roll-yield) arises when investors want to maintain a crude oil futures position. This can be easily done, by selling the expiring contract and use the proceed to buy another futures contract for delivery at a more distant date.

In the case of backwardated market oil speculators can earn a positive roll-yield, even if the spot price does not change. However the roll-yield (and hence the crude oil risk premium received by the oil speculators) could partially decline because of the arbitrageurs’ attempt to eliminate any possible mispricing triggered by index funds or other types of speculators during the rolling period.

Even if provisionally, roll yield opportunities for commodity investors might cause a reduction of the expiring futures price, below its equilibrium. Conversely the buying pressure of the next-to-expire contracts might cause a rise above the their economic fundamental prices.

As a result, a market price anomaly could be easily exploited by a long-short strategy from the arbitrageurs. They can simultaneously short the nearby maturity contract and long the more distant one by earning a profit from the calendar spread. The arbitrageurs will close-out their positions by longing the short-maturity contract and shorting the long-maturity contract.

Basically, the arbitrageurs’ gain causes a drop of the crude oil risk premium which is mainly reflected by the decline of the roll-yield.

In general crude oil plays a primary role in determining the performance of a commodity index or for financial portfolios. Thus, understanding the path response of crude oil risk premium to unexpected changes in the price of oil can help how best to perform forward looking asset allocation analysis. In order to define a proper set of forward-looking efficient frontiers, oil speculators should combine assets weight and forecasts return at the net of the risk premium reduction, as documented in this analysis.
Figure 4: Median impulse responses of risk premium estimates to oil market shocks

Note: See figure 3.
Figure 4 shows that the crude oil risk premium responds to oil price shocks differently, depending on the cause behind the shocks.

An oil supply disruption causes a slight decline in the risk premium but much of the initial drop is reversed within the first ten months.

A positive aggregate demand shock, driven by unexpected fluctuation in the global business cycle, causes a large reduction in the crude oil risk premium. In the global economy if the aggregate demand expands quickly then we would expect to see a rise in the level of inflation. Although numerous studies document that commodity diversified portfolios represent one of the best ways to hedge against inflation risks our results suggest that the efficacy of this strategy could be adversely affected by the reduction of crude oil risk premium.

A positive precautionary demand shock causes a persistent reduction of the risk premium. According to Kilian (2009) and Alquist and Kilian (2010) this shock might reflect an unanticipated increase in the demand for storage. The latter might provide useful information about what the term structure of futures prices will look like in the future.

Our results suggest that whenever the shape of the term structure is downward-sloping because of a positive precautionary demand shock the crude oil risk premium earned by a long investor could decline, even during backwardated futures market.

In general the response of the crude oil risk premium to demand shocks is greater than the supply shocks. Moreover, precautionary and aggregate demand shocks produce similarly effects on the risk premium.

Figure 5 plots the median impulse response of the endogenous variables to different proxies for positive risk premium shock.

The first piece of evidence is that the oil risk premium is the only variable to increase in response to unanticipated positive risk premium shocks.

Other macroeconomic and global oil market variables are not simultaneously affected by the risk premium shock, according to the identification structure implied by the model.

For this reason, risk premium shocks are not driven by economic fundamentals, as is typical of the global market for crude oil. For commercial firms positive shocks to the risk premium reflect a rise in the cost of hedging for reasons that are independent from the
Figure 5: Median impulse responses of endogenous variables to risk premium shocks

Note: See figure 3.
global market for crude oil. Figure 5 shows that, beyond the impact period, an unan-
ticipated positive risk premium shock causes a rise in the real price of oil only for the
first-two risk premium estimates.
In contrast to previous structural shocks this result provides evidence of a positive rela-
tionship between the price of oil and the risk premium upon impact.
However this result does not hold for the last-two risk premium estimates. Therefore the
effects of positive risk premium shocks on the real price of oil upon the impact period is
mixed.
Interestingly, when the estimation of the crude oil risk premium is performed by linear
regression model positive risk premium shocks cause price, production and real economic
activity to increase.
In the case where the risk premium estimate is derived from the term affine structure
model there is little evidence that shocks to risk premium cause significant changes in the
endogenous variables of the global market for crude oil.
Figure 6 plots the historical decomposition of the crude oil risk premium and the real
price of oil with 68\% posterior credible set. There is empirical evidence that, from early
2003 until mid-2008, shocks to aggregate demand (likely driven by Emerging Asia and
OECD countries) have represented the main economic factors behind the decline of the
oil futures risk premium.
This implies that economic fundamentals represent the rational drivers behind any invest-
ment strategy taken on by speculators, which direct impacts the aggregate measure for
crude oil risk premium.
It is important to point out that this finding is still consistent with the claim that the
growth of commodity index investments have caused a reduction in the crude oil risk
premium during the financialization of commodity markets.
Interestingly, the historical effect of the risk premium shocks on the real price of oil is
negligible. This result tells us that specific shocks to speculators (independent from the
aggregate demand and/or supply of oil) were not the main drivers in explaining the path
of the real price of oil, during the financialization of commodity markets. This is consist-
tent with the empirical results of Kilian and Lee (2014).
The role of speculation in driving the oil prices became very popular and important for policy implications when the spot price of oil dropped from historic highs of $144 in July 2008 to $33, five months later.

In the first half of 2008, figure 6 provides indication that the drop of the crude oil risk premium was associated with an increase in the real price of oil mainly driven by shocks to precautionary demand for oil. They were triggered by a full of exogenous events, as discussed in Smith (2009). For example, in March 2008 there was the sabotages of the two main oil export pipelines in the south of Iraq, in April 2008 the strike of Nigerian union workers and finally, in June 2008 there was the closure of the North Forties pipeline in the UK and the mass rioting in Nigeria.

Figure 7 plots the hedging pressure indicator six months before crude oil reached a peak of $147 per barrel in July 2008, an all-time high.

The significant increase in the price of oil caused a reduction of the net-hedging demand from commercial firms followed by a decline of the crude oil risk premium paid to the speculators as a form insurance against down-trended prices.

On the other side, higher oil prices required more money to invest in the futures market to buy the same amount of contracts. This caused a reduction in the crude oil risk premium. Moreover figure 2 shows that positive precautionary demand shocks were partially responsible for the reduction of crude oil risk premium between 2010 and 2012. These shocks might be triggered by some concerns about possible international oil supply disruptions. According to Bastianin et al. (2017) the negative oil supply shock caused by Libyan civil war of February 2011 took about 1.5 million barrels per day off the global market. Then, the political tensions related to Iran’s nuclear program lead to the European Union foreign ministers to agree on a ban on the transport, purchase and import into Europe of Iranian crude oil.

In early 2012, the Europe’s sovereign debt crisis represented another possible factor that contributed to decline the crude oil risk premium through precautionary demand shocks. On the other hand, a sequence of positive and negative aggregate demand shocks was responsible for high level of risk premium until the end of 2013. As a result the effect of precautionary and aggregate demand shocks on oil risk premium was offsetting.
Figure 6: Historical decomposition of crude oil risk premium

Note: Historical contribution of the structural shocks (black lines) with 64% posterior credibility sets (red-dashed lines) implied by the identification structure of model 5.
Between June and December 2014, the drop of the price of crude oil caused a systematic upward trend of the oil risk premium, which was mainly caused by a combination of unanticipated positive shocks to the global oil production and negative aggregate demand shocks. The latter could reflect the unexpected slowdown of the global economy, mainly driven by the decline of the Chinese manufacturing industry, as reported by the Caixin manufacturing index.

**Figure 7:** Hedging pressure indicators.

Note: Figure 7 plots the WTI spot prices, and the risk premium estimate combined with the hedging pressure indicators over the period January 2008 - July 2018. Green bars refer to the net-hedging demand from commercial firms. Negative values indicate that hedgers are net-short. Yellow bars refer to the net-speculative demand from money managers. Positive values indicate that speculators are net-long.
6 Conclusions

The benefit of including oil futures contracts as an asset of financial investment portfolio is widely accepted in both the academic literature and the oil industry. Crude oil futures contracts allow investors to get more attractive portfolio’s diversification with a higher ratio of return to risk than traditional investments made of equities and bonds, only.

Therefore if the current futures price is below the expected future spot price, arbitrage mechanism implies convergence of the futures price to the spot price at maturity and this turns into positive excess returns.

The expected returns of crude oil futures investment represent simply bets because spot and roll returns follow an uncertain scenario. However, a possible way to derive a forward-looking measure of risk premium relies on the impulse response estimates implied by the SVAR model proposed in this analysis.

Moreover, impulse response analysis suggests the existence of a negative relationship between the impact responses of the real price of oil and oil futures risk premium to shocks of the economic fundamentals of the global oil market. As a consequence investors could exploit these findings as an asset allocation strategy.

Finally, we highlight that the main economic factors behind the historical decline of the risk premium can be explained by shocks to aggregate and precautionary demand for crude oil.

In this way the effects of the financialization of commodity markets on the risk premium are modelled endogenously with respect to the global market for crude oil.

Our results suggest that shocks to economic fundamentals play a crucial role in explaining the path of the oil futures risk premium. They are also more important than specific shocks to the non-diversifiable component of the long investors. This confirms the economic idea that oil speculators do not represent mindless traders.
References


Appendix A  Identification strategy

This section reports a short description of the algorithm proposed by Baumeister and Hamilton (2015a) for the estimation of model 5 under a Bayesian perspective. The implementation of the estimation algorithm is based on three main steps.

First stage consists of a specification of informative prior beliefs, represented in form of density functions about the matrix $A$, the vector collecting the structural disturbances $v_t$ and the matrix $B_j$, for $j = 1, \cdots, 12$.

Prior for the elements of the contemporaneous structural matrix that are not known with certainty are collected in a vector $(\alpha)$. Thus, let $p(A)$ be the joint prior distribution which is made by the product of Student $t$ distributions of the elements collected in $\alpha$. Then, we need to specify priors for the inverse diagonal elements of the variance-covariance matrix of the structural errors $D$ conditional on $A$. The priors for $d^{-1}_{ii}$ (which denotes the element in row and column $i$ of matrix $D$) conditional on $A$ is given by a $\Gamma(\kappa_i, \tau_i)$ distribution, as follow:

$$p(D|A) = \prod_{i=1}^{n} p(d_{ii}|A)$$

where $\kappa_i/\tau_i$ and $\kappa_i/\tau_i^2$ represent the first and second moments of $d^{-1}_{ii}$, respectively. Notice that, the parameter $\tau_i$ depends on $A$ whereas $k_i$ does not.

Following Baumeister and Hamilton (2015a) we calibrate the diagonal elements of $D$ from the residuals obtained by running ols regressions from the univariate autoregressive models of order 12. Moreover, we set the prior mean for $d^{-1}_{ii}$ equals to the reciprocal of the diagonal element of a matrix $ASA'$, where $S$ represents the sample variance covariance matrix of the univariate autoregressive models performed for each time-series.

We postulate $\tilde{b}_i$ is a row vector of random structural coefficients following a conditional normal multivariate distribution, $\tilde{b}_i|A, D \sim N(m_i, d_{ii}M_i)$ where $m_i$ can be interpreted as the best guess about $\tilde{b}_i$ before seeing the data and $M_i$ represents the level of uncertainty about the standard Minnesota prior.

We follow the approach proposed by Doan et al. (1984) in which the behaviour for a generic time-series can be represented by a random walk process with $m_i = 0$ and great
confidence to expect that coefficients related to higher lags are zero.\footnote{Following \cite{Baumeister2015b} we need to set three different values for the hyper-parameters of the random walk prior for the lagged coefficients. Thus, we set the parameter controlling the overall tightness of the prior to 0.5. We set the parameter that governs how quickly the prior for lagged coefficients tightness to zero as lag increase to 1. Finally, we put prior on the parameter governing the tightness of the prior for the constant term to 100. The latter is used to make the prior on the constant term irrelevant.}

In the end, the joint probability distribution of the prior information about the plausible values of the parameters is defined as:

\[
p(A, D, B_j) = p(A)p(D|A)p(B_j|A, D) \quad \text{for } j = 1, 2, \ldots, 24 \tag{12}
\]

In the second step, the \cite{Baumeister2015a}'s algorithm searches for a vector of values \( \hat{\alpha} \) that solves numerically a maximization problem of the target function \( q(\alpha) \). Thus, the vector \( \hat{\alpha} \) provides a reasonable guess for the posterior mean of \( \alpha \) while the matrix of second derivatives of \( q(\alpha) \) with respect to \( \alpha = \hat{\alpha} \) exploits information about the shape of the posterior distribution of \( \alpha \).

In other words, the second stage sets the initial values for the elements of \( A \) in order to inform the random-walk Metropolis Hasting algorithm, that is performed in the third step.

The last stage is designed to construct the joint posterior distribution of the parameters, that is \( p(A, D, B|Y_T) \), where \( Y_T \) represents the sample-data. According to \cite{Baumeister2015a}, we proceed as follow.

First, we use the Metropolis Hasting algorithm to generate draws from the posterior distribution of the contemporaneous structural matrix, that is \( p(A|Y_T) \). The iteration starts setting \( \alpha^1 = \hat{\alpha} \) and for a generic step \( l + 1 \) we generate a candidate \( \tilde{\alpha}^{(l+1)} \) as a sum of \( \alpha^l \) and the product between three components: (1) a vector of independent standard \textit{student t} variables with 2 degrees of freedom, (2) a scalar tuning parameter for 30% acceptance ratio and (3) the Cholesky factorization of the matrix capturing the curvature of the posterior distribution of the vector of unknowns parameters \( A \).

Then, we compare the value of the target function evaluated in \( \tilde{\alpha}^{(l+1)} \) and \( \alpha^{(l)} \), respectively. If \( q(\tilde{\alpha}^{(l+1)}) < q(\alpha^{(l)}) \), we set \( \alpha^{(l+1)} = \alpha^{(l)} \) with probability \( 1 - \exp[q(\tilde{\alpha}^{(l+1)}) - q(\alpha^{(l+1)})] \); otherwise we set \( \alpha^{(l+1)} = \tilde{\alpha}^{(l+1)} \). The value \( l \) indicates the number of iterations with the
first $D$ burn-in draws included. Thus, starting with $l = D + 1$, for each $\alpha^l$ we generate $\delta_i^l \sim \Gamma(k_i^*, \tau_i^* (A(\alpha^l)))$ for $i = 1, 2, 3, 4$ and take $D^l$ to be diagonal matrix whose elements $d_{ii}^l = 1/\delta_{ii}^l$.

Finally, from the posterior distribution of the variance covariance matrix of the structural error terms we can further generate $\hat{b}_i^l \sim \mathcal{N}(m_i^*, d_{ii}^l M_i^*)$ for $i = 1, 2, 3, 4$, where $\hat{b}_i^l$ is the row vector of lagged structural parameters referred to the $i$th variable.

In the end, the triple $\{A(\alpha^l), D^l, B^l\}_{l=1}^{D+N}$ represents a sample size $N$ of posterior distribution:

$$p(A, D, B | Y_T) = p(A | Y_T)p(D | A, Y_T)p(B | A, D, Y_T)) \quad (13)$$

with the first $D$ burn-in draws equals to 200,000 and $N = 200,000$. 

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