Debt and Transfer Pricing: Implications on Business Tax Policy

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**Summary**

In this article we introduce model to describe the behavior of a multinational company (MNC) that operates transfer pricing and debt shifting, with the purpose of incrementing its value, intended as the sum of equity and debt. We compute, in a stochastic environment and under default risk, the optimal shares of profit and debt to be shifted and show how they are affected by exogenous features of the market. In addition, by means of a numerical analysis, we simulate and quantify the benefit arising from the exploitation of tax avoidance practices and study the corresponding impact on MNC's fundamental indicators. A wide sensitivity analysis on model's parameters is also provided.

**Keywords**: Capital Structure, Default Risk, Business Taxation and Welfare

**JEL Classification**: H25, G33, G38

We wish to thank the participants at the First Joint Workshop on Business Taxation, University of Brescia - University of Mannheim, held in Brescia in January 2020, for their useful comments. We are responsible for any remaining error.

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Abstract

In this article we introduce model to describe the behavior of a multinational company (MNC) that operates transfer pricing and debt shifting, with the purpose of incrementing its value, intended as the sum of equity and debt. We compute, in a stochastic environment and under default risk, the optimal shares of profit and debt to be shifted and show how they are affected by exogenous features of the market. In addition, by means of a numerical analysis, we simulate and quantify the benefit arising from the exploitation of tax avoidance practices and study the corresponding impact on MNC’s fundamental indicators. A wide sensitivity analysis on model’s parameters is also provided.

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1 Introduction

The empirical literature on debt shifting started with studies on US and Canadian companies (see, e.g., the pioneering articles by Collins and Shackleford (1992) and Froot and Hines (1995)), whereas the research in Europe started later.\(^1\) Similarly, the research on transfer pricing started in the USA (see, e.g., Grubert and Slemrod (1998)) and has more recently shown that MNCs shift income to low-tax subsidiaries in order to minimize their overall tax expenses around the world (see, e.g., Dischinger et al. (2014); Dischinger (2010); Devereux and Maffini (2007); Hines and Rice (1994)). Quite interestingly, this is a worldwide phenomenon.

Despite the existence of several empirical articles, only a few of them have jointly analyzed profit and debt shifting.\(^2\) For this reason, we develop a stochastic model where both options are feasible. In doing so, we depart from Schindler and Schjelderup (2016), who use a deterministic model to show that concealment costs may be complementary (substitutes), i.e., a higher leverage reduces (increases) marginal concealment costs of transfer pricing. Of course, concealment cost substitutability exists if marginal concealment costs related to transfer pricing rise (drop) when debt shifting increases (decreases).

Our aim is to show that, assuming a stochastic Earning Before Interests and Taxes (EBIT), default risk must be accounted for and results crucially

\(^1\)For instance, Ramb and Weichenrieder (2005) showed that the tax rates of the parent companies have no statistically significant effect on their subsidiaries’ leverage, whereas Overesch and Wamser (2014) studied the effects of parent companies’ tax rates on their own capital structure. Moreover, using the effective cross-border tax rates Huizinga et al. (2008) estimated a negative impact of parent company taxation. As shown by Miniaci et al. (2014) however, the effects of a change in parent company tax rate are much more complex, because taxes affect both a MNC’s borrowing decision and the distribution of debt among its entities. Accordingly, the meta-analysis of the empirical literature on corporate capital structure by Feld et al. (2013) emphasized the complexity of tax effects at a multinational level. Based on 48 studies, they estimate a marginal tax effect on the debt ratio of about 0.27, that is, the debt-to-assets ratio rises by 2.7% if the marginal tax rate increases by 10%. When however they focus on the capital structure of foreign subsidiaries, taxation has a more complex impact, as the tax sensitivity of inter-company debt financing is particularly strong. Overall, their meta-analysis does not support the idea that the international tax system affects the financing decisions of multinational firms. These results show that there is room for further research aimed at focusing on firms’ heterogeneity.

\(^2\)This point is stressed by Schindler and Schjelderup (2016), who maintain that the existing tax literature studies debt shifting and transfer pricing separately.
depend on several parameter values. In particular, we show that a change in multinational company’s (MNC’s) EBIT has a twofold effect: on the one hand, it increases the potential benefit arising from transfer pricing and, 

ceteris paribus, reduces the default risk. This latter effect allows to take a better benefit from debt shifting, as already pointed out by empirical literature. Schenkelberg (2020) for instance studies both transfer pricing (TP) and debt shifting (DS) and finds that the former device is on average 85% of the increase in pre-tax earnings while less than 15% is attributable to debt shifting activities. These findings are substantially confirmed by our study. On the other hand, a decrease in MNC’s profitability leads to an increased default risk. Of course, these tax saving opportunities crucially depend on the characteristics of concealment costs. Since there is no evidence about the characteristics of these cost functions, we will let them be separate and convex.\(^3\) We will also assume that the optimal shares of transfer pricing and debt shifting, together with the default threshold level of default are optimally chosen by shareholders (this assumption is in line with Leland (1994) and Goldstein et al. (2001)). The optimal debt level is obtained by maximizing the levered value of a representative MNC. In doing so, we will allow lenders and shareholders to decide the leverage ratio together.

The remaining part of this article is structured as follows. Section 2 introduces the model describing the behavior of a representative MNC. Section 3 provides a numerical analysis. A set of sensitivity analyses is also added to show the robustness our results. Section 4 summarizes our findings and discusses policy implications.

2 The model

2.1 Random process

In this section we introduce a continuous-time model, in line with Goldstein et al. (2001). We will then focus on a representative MNC’s EBIT, that allows us to deal with volatility and the risk of default. By assumption, EBIT, that is denoted as \(\Pi\), follows a Geometric Brownian Motion (GBM):

\[
\frac{d\Pi_t}{\Pi_t} = \mu dt + \sigma dz_t, \tag{1}
\]

\(^3\)The use of a joint concealment cost, depending on both transfer pricing and debt shifting, is left for future research.
where $\Pi_0 > 0$ is its initial value, $\mu$ and $\sigma$ are the drift and the instantaneous standard deviation, respectively. Moreover, $dz_t$ is the increment of a Weiner process. In line with Dixit and Pindyck (1994), we let $\delta = r - \mu$ be positive. In this framework we also assume that the firm can borrow from a perfectly competitive credit sector, where the discount factor is the risk-free interest rate $r$. Moreover, we introduce the following:

**Assumption 1** At time 0, shareholders set the profit threshold $\bar{\Pi}$ below which the default occurs, together with the optimal level of transfer pricing and debt shifting, to maximize the value of equity.

**Assumption 2** Still at time 0, the firm borrows some resources thereby paying a non-renegotiable coupon $C$, which is set by the lenders to maximize the value of the whole company.

**Assumption 3** If the firm does not meet its obligations, default occurs and hence the firm is expropriated by the lender and looses access to credit market.

Assumption 1 implies that shareholders behave as if they own a put option, whose exercise leads to default.\(^4\)

Assumption 2 means that the firm sets a coupon and then computes the debt market value. Without arbitrage, this is equivalent to first setting the debt value and then calculating the effective interest rate. For simplicity, we assume that debt cannot be renegotiated: this means that we apply a static trade-off approach where the firm’s financial policy cannot be reviewed later.\(^5\)

Assumption 3 introduces the risk of default, which occurs if the firm’s EBIT falls to a given threshold value $\bar{\Pi}$. In this case, the MNC is expropriated by the lender who bears the cost of default and then becomes shareholder: our firm’s operations continue generating further EBIT. It is worth noting that a tax saving due to debt-finance arises as long as the business tax rate is higher than the lender’s rate (see, e.g., Panteghini (2007b)). For simplicity and without loss of generality, we let the lender’s pre-default tax rate be nil. When, however, default takes place, the lender becomes shareholder and is therefore subject to corporate taxation.

\(^4\)For further details on the characteristics of default conditions see, e.g., Leland (1994) and Panteghini (2007a).

\(^5\)The analysis of a dynamic trade-off model, where firms can subsequently adjust their capital structure, is left for future research.
2.2 Net profit of the multinational company

For simplicity, we assume that our representative MNC holds two branches: A and B, located in two different countries, where relevant tax rates are \( \tau_A \) and \( \tau_B \), respectively. Both subsidiaries contribute to holding’s overall profit \( \Pi \) and coupon \( C \). For this reason we assume that a portion \( \theta \in (0, 1) \) of EBIT is produced by the subsidiary located in A. The remaining portion \( (1 - \theta) \) is produced in country B.

In line with the empirical literature, we let the MNC shift a share \( \alpha \in [0, 1] \) of \( \Pi \) from the high-tax country to the low-tax one. Analogously, a share \( \gamma \in [0, 1] \) of \( C \) can be shifted from the low-tax country to the high-tax one, under the assumption the interest expenses are at least partially deductible. It is well-known that shifting EBIT and debt is costly. For simplicity we assume that both the cost of transfer pricing, denoted as \( \phi(\alpha) \), and the one of debt shifting, i.e, \( \nu(\gamma) \), have a quadratic form:\(^6\)

\[
\phi(\alpha) = \frac{m}{2} \alpha^2 \quad \text{and} \quad \nu(\gamma) = \frac{n}{2} \gamma^2, \tag{2}
\]

where \( m \) and \( n \) are scale parameters. Given these assumptions, our MNC’s overall net profit \( \Pi^N \) is equal to:

\[
\Pi^N = \left[1 - \tilde{\tau} (\tau_A, \tau_B)\right] (\Pi - C) + \left[(\tau_B - \tau_A) \alpha - \phi(\alpha)\right] \Pi + \left[(\tau_A - \tau_B) \gamma - \nu(\gamma)\right] C, \tag{3}
\]

where \( \tilde{\tau} \) is the effective tax rate without tax avoidance:\(^7\)

\(^6\)This choice is motivated by the lack of empirical evidence about the (hidden) cost of such operations. However, despite its simplicity, the functional form we propose introduces a penalty which is more than proportional to the shifted share \( \alpha \) or \( \gamma \), implicitly setting a limit to the exploitation of these techniques.

\(^7\)Let A be the branch toward which both transfer pricing and debt shifting are carried out. Its shares of MNC’s profit and coupon, equal to \( \theta \Pi \) and \( \theta C \) without transfer pricing or debt shifting, thanks to these practices are increased to \( (\theta + \alpha) \Pi \) and \( (\theta + \gamma) C \) respectively. In reverse, the same shares of branch B become \( (1 - \theta - \alpha) \Pi \) and \( (1 - \theta - \gamma) C \). As the cost of these operations, as from equation (2), are \( \phi(\alpha) \Pi \) and \( \nu(\gamma) C \) respectively, \( \Pi^N \) is defined as:

\[
\Pi^N = (1 - \tau_A) [(\theta + \alpha) \Pi - (\theta + \gamma) C] + (1 - \tau_B) [(1 - \theta - \alpha) \Pi - (1 - \theta - \gamma) C] - \phi(\alpha) \Pi - \nu(\gamma) C. \tag{4}
\]

The effective tax-rate without transfer pricing or debt shifting \( \tilde{\tau} \), as a function of \( \tau_A \),
\[ \tilde{\tau} (\tau_A, \tau_B) = \tau_B - (\tau_B - \tau_A) \theta. \] (6)

In addition, we notice that when DS is possible, effective tax rate is higher than in the other case, as \((\tau_A - \tau_B)\gamma - \nu (\gamma) > 0.\)

### 2.3 The value of equity

A MNC’s value coincides with the value of equity, \(E (\Pi)\), if debt is nil. Otherwise, using the notation of Dixit and Pindyck (1994), \(E (\Pi)\) is equal to the sum of instantaneous profit plus the expected capital gains, before default. When however, the MNC is debt financed and default occurs, \(E (\Pi)\) goes to zero, i.e.,:

\[
E (\Pi) = \begin{cases} 
0 \quad \text{a.d.} \\
\Pi^N dt + e^{-rdt} \mathbb{E} [E (\Pi + d\Pi)] \quad \text{b.d.}
\end{cases}
\] (7)

where \(\mathbb{E}\) is the expected value operator. Labels “a.d.” and “b.d.” stand for “after default” and “before default”, respectively. As proven in Appendix A.1.1, equation (7) can be rewritten as:

\[
E (\Pi) = \begin{cases} 
0 \quad \text{a.d.} \\
[1 - \tilde{\tau} (\tau_A, \tau_B)] \left(\frac{\Pi}{\delta} - \frac{\gamma}{\tau} \right) + \left[(\tau_B - \tau_A) \alpha - \phi (\alpha) \right] \frac{\Pi}{\delta} \\
+ [(\tau_A - \tau_B) \gamma - \nu (\gamma)] \frac{\Pi}{\tau} + \sum_{i=1}^{2} A_i \Pi^{\beta_i} \quad \text{b.d.}
\end{cases}
\] (8)

As shown in Appendix A.1.2, we set \(A_1 = 0\) to avoid financial bubbles. Moreover, we solve the equation for \(A_2\) at point \(\Pi = \Pi\) and obtain:

\[
E (\Pi) = [1 - \tilde{\tau} (\tau_A, \tau_B)] \left(\frac{\Pi}{\delta} - \frac{\gamma}{\tau} \right) + (\tau_B - \tau_A) \left(\alpha \frac{\Pi}{\delta} - \gamma \frac{\Pi}{\tau} \right) - \phi (\alpha) \frac{\Pi}{\delta} - \nu (\gamma) \frac{\Pi}{\tau}
- \left[1 - \tilde{\tau} (\tau_A, \tau_B) \right] \left(\frac{\Pi}{\delta} - \frac{\gamma}{\tau} \right) + (\tau_B - \tau_A) \left[\alpha \frac{\Pi}{\delta} - \gamma \frac{\Pi}{\tau} \right] - \phi (\alpha) \frac{\Pi}{\delta} - \nu (\gamma) \frac{\Pi}{\tau}
\right) \left(\frac{\Pi}{\tau}\right)^{\beta_2}
\] (9)

\(\tau_B\) and \(\theta\), is obtained by solving the following equation:

\[
1 - \tilde{\tau} (\tau_A, \tau_B) = (1 - \tau_A) \theta + (1 - \tau_B) (1 - \theta). \] (5)

Finally, by rewriting \(\Pi^N\) as a function of \(\tilde{\tau}\) and rearranging, equation (3) follows.

8The after-tax cost of debt, as from equation (3), is \(1 - \tilde{\tau} (\tau_A, \tau_B) + (\tau_A - \tau_B) \gamma + \nu (\gamma)\). Thanks to the definition of \(\gamma^*\) later derived, the last two addends can be rewritten as \((\tau_A - \tau_B)^2 (2n)^{-1}\), which is always positive, given the existence of a tax differential.
According to Goldstein et al. (2001), shareholders are assumed to choose the optimal default timing. Moreover, we also let them choose the optimal tax avoiding strategy. Their problem is therefore the following:

$$\max_{\Pi, \alpha, \gamma} E(\Pi).$$ \tag{10}

As shown in Appendix A.1.3, the solution of this problem leads to the optimal controls for $\alpha$ and $\gamma$:

$$\alpha^* = \frac{\tau_B - \tau_A}{m} \quad \text{and} \quad \gamma^* = \frac{\tau_A - \tau_B}{n}. \tag{11}$$

As can be seen an increase in $m$ and $n$ reduces the absolute value of $\alpha^*$ and $\gamma^*$. Moreover, the trigger point below which default take place will then be:

$$\Pi^* = \frac{\beta_2}{\beta_2 - 1} \left[ 1 - \tilde{\tau}(\tau_A, \tau_B) \right] - \frac{(\tau_A - \tau_B)^2}{2n} \delta \equiv \Delta C,$$ \tag{12}

where $\Delta < 1$. It is worth noting that, coeteris paribus, $m$ and $n$ affect not only the absolute value of $\alpha^*$ and $\gamma^*$, but also the optimal threshold in $\Pi^*$. In particular, an increase (decrease) in either $m$ or $n$ raises (reduces) $\Pi^*$, thereby increasing (decreasing) the probability that $\Pi$ hits $\Pi^*$. In other terms, an increase (decrease) in either $m$ or $n$ raises (reduces) the default risk. A sensitivity analysis about the effects of parameter changes will be provided in section 3.3.

Given, these results, we can rewrite 9 as:

$$E(\Pi) = \left[ 1 - \tilde{\tau}(\tau_A, \tau_B) \right] \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) + \frac{(\tau_B - \tau_A)^2}{2n} \Pi + \frac{(\tau_A - \tau_B)^2}{2m} \frac{C}{r}$$ \tag{13}

$$- \left\{ \left[ 1 - \tilde{\tau}(\tau_A, \tau_B) \right] \left( \frac{\Pi}{\delta} - \frac{C}{r} \right) + \frac{(\tau_B - \tau_A)^2}{2n} \Pi + \frac{(\tau_A - \tau_B)^2}{2m} \frac{C}{r} \right\} \left( \frac{\Pi}{\delta} \right)^{\beta_2}. $$

2.4 The value of debt

In order to calculate the value of debt $D(\Pi)$, we will follow the same procedure. That is, before default debt is equal to the sum of the instantaneous coupon plus its expected change due to default. It is worth noting that, after default, the value of $D(\Pi)$ does not fall to zero. As pointed out by assumption 3, the MNC goes on producing: in this case, the lender will benefit from
the future net profit flow. As proven by Comincioli et al. (2019), a second default cannot occur anymore. Thus, the value of debt after default is equal to a portion \( \Omega \in (0, 1) \) of the discounted perpetual rent of future net profit:

\[
D(\Pi) = \left\{ \begin{array}{ll}
\Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{C dt + e^{-rdt}\delta [D (\Pi + d\Pi)]} & \text{a.d.} \\
\frac{\delta}{r} + \sum_{i=1}^{2} B_i \Pi^\beta & \text{b.d.}
\end{array} \right.
\] (15)

As proven in Appendix A.2.1, equation (15) can be rewritten as:

\[
D(\Pi) = \left\{ \begin{array}{ll}
\Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{C} & \text{a.d.} \\
\frac{\delta}{r} + \sum_{i=1}^{2} B_i \Pi^\beta & \text{b.d.}
\end{array} \right.
\] (16)

Moreover, as shown in Appendix A.2.2, assuming the absence of financial bubbles (i.e., \( B_1 = 0 \)) and solving for \( B_2 \) at point \( \Pi = \bar{\Pi} \) gives:

\[
D(\Pi) = \left\{ \begin{array}{ll}
\Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{\delta} & \text{a.d.,} \\
\frac{\delta}{r} + \left( \Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{\delta} - \frac{C}{r} \right) (\frac{\Pi}{\bar{\Pi}})^{\beta_2} & \text{b.d.}
\end{array} \right.
\] (17)

After default, the lender chooses the optimal level of transfer pricing (see Appendix A.2.3):

\[
\max_{\alpha} D(\Pi) \quad \text{a.d.}
\] (18)

which coincides with (11), i.e., \( \alpha^* = \frac{\bar{\tau}_B - \bar{\tau}_A}{m} \). Hence, the value of debt is equal to:

\[
D(\Pi) = \left\{ \begin{array}{ll}
\Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{\delta} & \text{a.d.,} \\
\frac{\delta}{r} + \left( \Omega \frac{[1 - \tau_A (\theta + \alpha) + (1 - \tau_B (1 - \theta - \alpha) - \phi (\alpha)] \Pi}{\delta} - \frac{C}{r} \right) (\frac{\Pi}{\bar{\Pi}})^{\beta_2} & \text{b.d.}
\end{array} \right.
\] (19)

2.5 The value of multinational company

Let the overall MNC’s value be defined as the sum of its equity and debt:

\[\Pi^N = [(1 - \tau_A) (\theta + \alpha) + (1 - \tau_B) (1 - \theta - \alpha) - \phi (\alpha)] \Pi.\] (14)
\[ V(\Pi) = E(\Pi) + D(\Pi). \] (20)

This value can be made explicit by substituting the values of equity and debt before default, shown in equations (13) and (19), together with the definition of optimal trigger shown in equation (12):

\[
V(\Pi) = [1 - \bar{\tau}(\tau_A, \tau_B)] \left( \frac{1}{\beta} - \xi \right) + \frac{(\tau_B - \tau_A)^2}{2m} \left( \frac{U}{\beta} - \xi \right) + \frac{(\tau_A - \tau_B)^2}{2n} \left( \frac{C}{\beta} \right) + \left[ \left( \Omega - 1 \right) \left( 1 - \bar{\tau}(\tau_A, \tau_B) \right) \right] \left( \frac{\Delta C}{\beta} \right) \left( \frac{C}{\beta} \right) + \left( \frac{\tau_B - \tau_A}{2m} \right) \left( \frac{\Delta C}{\beta} \right) \left( \frac{C}{\beta} \right) \left( \frac{1}{\beta} \right).
\] (21)

As shown in Appendix A.3, the optimal control of \( C \) is such that (21) is maximized. The result is:

\[
C^* = \left[ - \frac{\bar{\tau} + \frac{(\tau_A - \tau_B)^2}{2n}}{r(1 - \beta^*_C)} \left( \frac{1}{\Omega - 1} \right) \left( 1 - \bar{\tau}(\tau_A, \tau_B) \right) \left( \frac{\Delta C}{\beta} \right) \left( \frac{C}{\beta} \right) \right]^{-\frac{1}{\beta^*_C}}.
\] (22)

As can be seen, parameter values have a non-linear impact on endogenous variables. For this reason, we will run a numerical analysis.

3 A numerical analysis

3.1 Purpose and parameters

The consequences of the exploitation of transfer pricing (TS) and debt shifting (DS) on the MNC are next investigated. To do so, we will use a numerical approach. Our analysis is focused on four MNC’s key indicators: the value of equity \( E \), the value of debt \( D \), the overall value \( V \) and the leverage ratio \( L \).\(^{10}\) The behavior of these indicators is studied with respect to the relevant tax rate in country B, namely \( \tau_B \), and with respect to \( \mu \), which proxies the dynamics of profitability.

More in detail, the purpose of this exercise is twofold. Firstly, we evaluate if and how much both transfer pricing and debt shifting affect MNC’s indicators. Section 3.2 contains our main results. Secondly, in Section 3.3 we perform a sensitivity analysis aimed at evaluating the impact of changes in some of the relevant parameter values. The parameters accounted for are:

\(^{10}\)The leverage ratio is defined as the ratio between the value of debt and overall value, namely \( L = DV^{-1} \).
future profitability $\mu$, EBIT’s diffusion $\sigma$, the relevant tax rate $\tau_B$ – which in turn affects the tax differential between countries $\tau_A - \tau_B$ – as well as the costs of transfer pricing and debt shifting.\footnote{For sake of simplicity, we focus on the MNC’s value. All other results are available upon request.}

The benchmark values of variables used for this analysis are shown in table 1. The starting value of relevant tax rates in country A and B are 0.15 and 0.25, respectively: this differential would make transfer pricing feasible. The drift $\mu$ and the diffusion $\sigma$ of the GMB are equal to 0.02 and 0.2, respectively (in line with Dixit and Pindyck (1994)). In order to normalize the results, the current value of $\Pi$ (2.5) and $r$ (0.02) are such that perpetual rent $\Pi/r$ is equal to 100. Unfortunately, the evidence on tax avoidance costs is poor. For this reason we arbitrarily set the scale parameters $m$ and $n$ equal to 0.05 and 0.1, respectively.\footnote{Although these values are arbitrary set, we let $n = 2m$ to reflect the increased cost of debt shifting, due to the widespread use of thin cap rules.} Finally, with no loss of generality, we assume that $\theta = 0.5$, and that the cost of default $1 - \Omega$ is equal to 0.2.

For all the parameters not object of sensitivity analysis we verified that their change does not affect the quality of results.

### 3.2 Effects of tax avoidance practices

In this section we provide a numerical analysis of the behavior of our representative MNC, when tax avoidance is feasible. This numerical simulation is based on the parameter values of table 1. The only exception is represented by the scale parameters of TP and DS costs, that have been properly set to define the following scenarios: (i) both transfer pricing and debt shifting are exploited, (ii) only debt shifting is feasible, (iii) only transfer pricing is allowed and (iv) tax avoidance is impossible (this may happen if both $m$ and $n$ are set to zero).

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<tr>
<td>$\theta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$1 - \Omega$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1: Benchmark values of parameters and variables used in the numerical simulations.


n are high enough. In what follows we show the effects of changes in both \( \mu \) and \( \tau_B \) on \( V \).

![Figure 1: Effects on value function, expressed as functions of future profitability \( \mu \) (left panel) and of effective tax rate \( \tau_B \) (right panel), of different availability of tax avoidance practices.](image)

In the left panel of Figure 1, \( V \) is shown as a function of future profitability \( \mu \). Since the higher the drift, the higher the future expected profitability is, \( V \) is always increasing in \( \mu \). Moreover, the highest (lowest) value is found when both (neither) tax avoidance practices are available. Interestingly, we also see that the effect of TP is always more relevant than the one of DS. The greater effect of TP has already been highlighted in scientific literature. This result is in line with Schenkelberg (2020), who estimated that about 85% of the tax avoidance benefit is due to TP. The remaining 15% is due to debt shifting activities. Here, we find similar values. With all parameters set to their benchmark values and with \( \mu = 0.01 \), the portion of benefit arising from TP (DS) is 83.3% (16.7%). In the right panel, we focus again on \( V \) as a function of \( \tau_B \). Since, for simplicity, \( \tau_A \) is set equal to 0.15, a change in \( \tau_B \) leads to a change in the tax rate differential. Obviously, the greater the tax rate differential, the greater the tax benefit is. If however, the equality \( \tau_A = \tau_B \) holds, no benefit is ensured. Like the previous case, \( V \) is higher (lower) when both (neither) practices are allowed.

\[ \text{Notice that when either } m \text{ or } n \text{ are higher than 3, tax avoidance is no longer feasible.} \]
Figure 2: Effects on value function, expressed as function of future profitability $\mu$ (left panel) and effective tax rate $\tau_B$ (right panel), of different values of $\tau_B$ and $\mu$.

### 3.3 Sensitivity analysis

To better understand our results we also run some sensitivity analysis.

Figure 2 shows how $V$ is affected by $\mu$ and $\tau_B$ (on the horizontal axis). The left panel shows that $V$ is increasing in $\mu$ for any value of $\tau_B$: of course, this is in line with our previous findings. Moreover, the higher the rate $\tau_B$, the greater the tax differential and the higher the value $V$ is. This happens as obviously a higher tax benefit leads to an increased $V$. For example, in $\mu = 0.01$, we find that an increase of $\tau_B$ its benchmark value (.25) to .30 increases $V$ by 17.8%. $^{14}$ The right panel shows that $V$ is increasing in both tax differential, regardless of its sign, and $\mu$. For example, in $\tau_B = 0.225\%$, an increase in $\mu$ from its benchmark value (0.02) to 0.021 leads to a dramatic increase in $V$ equal to 25.1%. The shape of $V$ is due the fact that the higher the tax differential, the higher the MNC’s value is. In all cases, the minimum is obtained when $\tau_A = \tau_B$.

Figure 3 shows the effect of $m$ on $V$. To do so we set $\mu$ (left panel) and $\tau_B$ (right panel) on the horizontal axis, respectively. As can be seen, $V$ is decreasing in $m$. This depends on the fact that the higher the parameter $m$, the more costly the TP is. For example, in $\mu = 0.01$, an increase in $m$ from its benchmark value (0.05) to 0.5 leads to a decrease in $V$ equal to $-10\%$.

Figure 4 shows the effect of $n$ on $V$. To do so we set $\mu$ (left panel) and

$^{14}$When the tax differential is small enough, i.e., below 5%, the tax avoidance benefit vanishes.
Figure 3: Effects on value function, expressed as functions of future profitability $\mu$ (left panel) and of effective tax rate $\tau_B$ (right panel), of different values of scale parameter cost of transfer pricing $m$.

Figure 4: Effects on value function, expressed as functions of future profitability $\mu$ (left panel) and of effective tax rate $\tau_B$ (right panel), of different values of scale parameter cost of debt shifting $n$. 
τ_B (right panel) on the horizontal axis, respectively. As can be seen, V is decreasing also in n, since the higher the parameter n, the more costly the DS is. For example, in μ = 0.01, an increase in n from its benchmark value (0.1) to 1 leads to a decrease in V equal to −2.2%, thus we notice that V is more sensitive to variations in m than in n.

Figure 5: Effects on value function, expressed as functions of future profitability μ (left panel) and of effective tax rate τ_B (right panel), of different values of GBM’s diffusion σ.

5 finally shows the effect of σ on V. To do so we set μ (left panel) and τ_B (right panel) on the horizontal axis, respectively. As can be seen, V is slightly decreasing in σ, since the higher the volatility of profit, the lower the value of E is, that also overcomes the modest benefit taken by D. For example, in μ = 0.01, an increase in σ from its benchmark value (0.2) to 0.25 leads to an increase in V equal to only −1.6%.

4 Conclusion

In this article we have shown that, in a framework characterized by a stochastic profit and by the presence of default risk, the exploitation of tax avoidance practices produces a measurable benefit to a representative MNC. This conclusion is not only the result of the derivation of a theoretical model, but it has also been confirmed by an extensive numerical analysis.

Despite the limit of separate concealment costs, unlike most literature, our study has the advantage of allowing the simultaneous exploitation of both
transfer pricing and debt shifting. Thanks to this, we were able to assess the contribution that the exploitation of these practices makes to the value of MNC. In addition, these results obtained through numerical simulations have proved to be extremely aligned with the empirical evidence in the literature.

A Appendix

A.1 The value of equity

A.1.1 The calculation of (8)

In order to derive equation (8), it is necessary to rearrange the net profit, defined in equation (3), as:

$$\Pi^N = [- (1 - \tilde{\tau}) + (\tau_B - \tau_A) \gamma - \nu (\gamma)] C + [(1 - \tilde{\tau}) + (\tau_B - \tau_A) \alpha - \phi (\alpha)] \Pi,$$

that is as the sum of a constant term, $a \equiv [- (1 - \tilde{\tau}) + (\tau_B - \tau_A) \gamma - \nu (\gamma)] C$, and another one proportional to $\Pi$, $b \equiv [(1 - \tilde{\tau}) + (\tau_B - \tau_A) \alpha - \phi (\alpha)]$. After having applied Itô’s lemma to define the increment $dE(\Pi)$, equation (7) can be rewritten as the following second order differential equation:\footnote{The dependency of $E$ on $\Pi$ is omitted to lighten the notation.}

$$\frac{\sigma^2}{2} \Pi^2 E_{\Pi\Pi} + \mu \Pi E_{\Pi} - r E = -a - b \Pi. \quad (23)$$

The solution of equation (23), given the functional form of the forcing term, can be guessed to be:

$$E = H_0 + H_1 \Pi + A \Pi^\beta,$$

that, by substitution, leads to:

$$\frac{\sigma^2}{2} \Pi^2 \beta (\beta - 1) A \Pi^{\beta - 2} + \mu \Pi (H_1 + \beta A \Pi^{\beta - 1}) - r (H_0 + H_1 \Pi + A \Pi^\beta) + a + b \Pi = 0,$$

which is satisfied if:

$$\begin{cases}
\frac{\sigma^2}{2} \beta (\beta - 1) + \mu \beta - r = 0 \\
\mu H_1 - r H_1 + b = 0 \\
-r H_0 + a = 0
\end{cases}.$$
From the second and the third equations it easily follows that $H_0 = ar^{-1}$ and $H_1 = b(r - \mu)^{-1}$, while the first equation leads to:

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}},$$

where $\beta_1 > 1$ and $\beta_2 < 0$. It finally follows that the explicit form of the general solution of equation (23) is:

$$E(\Pi) = \left[-(1 - \tilde{\tau}) + (\tau_B - \tau_A)\gamma - \nu(\gamma)\right] \frac{C}{r} + [(1 - \tilde{\tau}) + (\tau_B - \tau_A)\alpha - \phi(\alpha)] \frac{\Pi}{r - \mu} + \sum_{i=1}^{2} A_i \Pi^{\beta_i},$$

that, after some rearrangements, leads to equation (8).

A.1.2 The calculation of (9)

To avoid the presence of financial bubbles, the constant associated to $\beta_1 > 1$ must be set equal to 0, leaving only the constant $A_2$ to be computed. Since in correspondence of $\Pi$ the value of equity before and after default must be equal, it holds that:

$$E(\Pi) = [1 - \tilde{\tau}(\tau_A, \tau_B)] \left(\frac{\Pi}{r} - \frac{C}{r}\right) + (\tau_B - \tau_A) \left[\alpha \frac{\Pi}{r} - \gamma \frac{C}{r}\right] - \nu(\gamma) \frac{C}{r} - \phi(\alpha) \frac{\Pi}{r} + A_2 \Pi^{\beta_2} = 0,$$

from which the value of $A_2$ easily follows:

$$A_2 = -\left\{[1 - \tilde{\tau}(\tau_A, \tau_B)] \left(\frac{\Pi}{r} - \frac{C}{r}\right) + (\tau_B - \tau_A) \left[\alpha \frac{\Pi}{r} - \gamma \frac{C}{r}\right] - \nu(\gamma) \frac{C}{r} - \phi(\alpha) \frac{\Pi}{r}\right\} \Pi^{-\beta_2},$$

which, once substituted in equation above, leads to equation (9).

A.1.3 The calculation of (11), (12) and (13)

To find the optimal controls of $\Pi$, $\alpha$ and $\gamma$ that maximize the value of equity shown in equation (9), it is necessary to set all the partials equal to zero. To ease the calculations, we exploit the fact that the derivatives of $E(\Pi)$ and $A_2(\Pi)$ with respect to $\Pi$ are equal:

$$\frac{\partial E(\Pi)}{\partial \Pi} = \frac{\partial A_2(\Pi)}{\partial \Pi} = (\beta_2 - 1) \frac{\Pi}{r} \{[1 - \tilde{\tau}(\tau_A, \tau_B)] + (\tau_B - \tau_A)\alpha - \phi(\alpha)\} - \beta_2 \left\{[1 - \tilde{\tau}(\tau_A, \tau_B)] \frac{C}{r} + (\tau_B - \tau_A)\gamma \frac{C}{r} + \nu(\gamma) \frac{C}{r}\right\} = 0,$$

from which it follows that:
that after some rearranging, leads to:

\[ \Pi^* = \frac{\beta_2}{\beta_2 - 1} \{ [1 - \bar{\tau}(\tau_A, \tau_B)] + (\tau_B - \tau_A) \alpha - \phi(\alpha) \} C. \tag{24} \]

With regard to optimal share of transfer pricing \( \alpha \) and debt shifting \( \gamma \), it must hold:

\[ \frac{\partial E(\Pi)}{\partial \alpha} = [(\tau_B - \tau_A) - m\alpha] \left[ \frac{\Pi}{\delta} - \frac{\Pi}{\delta} \left( \frac{\Pi}{\Pi} \right)^{\beta_2} \right] = 0 \]

and:

\[ \frac{\partial E(\Pi)}{\partial \gamma} = - (\tau_B - \tau_A + n\gamma) \left[ 1 + \left( \frac{\Pi}{\Pi} \right)^{\beta_2} \right] = 0. \]

whose solution easily leads to equation (11). By substituting into equation (24) the optimal controls for \( \alpha \) and \( \gamma \), it follows equation (12). Finally, by substituting into equation (9) the values of \( \alpha^* \) and \( \gamma^* \), the explicit definition of \( E(\Pi) \), as in equation (13), finally follows.

A.2 The value of debt

A.2.1 The calculation of (16)

After having applied Ito’s lemma to define the increment \( dD(\Pi) \), the value of debt before default in equation (15) can be rewritten as the following second order differential equation:\(^{16}\)

\[ \frac{\sigma^2}{2} \Pi^2 D_{\Pi\Pi} + \mu \Pi D_{\Pi} - rD = -C. \tag{25} \]

The solution of equation (25) can be guessed to be:

\[ D = K + B\Pi^\beta, \]

that, by substitution and after some rearrangements, leads to:

\(^{16}\)The dependency of \( D \) on \( \Pi \) is omitted to to lighten the notation.
\[
\left[\frac{\sigma^2}{2} \beta (\beta - 1) + \mu \beta - r\right] B \Pi^\beta - r K + C = 0,
\]

which is satisfied if:
\[
\begin{cases}
\frac{\sigma^2}{2} \beta (\beta - 1) + \mu \beta - r = 0 \\
-r K + C = 0
\end{cases}.
\]

From the second equation, it easily follows that \( K = Cr^{-1}, \) while the first one is equal to the one in the case of equity and then leads to the same \( \beta_1 \) and \( \beta_2. \) It finally follows that the explicit form of the general solution of equation (25), that immediately leads to the value of debt before default in equation (16).

**A.2.2 The calculation of (17)**

For the same reason detailed in section A.1.2, \( B_1 \) must be set equal to 0, leaving only the constant \( B_2 \) to be computed. Since in correspondence of default trigger \( \Pi \) the value of debt before and after default must be equal and set to zero, it holds that:

\[
D(\Pi) = \frac{C}{r} + B_2 \Pi^{\beta_2} = \Omega \left[ (1 - \tau_A) (\theta + \alpha) + (1 - \tau_B) (1 - \theta - \alpha) - \phi(\alpha) \right] \frac{\Pi}{\delta},
\]

from which, also recalling the definition of effective tax rate shown in equation (5), the value of \( B_2 \) easily follows:

\[
B_2 = \left[ \Omega \left[ (1 - \tilde{\tau} (\tau_A, \tau_B)) + (\tau_B - \tau_A) \alpha - \phi(\alpha) \right] \frac{\Pi}{\delta} - \frac{C}{r} \right] \Pi^{\beta_2},
\]

which once substituted in equation above leads to equation (17). The value of debt after default can be simplified, in the same way, by the definition of effective tax rate.

**A.2.3 The calculation of (19)**

The derivative with respect to \( \alpha \) of the value of debt after default, defined in equation (17), is:

\[
\frac{\partial D(\Pi)}{\partial \alpha} = \Omega \left[ (1 - \tau_A) - (1 - \tau_B) - m \alpha \right] \frac{\Pi}{\delta},
\]

which, once set equal to zero, immediately leads to \( \alpha^* \), which is the same solution of shareholders’ problem before default. By substituting this result into equation (17) and after some rearrangement, equation (19) easily follows.
A.3 The value of multinational company

The value of the MNC defined in equation (21) can be rearranged as:

\[
V(\Pi) = \left\{ \left[1 - \tau(\tau_A, \tau_B)\right] + \frac{(\tau_B - \tau_A)^2}{2m} \right\} \frac{\Pi}{\delta} + \left\{ \tau(\tau_A, \tau_B) + \frac{(\tau_A - \tau_B)^2}{2n} \right\} \frac{C}{\delta}
+ \left\{ \left(\Omega - 1\right) \left[1 - \tau(\tau_A, \tau_B)\right] + \frac{(\tau_B - \tau_A)^2}{2m} \right\} \frac{\Delta}{\delta} - \left\{ \tau(\tau_A, \tau_B) + \frac{(\tau_A - \tau_B)^2}{2n} \right\} \frac{1}{\delta} C^{1-\beta_2} \left(\frac{\Pi}{\delta}\right)^{\beta_2},
\]

whose derivative with respect to \(C\) is:

\[
\frac{\partial V(\Pi)}{\partial C} = \left\{ \tau(\tau_A, \tau_B) + \frac{(\tau_A - \tau_B)^2}{2n} \right\} \frac{1}{\delta}
+ \left(1 - \beta_2\right) \left(\frac{\Pi}{\delta}\right)^{\beta_2} \left\{ \left(\Omega - 1\right) \left[1 - \tau(\tau_A, \tau_B)\right] + \frac{(\tau_B - \tau_A)^2}{2m} \right\} \frac{\Delta}{\delta} - \left\{ \tau(\tau_A, \tau_B) + \frac{(\tau_A - \tau_B)^2}{2n} \right\} \frac{1}{\delta} C^{-\beta_2}.
\]

By setting it equal to zero and rearranging, (22) follows.
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