Residential and Industrial Energy Efficiency Improvement: A Dynamic General Equilibrium Analysis of the Rebound Effect

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Summary

The aim of this paper is to investigate bi-directional spillovers into residential and industrial sectors induced by energy efficiency improvement (EEI) in both the short- and long-term, and the impact of nesting structure as well as the size of elasticities of substitution of production and utility functions on the magnitude and the transitional dynamic of rebound effect. Developing a dynamic general equilibrium model, we demonstrate that residential EEIs spillovers into the industrial sector through the labor supply channel and industrial EEIs spill-overs into the residential sector through the conventional income channel. Numerical simulations calibrated on the U.S. suggest that not taking into account these spillover effects could lead to mis-estimate the rebound effect especially of residential sector EEIs. We also demonstrate how the size and the duration of the rebound effect depend on the value of elasticities of substitution. Especially, the elasticity of substitution between energy and non-energy consumption in household utility and the elasticity of substitution between physical capital and labor in production play a major role. Numerical simulations suggest that alternative sets of value for the elasticities of substitution may give sizable different patterns of rebound effects in both the short- and long-term. In policy terms, our results suggest that energy efficiency policies should be implemented simultaneously with rebound effect offsetting policies by considering short- and long-term wide-economy feedbacks. As a consequence, they recall for considering debates about what type of policy pathways is more effective in mitigating the rebound effect.

Keywords: Energy efficiency, Rebound effect, Transitional dynamics, Residential energy consumption, Industrial energy consumption

JEL Classification: D58, Q43

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Residential and industrial energy efficiency improvements: A dynamic general equilibrium analysis of the rebound effect

Sondès Kahouli* and Xavier Pautrel†

November 25, 2020

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1 Introduction

While the rebound effect due to energy efficiency improvement (EEI) has been extensively stud-
ied, empirically and theoretically, in both manufacturing and residential sector, all or almost all these
works, at our best knowledge, dealt with rebound effect in each sectors separately. Nevertheless, en-
ergy efficiency improvements in one sector could spill-over into the second sector, affecting the rebound
effect on sectoral energy consumption and on global energy consumption. The purpose of the paper
is to fill the gap by building a theoretical model in order to investigate the channels through which
energy efficiency improvements in industrial sectors could affect energy consumption in the residential
sector, and vice-versa, both in the short- and long-term. It also focuses on the impact of nesting
structure as well as the size of elasticities of substitution of production and utility functions on the
magnitude and the transitional dynamic of rebound effect.

There is a large body of theoretical and empirical literature dealing with energy efficiency and
rebound effect that are expected to happen. In general, energy efficiency refers to the amount of
output that can be produced with a given input of energy. It is usually measured as the amount of
energy output for a given energy input and listed as a percentage between 0% and 100%.

In many cases, improvements in energy efficiency results in energy savings that are lower than
expected. Sometimes, it can even induce an increase in energy consumption. This phenomenon is
called the rebound effect or Jevons’ paradox (Jevons 1865). It appears when consumers do not sim-
ply replace an old product with a more efficient one which fulfills the same function, but upgrade to
a larger product and often involves supplementary energy consumption that offset part of the energy
savings. So far, the rebound effect has been mainly associated with energy use and the question
how energy efficiency improvements affect energy consumption. Khazzoom (1980, 1987) and Brookes
(1990) proposed a precise definition of the rebound effect, which can easily be applied to resource use
in general. According to this definition, if technological progress makes equipment more energy effi-
cient, less energy is needed to produce the same amount of product or service. However, the amount
of product or service usually does not stay the same. Because the equipment becomes more energy
efficient, the cost per unit of product or service that is produced with this equipment falls which,
in turn, increases the demand for the product or the service. Since 1980s, several empirical studies
confirm the existence of rebound effect with respect to, both, residential and firms energy demand.
The economic literature recognize the existence of rebound effect but disagree about its magnitude.

The definition of the rebound effect encompasses different mechanisms that may reduce energy
savings derived from the improvements in energy efficiency (Greene et al. 1999, Sorrell and Dim-
tropolous 2008, Orea et al. 2015). Through three typologies of rebound effects, this definition
distinguishes a microeconomic and macroeconomic views of such phenomenon:

• **Direct rebound effects.** It is a pure price effect. It refers to the fact that improved energy
efficiency for a particular energy service will decrease the effective price of that service and
should therefore lead to an increase in its consumption (Khazzoom 1980). This will tend to offset
the reduction in energy consumption provided by the efficiency improvement. For consumers,
the direct effect of a price decrease may be decomposed into a substitution effect and an income
effect.

• **Indirect effects.** The lower effective price of the energy service may lead to an increase in the
demand for other goods, services. It also may lead to economic growth since it may induce an
increase in factors of production that require energy for their provision. The size of the indirect
effect for a consumer is dependent on the share of the consumer’s total income spent on energy

---

1Nevertheless, other types of output can also be used. For example, one can define energy efficiency as the ratio
of output of performance, service, goods or energy, to input of energy for a system. To illustrate these kinds of output,
thermal comfort in a building is an example of performance, transport of persons or of information is a service, a
smartphone is a good, the production of which requires energy. As for the system, it could be an individual energy
equipment, a building, an industrial process, a firm, a sector or an entire economy. Improvements in energy efficiency
may reduce the energy used by that system.

2Energy services refer to the commodity which is used or demanded, i.e. heating space and water, cooling space,
transportation, refrigeration.

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services. For firms in a given sector, indirect effect result from, both, the increased demand for non-energy inputs to their production process as a result of increased demand for production, and the effect of the lower cost of one sector’s output on production costs of other sectors.

- **Economy wide effects.** It captures the structural changes in the economy due to the variation of prices of goods and services subsequent to the improvement of energy efficiency. It means that a fall in the real price of energy services may reduce the price of intermediate and final goods throughout the economy, leading to a series of price and quantity adjustments, with energy-intensive goods and sectors likely to gain at the expense of less energy-intensive ones.

In addition to the large body of empirical literature estimating the size of the rebound effect in residential and industrial sectors⁴ the theoretical literature gives another comprehensive overview on the rebound effect. Most formal theoretical analyses of rebound effects have focused on partial equilibrium settings that hold some prices fixed (Dubin et al. (1986), Frondel et al. (2008), Greene et al. (1999), Klein (1987, 1988), Nadel (1993), Schwarz and Taylor (1995), West (2004), Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), and Van den (2011)). These studies assume that there are no changes in prices or nominal incomes following the efficiency improvement, and that the impacts are limited to the direct market for household energy use. This approach gives a partial equilibrium figure which is generally considered as the direct rebound effect. A few researches like for example Dufournaud et al. (1994) considered full general equilibrium economy-wide rebound effects from increased energy efficiency in the household sector. It examines the impacts of increasing efficiency in domestic wood stoves. Druckman et al. (2011), Freire-Gonzalez (2011) and Thomas and Azevedo (2013a, 2013b) use a fixed price input-output model to study indirect rebound effects resulting from household income induced by energy efficiency improvements and spent on non-energy commodities. This work considers changes in energy use in production, as well as household consumption. However, it is still studying the issue based on a partial equilibrium approach as it fails to incorporate endogenous prices, incomes or factor supply effects. As a consequence, there is a need to consider general equilibrium models to study in a more comprehensive context consequences of energy efficiency and the associated rebound effect.

Indeed, since a long time Jevons (1865) was especially worried about general equilibrium channels. More recently, his concern has been reinforced by insights from computable general equilibrium models suggesting a potential for strong rebound effects through economy-wide or alternatively indirect channels (Allan et al. 2009). In this context, many have called for theoretical research to illuminate these channels (Dimitropoulos 2007, Turner 2013, Borenstein 2015). Therefore, there has been extensive investigation of the economy-wide rebound effects resulting from energy efficiency improvements in production using a computable general equilibrium (CGE) modelling approach (Dimitropoulos 2007, Turner 2013). However, very few studies have attempted to measure the economy-wide impacts of energy efficiency improvements in the household sector (Lecca et al. 2014). More interestingly, there is also a very few studies of rebound effect by considering interactions between production and consumption sectors. For example, Lemoine (2020) develops an analytically general equilibrium framework to study the implications of improved energy efficiency⁵ but he did not take into account feedbacks between production and consumption sectors.

This paper is aimed at filling the gap by building a general equilibrium framework where we allow for interaction between residential and industrial sectors in order to study consequences of energy efficiency improvement, in particular, the magnitude and the short- and long-run transitional dynamic of the rebound effect. We consider a variety of possible feedbacks affecting energy use in both residential (household) and production (industry) sectors. In particular, we focus on bi-directional energy spillover effects expected to spread in the residential and industrial sectors and we look at general

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³The empirical literature usually focuses on the estimation of the direct rebound effect in the residential sector. However, measurements of the direct rebound effect for industrial and commercial sectors are very limited and focus on the short-run (Nadel (1993), Eto et al. (1995)).

⁴Lemoine (2020)’s theoretical contribution is a decomposition of the general equilibrium response to improved efficiency. He fully endogenize all prices, the supply of labor, and the supply of energy in an economy with an arbitrary number of consumption good sectors.
equilibrium channels through which these spillover operates. We quantify the rebound effect through numerical simulation after calibrating our model on the 2018 U.S. economy.

In this framework, we also investigate how important is the choice of, both, the nested structure of function, i.e. production function for firms and utility function for households, as well as the size of the elasticity of substitution between energy, labour, and physical capital in this production function and between energy and non-energy consumption in household utility function. This choice is particularly important for analyzing the magnitude and the short- and long-run transitional dynamics of rebound effect through its impact on the demand for energy and other inputs. Indeed, by studying all possible nesting structure for the constant elasticity of substitution, Van der Werf (2008) argues that the sizes of elasticities in the production function is a crucial parameter for climate policy modeling. This may also hold for energy and rebound effect modeling. For example, in Sorrell (2014) the ease of substitution between energy and other inputs and the size of the rebound effects is investigated thoroughly. By the same, Locca et al (2014) argue that a key parameter that drives rebound analysis is the elasticity of substitution between aggregate energy and non-energy goods and services in the household’s utility function.

Our main contributions are twofold. First, our theoretical model demonstrates that energy efficiency improvements in one sector may spillover into the other sector directly affecting its energy consumption. It also may affect the wide-economy energy consumption as well as the magnitude and the duration of the rebound effect in the sector in which the energy efficiency improvement occurred. The spillovers of EEIs in residential sector into industrial sector go through the labor supply channel and the spillovers of EEIs in industrial sector into residential sector go through the income channel. Numerical simulations calibrated on the U.S. economy show that such a spillovers affect the rebound effect in both the short- and long-term. They also show that not taking into account these spillovers effects could lead to mis-estimate the rebound effect especially for EEIs in the residential sector, in both the short- and long-term.

Our second main contribution is to demonstrate that the size and the duration of the rebound effect does not only depend on the value of elasticity of substitution between physical capital/labor and energy in the production function. Interestingly, they also depend on the value of elasticity of substitution between energy and non-energy consumption in household utility function and on the value of the elasticity of substitution between physical capital and labor. This result highlights the importance to introduce physical capital in the general equilibrium analysis of the rebound effect. Performing a comparative analysis of the short- and long-term effect of EEIs depending on whether we select values estimated by Van der Werf (2008) or values estimated by Bosetti et al. (2006) reveals sizable differences in the size and the duration of the rebound effect. This result calls for systematic evaluations of the elasticities of substitution between physical capital and labor in production and between non-energy and energy consumption in utility, in a similar way that it has been done for the elasticity of substitution between non-energy and energy inputs in production.

The paper unfolds as follows. In section 2, we present the theoretical model and we derive analytical results about the steady-state. In section 3, we compute numerical simulations (calibrated in the 2018 U.S. economy) for both the short- and long-term and investigate the role played by the elasticity of substitution between energy and physical capital/labor on one hand, and between physical capital and labor on the other hand. Finally, in Section 4 we present conclude and give some policy implications.

2 The model

In this section, we present the model, we derive the intertemporal general equilibrium and study the steady-state equilibrium.

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1By extension, in some modeling exercise taking into account the sector of the production of energy, it was shown that the key parameters in determining the size of rebound effects are elasticities of substitution in production of energy for other inputs as well as price elasticities of direct and derived demands for energy through trade – particularly in small open economies where energy itself is traded- Saunders (1992), Allan et al. (2007), Hanley et al. (2009).
2.1 Firms

We assume that firms produce an homogenous good, denoted \( y \), used to final and energy consumption in residential and producing sectors as well as to physical capital accumulation. They operate under perfect competition with a technology defined by the following nested Constant Elasticity Substitution (CES) production function:

\[
y(t) = A \left[ (1 - a)Z(t)^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + a \left( \varepsilon_f E_f(t) \right)^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \right]^{\frac{\sigma_{k,l,e}}{\sigma_{k,l,e} - 1}}
\]

with
\[
Z(t) \equiv \left[ \beta K^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + (1 - \beta)L^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \right]^{\frac{1}{\sigma_{k,l,e} - 1}}, \quad (a, \beta) \in [0, 1],
\]

where \( K \) denotes the aggregate stock of physical capital, \( L \) the labor supply, \( E_f \) the energy consumption in production and \( \varepsilon_f \) its efficiency.

We note \( \sigma_{k,l,e} \geq 0 \) the elasticity of substitution between capital/labor and energy, and \( \sigma_{k,l} \geq 0 \) is the elasticity of substitution between capital and labor.

Firms maximize their profits \( y(t) - r(t)K(t) - w(t)L(t) - p_f E_f(t) \), where \( p_f \) is the price of energy in production, leading to the first order conditions:

\[
r = A \left[ (1 - a)Z^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + a \left( \varepsilon_f E_f \right)^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \right]^{\frac{1}{\sigma_{k,l,e} - 1}} (1 - a)Z^\frac{1}{\sigma_{k,l,e}} \frac{\beta K^{\frac{1}{\sigma_{k,l,e} - 1}}}{\beta K^{\frac{1}{\sigma_{k,l,e} - 1}} + (1 - \beta)L^{\frac{1}{\sigma_{k,l,e} - 1}}} (2)
\]

\[
p_f = A \left[ (1 - a)Z^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + a \left( \varepsilon_f E_f \right)^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \right]^{\frac{1}{\sigma_{k,l,e} - 1}} a \varepsilon_f \frac{1}{\left( \varepsilon_f E_f \right)^{\frac{1}{\sigma_{k,l,e} - 1}}} (3)
\]

\[
w = A \left[ (1 - a)Z^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + a \left( \varepsilon_f E_f \right)^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \right]^{\frac{1}{\sigma_{k,l,e} - 1}} (1 - a)Z^\frac{1}{\sigma_{k,l,e}} \frac{(1 - \beta)L^{\frac{1}{\sigma_{k,l,e} - 1}}}{\beta K^{\frac{1}{\sigma_{k,l,e} - 1}} + (1 - \beta)L^{\frac{1}{\sigma_{k,l,e} - 1}}} (4)
\]

Using equation (2) we obtain:

\[
\frac{Z^{\frac{1}{\sigma_{k,l,e} - 1}}}{\varepsilon_f E_f} = \Omega(\varepsilon_f, p_f; \sigma_{k,l,e}) \equiv \left( \frac{\left( \frac{p_f}{a \varepsilon_f} \right)^{\sigma_{k,l,e} - 1}}{\sigma_{k,l,e} - 1} - a \right)^{\frac{1}{\sigma_{k,l,e} - 1}} > 0 \quad (5)
\]

**Lemma 1.** \( \partial \Omega(\varepsilon_f, p_f; \sigma_{k,l,e})/\partial \varepsilon_f < 0 \) and \( \partial \Omega(\varepsilon_f, p_f; \sigma_{k,l,e})/\partial p_f > 0 \), \( \forall \sigma_{k,l,e} \).

**Proof.** From equation (5). \( \square \)

Furthermore, equations (2) and (4) give the ratio physical capital to labor \( K/L \):

\[
\left( \frac{K}{L} \right)^{1/\sigma_{k,l,e}} = \left( \frac{\beta}{1 - \beta} \right) \frac{w}{r} \quad (6)
\]

Using equations (3) and (2), it comes:

\[
r = \left( \frac{1 - a}{a} \right) p_f \Omega(\varepsilon_f, p_f; \sigma_{k,l,e})^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} \frac{\beta E_f K^{\frac{1}{\sigma_{k,l,e} - 1}}}{\beta K^{\frac{1}{\sigma_{k,l,e} - 1}} + (1 - \beta)L^{\frac{1}{\sigma_{k,l,e} - 1}}} \left( A \varepsilon_f \right)^{1 - \sigma_{k,l,e}} \quad (7)
\]

Equation (3) enables us to rewrite equation (1) as:

\[
y = \frac{p_f E_f}{a} \left[ (1 - a)\Omega(\varepsilon_f, p_f; \sigma_{k,l,e})^{\frac{\sigma_{k,l,e} - 1}{\sigma_{k,l,e}}} + a \right] = \left( \frac{p_f}{a} \right)^{\sigma_{k,l,e}} \left( A \varepsilon_f \right)^{1 - \sigma_{k,l,e}} E_f \quad (8)
\]

From equations (3) and (4) we have:

\[
w = A(\varepsilon_f, p_f; \sigma_{k,l,e})(1 - \beta) \left[ \beta K^{\frac{1}{\sigma_{k,l,e} - 1}} + (1 - \beta)L^{\frac{1}{\sigma_{k,l,e} - 1}} \right]^{\frac{1}{\sigma_{k,l,e} - 1}} L^{\frac{1}{\sigma_{k,l,e} - 1}} \quad (9)
\]

where \( A(\varepsilon_f, p_f; \sigma_{k,l,e}) \equiv \frac{(1 - a)p_f}{\varepsilon_f} \Omega(\varepsilon_f, p_f; \sigma_{k,l,e})^{\frac{1 - a}{\sigma_{k,l,e}}} \).
Lemma 2. We have $\partial A(\varepsilon, p_f; \sigma_{c,e}) / \partial \varepsilon > 0$ and $\partial A(\varepsilon, p_f; \sigma_{c,e}) / \partial p_f < 0$, $\forall \sigma_{c,e}$. 

Proof. From Lemma 1.

Then, equations (6) and (7) give:

$$r = A(\varepsilon, p_f; \sigma_{c,e}) \beta \left[ \beta K^{1 - \frac{1}{\sigma_{c,e}}} + (1 - \beta)L^{1 - \frac{1}{\sigma_{c,e}}} \right]^{\frac{1}{\sigma_{c,e}}} K^{-\frac{1}{\sigma_{c,e}}}$$

(10)

Using equations (10) and (7) gives:

$$E_f = \frac{\beta K^{1 - \frac{1}{\sigma_{c,e}}} + (1 - \beta)L^{1 - \frac{1}{\sigma_{c,e}}}}{\varepsilon_f \Omega(\varepsilon, p_f; \sigma_{c,e})}$$

(11)

2.2 Households

There is a representative household that is infinitely lived, with preferences given by:

$$\int_{t=0}^{\infty} e^{-\rho t} U(\tilde{C}(t), \ell(t)) \, dt$$

(12)

where $\ell(t) \in [0, 1]$ is the fraction of his unitary time endowment devoted to leisure and $\tilde{C}(t)$ is aggregate consumption. $\rho > 0$ is the time discount rate and $U$ is the period utility function. Aggregate consumption is defined as:

$$\tilde{C}(t) = \left( 1 - b \right)c(t)^{\frac{\sigma_{c,e} - 1}{\sigma_{c,e}}} + b(\varepsilon_r E_r(t))^{\frac{\sigma_{c,e} - 1}{\sigma_{c,e}}}$$

(13)

with $b \in [0, 1[$

where $c(t)$ is the household consumption in final goods and $E_r(t)$ is the residential consumption of energy whose efficiency is captured by the elasticity $\varepsilon_r$. As for $\sigma_{c,e} \geq 0$, it represents the elasticity of substitution between final goods and energy consumptions.

The representative household owns physical capital that he rents to firms. He supplies $L = 1 - \ell$ of his time endowment as labor. His budget constraint is:

$$\dot{K}(t) = r(t)K(t) + w(t)(1 - \ell(t)) - c(t) - p_r E_r(t)$$

(14)

where $p_r > 0$ denotes the unitary price of residential energy services.

Household’s choices are represented by the following consumer’s problem:

$$\max_{c,\ell,K,E_r} \int_{t=0}^{\infty} e^{-\rho t} U(\tilde{C}(t), \ell(t)) \, dt \quad s.t. \quad \begin{cases} \dot{K}(t) = r(t)K(t) - w(t)(1 - \ell(t)) - c(t) - p_r E_r(t) \\ \tilde{C}(t) = \left( 1 - b \right)c(t)^{\frac{\sigma_{c,e} - 1}{\sigma_{c,e}}} + b(\varepsilon_r E_r(t))^{\frac{\sigma_{c,e} - 1}{\sigma_{c,e}}} \end{cases}$$

(15)

Lagrangian is written as:

$$\mathcal{H} = U(\tilde{C}, \ell) + \mu_k [rK - w(1 - \ell) - c - p_r E_r]$$

(16)

The first-order conditions are:

$$\frac{\partial \mathcal{H}}{\partial c} = 0 \quad \Rightarrow \quad U_c \tilde{C}_c - \mu_k = 0$$

(17)

$$\frac{\partial \mathcal{H}}{\partial \varepsilon_r} = 0 \quad \Rightarrow \quad U_c \tilde{C}_e - \mu_k p_r = 0$$

(18)

$$\frac{\partial \mathcal{H}}{\partial \ell} = 0 \quad \Rightarrow \quad U_\ell - \mu_kw = 0$$

(19)

$$\frac{\partial \mathcal{H}}{\partial K} = -\mu_k + \rho \mu_k \quad \Rightarrow \quad r \mu_k = -\mu_k + \rho \mu_k$$

(20)

Equations (17) and (18) give:

$$E_r = S(\varepsilon_r, p_r; \sigma_{c,e})c \quad \text{with} \quad S(\varepsilon_r, p_r; \sigma_{c,e}) = \left( \frac{b}{1 - \varepsilon_r} \right)^{\sigma_{c,e}} p_r^{-\sigma_{c,e}} \varepsilon_r^{\sigma_{c,e} - 1}$$

(21)
Lemma 3. We have \( \partial S(\varepsilon_r, p_r; \sigma_{c,e})/\partial \varepsilon_r \geq 0 \) when \( \sigma_{c,e} \geq 1 \) and \( \partial S(\varepsilon_r, p_r; \sigma_{c,e})/\partial p_r < 0 \ \forall \sigma_{c,e} > 0 \).

Proof. From equation (21).

As a consequence, utility given by (13) can be written as:

\[
\bar{C} = U(\varepsilon_r, p_r; \sigma_{c,e})c \quad \text{with} \quad U(\varepsilon_r, p_r; \sigma_{c,e}) = \left[ (1-b) + b (\varepsilon_r S(\varepsilon_r, p_r; \sigma_{c,e}))^{\sigma_{c,e}-1} \right]^{\sigma_{c,e}-1}
\]

(22)

Furthermore, leisure is expressed as a function of consumption and wage:

\[
U_t = w U C \bar{C}
\]

(23)

and the law of motion of consumption is given by

\[
U_{c,c} = (r - \rho) U_c - U_c \ell 
\]

(24)

2.3 Intertemporal general equilibrium

The aggregate resource constraint is:

\[
y = c + \bar{K} + p_r E_r + p_f E_f
\]

(25)

Equilibrium in labor market is described by:

\[
L = 1 - \ell
\]

(26)

and the aggregate energy use \( E \) is defined as:

\[
E = E_f + E_r
\]

(27)

Assuming for simplicity that:

\[
U(\bar{C}, \ell) \equiv \log \bar{C} + \varphi \log \ell
\]

(28)

then from equation (23), we obtain:

\[
\ell = \frac{\varphi c}{w}
\]

(29)

Using equations (25), (8), (11) and (29), we obtain the law of motion of \( K \):

\[
\dot{K} = A(\varepsilon_f, p_f; \sigma_{k,l}) \left[ \beta K \frac{\sigma_{k,l}^{-1}}{\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell) \right]^{\sigma_{k,l}^{-1}} \ell
\]

\[
\times \left\{ 1 - \left[ \frac{1 + p_r S(\varepsilon_r, p_r; \sigma_{c,e})} {\beta K \frac{\sigma_{k,l}^{-1}}{\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell)} \right]^{\sigma_{k,l}^{-1}} \right\}
\]

(30)

Using equation (10), equation (24) gives:

\[
\dot{c} = \left( A(\varepsilon_f, p_f; \sigma_{k,l}) \left[ \beta K \frac{\sigma_{k,l}^{-1}}{\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell) \right]^{\sigma_{k,l}^{-1}} \right) \beta K^{-\sigma_{k,l}^{-1}} \left( \beta K^{-\sigma_{k,l}^{-1}} - \rho \right) c
\]

(31)

Differentiating equation (29) with respect to time and using equation (9), the law of motion of the portion of time endowed in production is:

\[
\frac{\dot{\ell}}{\ell} = \left[ 1 + \left( \frac{\sigma_{k,l}^{-1} \beta K \frac{\sigma_{k,l}^{-1}}{\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell) \sigma_{k,l}^{-1}} {\frac{1}{\sigma_{k,l}^{-1}} \beta K^{-\sigma_{k,l}^{-1}} - \rho} \right) \right]^{-1} \times
\]

\[
\left[ \beta A(\varepsilon_f, p_f; \sigma_{k,l}) \left[ \beta K^{-\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell) \right]^{\sigma_{k,l}^{-1}} \right] \beta K^{-\sigma_{k,l}^{-1}} \times
\]

\[
\left\{ \frac{\sigma_{k,l}^{-1} - (1 - \sigma_{k,l}^{-1})}{\sigma_{k,l}^{-1}} \left[ 1 + p_r S(\varepsilon_r, p_r; \sigma_{c,e}) \right] \left( \beta K^{-\sigma_{k,l}^{-1}} + (1-\beta)(1-\ell) \right]^{-1} \right\} - \rho
\]

(32)
2.4 Steady-state analysis

At the steady-state, \( K = K^* \) and \( \ell = \ell^* \) are constant. As a result, from equation (32), \( \dot{\ell} = 0 \) gives:

\[
K^* = \frac{1}{1 - \ell^*} \beta K^* \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}} + (1 - \beta) \left( 1 - \ell^* \right) \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}}
\]

which defines the physical capital-labor ratio as a function of the energy-efficiency \( \varepsilon_f \):

\[
\frac{K^*}{1 - \ell^*} = \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) \equiv \left[ \frac{\beta A(\varepsilon_f, p_f; \sigma_{x,\ell,\ell})}{1 - \beta} \right]^{\frac{1}{\sigma_{\ell,k}-1}} - \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}}
\]  

(33)

Lemma 4. We have \( \partial \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) / \partial \varepsilon_f > 0 \) and \( \partial \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) / \partial p_f < 0 \), \( \forall (\sigma_{x,\ell,\ell}, \sigma_{x,\ell,\ell}) \).

Proof. From equation (33) and Lemma 2.

From equation (30) and (34), \( K = 0 \) gives:

\[
\ell^* = \left[ 1 + \frac{[1 + \rho S(\varepsilon_r, p_r; \sigma_{c,e})] (1 - \beta)}{\phi [\beta \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell})] \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}} + (1 - \beta)} \right]^{-1}
\]

(35)

Using equation (11) and (33), we obtain:

\[
E_f^* = \frac{\beta \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}} + 1 - \beta}{\varepsilon_f \Omega(\varepsilon_f, p_f; \sigma_{x,\ell,\ell})} \left[ \frac{1 + \rho S(\varepsilon_r, p_r; \sigma_{c,e})}{[\beta \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell})] \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}} + 1 - \beta} + \frac{\phi}{1 - \beta} \right]^{-1}
\]

(36)

Proposition 1. At the steady-state equilibrium, the consumption of energy in the manufacturing sector can be expressed as:

\[
E_f^* = E_f(\varepsilon_f, \varepsilon_r; p_r, p_f)
\]

1. If \( \phi = 0 \), \( \partial E_f(\cdot) / \partial \varepsilon_r = 0 \) and \( \partial \varepsilon_f(\cdot) / \partial \varepsilon_f > 0 \) when \( \sigma_{x,\ell} > 1 \).

2. If \( \phi > 0 \), \( \partial \varepsilon_f(\cdot) / \partial \varepsilon_r \geq 0 \) when \( \sigma_{c,e} \geq 1 \) and \( \partial \varepsilon_f(\cdot) / \partial \varepsilon_f > 0 \) when \( \sigma_{x,\ell} > 1 \).

Proof. From equation (36), Lemma 1 and Lemma 4.

Proposition 1 states that, energy efficiency improvements in the residential sector may spillover into industrial sector, when labor is endogenous. Then, if the elasticity of substitution between non-energy consumption and energy consumption in utility function is higher than unity, i.e. \( \sigma_{c,e} > 1 \), energy efficiency in the residential sector affects positively energy consumption in the industrial sector. Indeed, industrial energy consumption directly relies on production function (equation (11), thus, on labor -one to one- which is inversely related to leisure time (equation (35)). When \( \sigma_{c,e} > 1 \), the increase in residential energy efficiency, i.e. \( \varepsilon_r \), increases the ratio between residential energy consumption and non-energy consumption in utility (equation (21)). As a consequence, consumption and leisure time diminish, everything being equal, leading to a rise of labor supply which increases production and, therefore, energy consumption in the industrial sector, i.e. \( E_f \).

From equations (6) and (34), it comes:

\[
w^* = \left( 1 - \frac{\beta}{1 - \beta} \right) \rho \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) \frac{1}{\sigma_{\ell,k}}
\]

(37)

Then, using equations (29) and (33) we obtain:

\[
c^* = \rho \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell}) \frac{1}{\sigma_{\ell,k}} \left[ \frac{\beta (1 + \rho S(\varepsilon_r, p_r; \sigma_{c,e}))}{[\beta \kappa(\varepsilon_f, p_f; \sigma_{x,\ell,\ell})] \frac{\sigma_{\ell,k}-1}{\sigma_{\ell,k}} + 1 - \beta} + \frac{\phi \beta}{1 - \beta} \right]^{-1}
\]

(38)
Lemma 5. At the steady-state equilibrium, the aggregate consumption is such that:

$$c^* = C(\varepsilon_r, \varepsilon_f)$$

with $\partial C(\cdot)/\partial \varepsilon_r \geq 0$ if $\sigma_{c,e} \leq 1$ and $\partial C(\cdot)/\partial \varepsilon_f > 0$, $\forall \sigma_{k,l}$.

Proof. From equations (21), (38) and Lemma 4.

Using equations (21) and (38), we obtain the expression of the residential consumption of energy at the steady-state:

$$E_{r}^* = \kappa(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{1}{\sigma_{k,l}} S(\varepsilon_r, p_r; \sigma_{c,e}) \frac{1 + p_r S(\varepsilon_r, p_r; \sigma_{c,e})}{\beta K(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{\sigma_{k,l}}{\sigma_{k,l} - 1} + 1 - \beta} + \frac{\varphi}{1 - \beta} \right]^{-1}$$

(39)

Proposition 2. At the steady-state equilibrium, the consumption of energy in the residential sector can be expressed as:

$$E_{r}^* = \mathcal{E}_r(\varepsilon_f, \varepsilon_r; p_r, p_f)$$

with $\partial \mathcal{E}_r(\cdot)/\partial \varepsilon_f > 0$ when $\sigma_{k,l} \geq 1$ and $\partial \mathcal{E}_r(\cdot)/\partial \varepsilon_r \geq 0$ if $\sigma_{c,e} \geq 1$.

Proof. From equation (39).

Proposition 3 states that energy efficiency improvements in industrial sector may spillover in residential sector: when the elasticity of substitution between physical capital and labor in production is higher than unity, i.e. $\sigma_{k,l} > 1$, the energy efficiency improvement in industrial sector increases residential energy consumption. Indeed, the increase in industrial energy efficiency rises non-energy consumption (from Lemma 5), therefore, residential energy consumption, i.e. $E_r$ which is related to non-energy consumption (equation (21)). Interestingly, these spillover effects depend on the shape of the production function.

Finally, global energy consumption at the steady-state is:

$$E^* = \left[ \frac{1 + p_r S(\varepsilon_r, p_r; \sigma_{c,e})}{\beta K(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{\sigma_{k,l}}{\sigma_{k,l} - 1} + 1 - \beta} + \frac{\varphi}{1 - \beta} \right]^{-1} \times \left\{ \kappa(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{1}{\sigma_{k,l}} S(\varepsilon_r, p_r; \sigma_{c,e}) \frac{1}{\beta K(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{\sigma_{k,l}}{\sigma_{k,l} - 1} + 1 - \beta} + \frac{\beta K(\varepsilon_f, p_f; \sigma_{k,l,e}) \frac{\sigma_{k,l}}{\sigma_{k,l} - 1} + 1 - \beta}{\varepsilon_f \Omega(\varepsilon_f, p_f; \sigma_{k,l,e})} \right\}$$

(40)

Proposition 3. At the steady-state equilibrium, the global consumption of energy in the economy can be expressed as:

$$E^* = \mathcal{E}(\varepsilon_f, \varepsilon_r; p_r, p_f)$$

with $\partial \mathcal{E}(\cdot)/\partial \varepsilon_r \geq 0$ if $\sigma_{c,e} \geq 1$ and $\partial \mathcal{E}(\cdot)/\partial \varepsilon_f > 0$ if $\sigma_{k,l} > 1$.

Proof. From equation (40) and Propositions 1 and 2.

3 Numerical simulations

In this section, we present numerical simulations of our theoretical model that we calibrate on the 2018 U.S. economy. In sub-section 3.1, we explain calibration of preference, technology, and energy parameters. In sub-section 3.2, we discuss results.
3.1 Calibration

Preference parameters: Following Turnovsky and Garcia-Penalosa (2008), we set the elasticity of leisure in utility to 1.75 which leads to a steady-state leisure time close to two-thirds of individual time as in Prescott (2004). The value of the elasticity of substitution between energy and non-energy consumption in utility comes from Lemoine (2020) and is very close to the value reported by de Miguel and Manzano (2011). The time discount rate $\rho$ is chosen to obtain an annual interest rate equal to 4% as usual in the literature. Finally, we calibrate energy consumption share in utility $b$ in order to match the 2018 personal consumption expenditures net of energy expenditures to residential energy consumption ratio in the U.S. economy. Personal consumption expenditures come from the FRED Economic dataset of the Saint-Louis Federal Reserve and residential energy consumption are extracted from the U.S. Energy Information Administration (EIA) State Energy Data set.

Technology parameters: We follow the Real Business Cycle literature to set the share of physical capital in production $\alpha$ at 1/3. As for elasticities of substitution in the production function, we consider the elasticity of substitution between energy $E_f$ and non-energy factors $Z$ as well as between physical capital and labor. The question of what values to attribute to these elasticities is the subject of recurrent debate in the current state of empirical literature. As a consequence, in our model, we use as benchmark values the estimations made by Van der Werf (2008) for the U.S. and we will also investigate alternative values estimated by other authors. Otherwise, we approximate the share of energy in industrial production $a$ by the ratio energy expenditures in the industrial sector to GDP, using the data from the U.S. EIA State Energy Data set. Finally, we calibrate the scale parameter $A$ in the production function to match the 2018 ratio of residential energy consumption estimates to the industrial energy consumption estimates as published in the U.S. EIA State Energy Data set.

Energy parameters: Values of energy efficiencies are extracted from the 2016 American Council for an Energy-Efficient Economy (ACEEE) report. As for energy prices, they come from the 2018 U.S. EIA State Energy Data set.

In Table 1 we summarize benchmark parameter values and their sources.

Table 1: Benchmark parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preference</td>
<td>Elasticity of leisure in utility $\varphi$</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>Elasticity of substitution between energy and non-energy consumption $\sigma_{c,e}$</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>Subjective rate of time preference $\rho$</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Energy consumption share $b$</td>
<td>0.0084</td>
</tr>
<tr>
<td>Technology</td>
<td>Share of physical capital in production $\beta$</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>Productivity parameter $A$</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>Share of energy in industrial production $a$</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>Elasticity of substitution between energy and non-energy in production $\sigma_{k,l,e}$</td>
<td>0.5470</td>
</tr>
<tr>
<td></td>
<td>Elasticity of substitution between physical capital and labor in production $\sigma_{k,l}$</td>
<td>0.3191</td>
</tr>
<tr>
<td>Energy</td>
<td>Energy efficiency in the residential sector $\varepsilon_r$</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>- Energy efficiency in the industrial sector $\varepsilon_f$</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>- Unitary price of residential energy services $p_r$</td>
<td>23.30</td>
</tr>
<tr>
<td></td>
<td>- Unitary price of firm energy services $p_f$</td>
<td>10.67</td>
</tr>
</tbody>
</table>

3.2 Main results

The transitional impact of an energy efficiency improvement in the industrial sector ($\varepsilon_f$) is shown in Figure 1(a). The one of an energy efficiency improvement in the residential sector ($\varepsilon_r$) depicted in Figure 1(b).

Figures 1(a) and 1(b) report in the three first quadrants the deviations from the steady-state values of energy consumption, respectively in industrial sector ($E_f$), in residential sector ($E_r$), and in the economy ($E$) -expressed as a % change-. They also report in the three first quadrants the change induced by a 1% increase in, both, residential energy efficiency ($\varepsilon_r$) and production energy efficiency.
(εf). The forth quadrant shows the rebound effect in short- and long-term. In our benchmark case, a 1% increase in energy efficiency in industrial sector has a large negative impact on residential energy consumption in % and on wide-economy energy consumption compared to the impact of the same change on the industrial energy consumption. In the short-term the negative impacts are small and rise during the transition which operates very quickly (around one and a half year). Our numerical simulation also highlights that the rebound effect evolves from an initial short-term value close to unity to a long-term value around 0.3. This means that in the very short-term, the 1% increase in industrial energy efficiency induce a high partial rebound effect which vanishes to a small partial rebound effect in the long-term.

Regarding impacts of variations in the residential energy efficiency, i.e. Figure 1(b), results show that 1% increase in residential energy efficiency reduces energy consumption in, both, the short- and long-term, where the negative short-term impact is lower than the long-term one. The short-term rebound effect on the increase in residential energy efficiency is almost full.

\[ \text{Rebound effect is computed as } 1 + \frac{\Delta E}{E_i} / \frac{\Delta \epsilon_i}{\epsilon_i} \text{ with } i = r, f. \]
Figure 1. (a) Dynamic impact of $\Delta^+ \varepsilon_f=1\%$ (Benchmark case), (b) Dynamic impact of $\Delta^+ \varepsilon_r=1\%$ (Benchmark case), (c) Dynamic impact of $\Delta^+ \varepsilon_f=1\%$ (Exogenous labor vs benchmark case -red dashed curves-), (d) Dynamic impact of $\Delta^+ \varepsilon_r=1\%$ (Endogenous labor vs benchmark case -red dashed curves-)
Figures 1(c) and 1(d) respectively show the transitional effects of energy efficiency improvements when labor is exogenous and endogenous. They corroborate the statement of Proposition 1 regarding the short-term steady-state equilibrium: endogenous labor supply generates energy efficiency spillover effects going from the residential sector ($\epsilon_r$) into the industrial sector in which the energy consumption increases ($E_f$). In particular, Figure 1(d) indicates that not taking into account endogenous labor, that is abstracting from intersectoral spillovers, would lead to underestimate the impact of a 1% increase in residential energy efficiency on industrial energy consumption. It would also lead to overestimate the short- and long-term reduction in residential energy consumption after an energy efficiency improvement. It would also induce an overestimation (respectively underestimation) of the reduction in total energy consumption in the short-term (respectively in the long-term). Finally, it would lead to underestimate (respectively overestimate) the short-term rebound effect in the residential sector (respectively long-term).

In Appendix B, we analyse the robustness of our results. We compute deviations from the steady-state values of energy consumption in residential ($E_r$) and industrial sectors ($E_f$) as well as in the economy ($E$). By the same, we also compute deviations from key parameter values after 1% increase in, both, residential ($\epsilon_r$) and industry energy efficiency ($\epsilon_f$). Appendix B shows that the sign of the impacts found in our benchmark case remain the same for all of our alternative parameter values. This means that our benchmark results are reliable. Furthermore, they also show that the magnitude of the effects we highlight could be greater with other choice of parameter values meaning than according to peculiar national situation, the misestimates we highlighted could be larger.

3.3 The influence of the elasticity of substitution between capital/labor and energy

Theoretical and empirical literature emphasized how the magnitude and the size of the rebound effect vary depending on the nested structure and the value of the elasticity of substitution in the industrial sector (Saunders (1992), Allan et al. (2007), Van der Werf (2008), Hanley et al. (2009)).

In this section we aim to study the impact of varying the value of elasticity of substitution between capital/labor and energy $\sigma_{kl,e}$. In particular, we investigate how the the transitional dynamic found in the benchmark case varies depending on different value of this elasticity of substitution. By referring to Hassler et al. (2012), we investigate two alternative values: $\sigma_{kl,e} = 0.8$ and $\sigma_{kl,e} = 0.02$.

Figures 2(a) to 2(d) depict comparative dynamics with respect to the benchmark case plotted as red dashed curves. The main feature is that the transitional effects of energy efficiency improvements are not affected similarly by a high and low elasticity of substitution between energy and non-energy inputs. Effects depends on the location of EEIs: in the residential or in the industrial sector. The influence of energy efficiency improvement in the residential sector ($\Delta^+ \epsilon_r$) on energy consumption is dimly influenced by the level of the elasticity of substitution between energy and non-energy inputs but the spillover effect on industrial energy consumption is amplified when the elasticity of substitution between energy and non-energy inputs is high. The influence of energy efficiency improvement in the industrial sector ($\Delta^+ \epsilon_f$) with high elasticity of substitution between energy and non-energy inputs is reduced in terms of decrease in energy consumption in the industrial sector but amplified in terms of decrease in energy consumption in residential sector and in the economy. High elasticity of substitution between between energy and non-energy inputs in production leads to a negative rebound effect, that is a super-conservation situation (Saunders 2008).
Figure 2. (a) Dynamic impact of $\Delta^+ \varepsilon_f=1\%$ ($\sigma_{KL,E} = 0.8$ vs benchmark case -red dashed curves-), (b) Dynamic impact of $\Delta^+ \varepsilon_r=1\%$ ($\sigma_{KL,E} = 0.8$ vs benchmark case -red dashed curves-), (c) Dynamic impact of $\Delta^+ \varepsilon_f=1\%$ ($\sigma_{KL,E} = 0.02$ vs benchmark case -red dashed curves-), (d) Dynamic impact of $\Delta^+ \varepsilon_r=1\%$ ($\sigma_{KL,E} = 0.02$ vs benchmark case -red dashed curves-).
3.4 The influence of the elasticity of substitution between capital and labor

While recent empirical studies found a value of physical capital/labor elasticity lower than unity especially for the U.S., several empirical articles estimating a nested production function with energy found a unitary elasticity of substitution \( \sigma_{k,l} \). We aim to study how the value of \( \sigma_{k,l} \) influences the impacts of an energy efficiency variation on energy consumption and rebound effect.

Figures 3(a) and 3(b) show the transitional and long-term effects when \( \sigma_{k,l} = 1 \), a conventional value, and \( \sigma_{k,l} = 0.87 \), the upper-bound value of the estimations by Knoblach and Stockl (2020), compared with the benchmark case where \( \sigma_{k,l} = 0.3191 \). These figures highlight two results. First, the higher elasticity of substitution between physical capital and labor rises the time of transition of the economy towards the steady-state, meaning that in the presence of physical capital in production with an elasticity of substitution with respect to labor at the top range of the estimations, the transitional effects of energy efficiency improvements matters. Second, the magnitude of effects of an energy efficiency improvement are quite different with respect to the benchmark case when the elasticity is high. In this context, we observe opposite effects in the short-term with respect to long-term in the case of industrial energy efficiency improvement. In particular, the Figure 3(a) shows that a 1% energy efficiency improvement in the industrial sector leads to a lower decrease in industrial energy consumption and even an increase in residential energy consumption, generating a rebound effect higher than unity in the very short-term and negative in the long-term.

Figure 3(b) shows that differences are less important in the case of a 1% energy efficiency improvement in the residential sector (with respect to a 1% energy efficiency improvement in the industrial sector) but that short- and long-term effects have the same sign whatever the targeted sector. They also show that the impact of energy efficiency improvement on the industrial energy consumption is quite different according to the value of \( \sigma_{k,l} \). This can be explained by the fact the energy efficiency improvement in the residential sector spillovers towards the industrial sector through the endogenous labor channel (Proposition 1) leading to a lower decrease in overall energy consumption and a higher rebound effect in both the short- and long-term. In addition to highlight the key role of elasticity of substitution between physical capital and labor in determining the magnitude and the transitional dynamic of rebound effect, our results also point out the crucial need to take into account the physical capital when modeling the rebound effect. In particular, not taking into account physical capital would lead to overestimate or underestimate the rebound effect both during the transition stage and in the long-term.

Finally, to illustrate how important is the choice of elasticities of substitution, especially the elasticity of substitution between physical capital/labor and energy, \( \sigma_{k,l,e} \), and between physical capital and labor, \( \sigma_{k,l} \), Figures 3(c) and 3(d) compare transitional and long-term effects using estimated values of \( \sigma_{k,l,e} \) and \( \sigma_{k,l} \) by Van der Werf (2008) (our benchmark case) and by Bosetti et al. (2006) (\( \sigma_{k,l,e} = 0.4 \) and \( \sigma_{k,l} = 1 \)). Differences are quite sizable in both the short- and long-term whatever the type of the energy efficiency improvement. With Bosetti et al. (2006)’ parameter values, with a lower \( \sigma_{k,l,e} \) and a higher \( \sigma_{k,l} \), transitional impacts of EEIs are longer meaning that during a long time impacts of EEIs on residential and industrial energy consumption, as well as, on global energy consumption may be mis-estimated with Van der Werf (2008)’ parameters with respect to Bosetti et al. (2006)’ parameters. For example, Figure 3(c) shows that with Bosetti et al. (2006) parameter values, a 1% increase in industrial energy consumption would rise residential energy consumption by more than 1% during 5-10 years while, with Van der Werf (2008) parameter values, it would decrease residential energy consumption by more than 2% from the first year. In the same time, both choices of parameter values lead to similar decreases in residential energy consumption in the very long-term. Conversely, with Bosetti et al. (2006) parameter values, this 1% increase in industrial energy consumption would lead to a rebound effect higher than unity during 1-2 years and to a rebound effect lower than zero after 25 years while, with Van der Werf (2008) parameter values, it would lead quick to a stable small positive rebound effect.

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7Performing a meta-analysis from 77 studies between 1961 and 2017 for the U.S. economy, Knoblach and Stockl (2020) estimate a long-run meta-elasticity ranging from 0.45 to 0.87 for the aggregate economy. Using the U.S. private sector data for the period 1948-1998, Antrás (2004) rejects that the U.S. aggregate production function is Cobb-Douglas.
Our simulations highlight that different estimated values for the elasticities of substitution in production may lead to sizable differences in the magnitude and the duration of the rebound effect. They also show that, besides the elasticity of substitution between physical capital/labor and energy ($\sigma_{kl,e}$) which has been extensively investigated, the elasticity of substitution between physical capital and labor $\sigma_{k,l}$ and between energy and non-energy consumption in utility $\sigma_{c,e}$ should be carefully estimated when the rebound effect is empirically evaluated.
Figure 3. (a) Dynamic impact of $\Delta^+ \varepsilon_f = 1\%$ ($\sigma_{k,l} = 1$ vs $\sigma_{k,l} = 0.87$ -green dot-dashed curves- vs benchmark case -red dashed curves-), (b) Dynamic impact of $\Delta^+ \varepsilon_r = 1\%$ ($\sigma_{k,l} = 1$ vs $\sigma_{k,l} = 0.87$ -green dot-dashed curves- vs benchmark case -red dashed curves-), (c) Dynamic impact of $\Delta^+ \varepsilon_f = 1\%$ (Bosetti et al. (2006) vs Van der Werf (2008) -red dashed curves-), (d) Dynamic impact of $\Delta^+ \varepsilon_r = 1\%$ (Bosetti et al. (2006) vs Van der Werf (2008) -red dashed curves-).
4 Conclusion and policy implications

In this paper, we propose a dynamic general equilibrium model to study effects of energy efficiency improvements and the associated short- and long-term rebound effect. We focus on feedbacks between residential and industrial sectors to identify channels through which an energy efficiency improvement (EEIs) in one sector affect rebound effect in the other sector. We stress the crucial role of the choice of the nesting structure as well as of the size of elasticities of substitution in the production function of firms and in the utility function of households. In the case of the production function, we particularly analyze the impact of varying the elasticity of substitution between capital/labor and energy and the elasticity of substitution between capital and labor.

Our main contributions are twofold. First, we demonstrate that energy efficiency improvements in one sector may spillover into the other sector to directly affect its energy consumption, thus, the magnitude of associated rebound effect. It also affects the magnitude of the rebound effect in the sector in which the energy efficiency improvement occurred as well as the wide-economy energy consumption. We demonstrate that the spillovers of EEIs in residential sector into industrial sector go through the labor supply channel and that the spillovers of EEIs in industrial sector into residential sector go through the income channel. Numerical simulations show that such a spillovers affect the rebound effect in both the short- and long-term.

Our second main contribution is related to the role played by the elasticities of substitution in the size and duration of rebound effects, whatever the sector in which EEIs are implemented. Our theoretical model demonstrates that the size and the sign of inter-sectorial spillovers depends on the values of the elasticity of substitution between physical capital and labor in production and of the elasticity of substitution between non-energy and energy consumptions in utility. This result is in sharp contrast with studies who emphasized the role of the elasticity of substitution between physical capital/labor and energy. It also highlights the importance to introduce physical capital in the general equilibrium analysis of the rebound effect. Finally, it states the role played by household preferences in terms of energy, even in the consequences of EEIs in industrial sector. Our numerical simulations emphasize the importance of the values chosen for these elasticities of substitution. A comparative analysis of the short- and long-term effect of EEIs depending on whether we select values estimated by Van der Werf (2008) or values estimated by Bosetti et al. (2006) reveals sizable differences in the size and the duration of the rebound effect. This result calls for systematic evaluations of the elasticities of substitution between physical capital/labor and energy. It also highlights the importance to introduce physical capital in the general equilibrium analysis of the rebound effect. Finally, it states the role played by household preferences in terms of energy, even in the consequences of EEIs in industrial sector. Our numerical simulations emphasize the importance of the values chosen for these elasticities of substitution. A comparative analysis of the short- and long-term effect of EEIs depending on whether we select values estimated by Van der Werf (2008) or values estimated by Bosetti et al. (2006) reveals sizable differences in the size and the duration of the rebound effect. This result calls for systematic evaluations of the elasticities of substitution between physical capital and labor in production and between non-energy and energy consumptions in utility. This result calls for systematic evaluations of the elasticities of substitution between physical capital and labor in production and between non-energy and energy consumptions in utility. In a similar way that it has be done for the elasticity of substitution between non-energy and energy inputs in production.

A possible extension of our model is to take into account the sector of energy production and multiple consumption goods rather than one. It would allow for a more comprehensive analysis of sectoral interactions, thus, a more detailed study of the rebound effect through a tiny decomposition of the general equilibrium response to an energy efficiency improvement. An another interesting extension is to investigate alternative utility functions to understand how the shape of the utility function and the way non-energy consumption and energy consumption enters it would affect our results.

In policy terms, in the context of recurrent worldwide debates about energy, environmental, and climate policies, our results suggest that the type of energy efficiency policies to be implemented should take into account sectoral feedbacks, in particular, the potential for rebound effect. Academic literature has credibly demonstrated that rebound effect that may appear in one or several sectors can offset benefits for the economy from an energy efficiency policies. For this reason, the policy inaction on the rebound effect was for a while criticized. The definition and implementation of energy efficiency policies in particular, and of environmental and climate policies in general, should be associated with rebound mitigation strategies. This recall for reconsidering the place of the rebound effect issue in the policy agenda as well as policy pathways that can be effective in offsetting negative impacts of rebound effect.
A Overview of economy-wide rebound effect in general equilibrium models

Table A.1: Overview of economy-wide rebound effect in general equilibrium models

<table>
<thead>
<tr>
<th>Reference</th>
<th>Aim</th>
<th>Nesting structure</th>
<th>Elasticity of substitution with energy</th>
<th>Main results</th>
</tr>
</thead>
</table>
| Dufouraud et al. 1994 | Study the economic adjustment of households to technical energy improvements in the residential sector in Sudan. The energy improvement considered is the 100%, 150% and 200% improvement in efficiency of wood-burning stoves | Household’s utility | 0.2 and 0.4                           | • Significant rebound effect: between 47-77%.
|                 |                                                                     |                    |                                        | • The magnitude of the rebound effect depends on the size of the elasticity of substitution at the households utility level. |
| Semboja 1994    | Analyse the effects of energy management policies on the Kenyan economy: two scenarios are considered namely an improvement of energy production efficiency and an improvement in energy use efficiency. | EKL                | 1                                      | • Rebound effect greater than 100%,
|                 |                                                                     |                    |                                        | • The effect of improved energy sector production efficiency is stimulation of economic growth, generation and/or saving of foreign currency, improvement in the balance of payments, reduction of dependency on foreign energy resources, increasingly efficient use of commercial energy and reduction or release of pressure on the use of conventional energy forms. |
|                 |                                                                     |                    |                                        | • The impact of an increase in oil fuel use efficiency is to increase domestic output quantities and reduce their prices in all sectors as well as a general reduction in production costs in all economic sectors. |
| Vikstrom 2004   | Study of the rebound effect in the Swedish economy between 1957 and 1962 by considering 15% increase in energy efficiency in non-energy sectors and 12% increase in energy sectors. | (KE)L              | 0.06-0.87                             | • Rebound effect: 50-60%. |

8 Nesting structure deals with household’s consumption or production function. In the case of the production function, it combines inputs into pairs, or “nests”. For example, a nested production function with capital (K), labour (L) and energy (E) inputs, could take one of three forms, namely: K(LE); (KL)E; (KE)L. As for the household’s consumption function, nested structure reflects the way of choosing simultaneously the quantities of all commodities in the consumption bundle.

9 Elasticity of substitution with energy means the elasticity of substitution between energy and other inputs. Interpretation depends upon the nesting structure. For example, for K(LE) the elasticity of substitution refers to the elasticity of substitution between L and E, while for (KL)E it refers to elasticity of substitution between (KL) and E.

10 If the elasticities of substitution at utility levels are set at 0.009 and 0.400, respectively, a greater rebound is shown to occur in a specific type of household.

11 Considered energies (E) are electricity and fuel.
<table>
<thead>
<tr>
<th>Study</th>
<th>Model</th>
<th>Parameters</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washida [2004] Economies-wide model for rebound effect in Japan by assuming 1% of energy efficiency improvement in all sectors modelled as change in efficiency factor for use of energy in production</td>
<td>(KL)E 0.54</td>
<td>• Rebound effect equals to 53 in the central scenario.</td>
<td></td>
</tr>
<tr>
<td>Grepperud and Rasmussen [2004] Assessment of rebound effect through the introduction of energy efficiency improvements into various sectors of the economy: doubling of growth rates of energy productivity, where four sectors have electricity efficiency doubled and two have oil efficiency doubled.</td>
<td>(KE)L 0.01</td>
<td>• Significant rebound effects only for manufacturing: small for oil but &gt;100% for electricity.</td>
<td></td>
</tr>
<tr>
<td>Allan et al. [2007] Assess the impact of a 5% across the board improvement in the efficiency of energy use in all industrial sectors in the UK.</td>
<td>(KL)(EM) 0.3</td>
<td>• Rebound effects: 30?50%,</td>
<td></td>
</tr>
<tr>
<td>Hanley et al. [2009] Study the impact of 5% improvement in efficiency of energy use across all industrial sectors in Scotland in order to determine the impact on environmental quality and sustainability.</td>
<td>(KL)(EM) 0.3</td>
<td>• No backfire effect (no increase in energy use). Results are sensitive to the assumed structure of the labour market, key production elasticities, the time period under consideration and the mechanism through which increased government revenues are recycled back to the economy.</td>
<td></td>
</tr>
<tr>
<td>Turner [2009] Study the conditions under which rebound effects may occur in response to 5% exogenous (and costless) increase in energy efficiency in all industrial sectors in the UK</td>
<td>(KL)(EM) 0.3</td>
<td>• Electricity rebound effect: 59.6-23.1%,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Production: 0.064-2%. Trade: 0.064-5%</td>
<td>• Non-electricity energy rebound effect: 54.7-30.9%,</td>
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<tr>
<td></td>
<td></td>
<td>• Rebound effects are lower in the long-run,</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>• Negative income, competitiveness and disinvestment effects may partially or wholly offset the positive pressure for rebound effects even where (direct and indirect) demands for energy are very price inelastic.</td>
<td></td>
</tr>
</tbody>
</table>

12 In the sensitivity analysis, the author varies elasticity of substitution from 0.3 to 0.7 jointly with other parameters.

13 The 5% energy efficiency shock in industrial sectors equates to a shock affecting 3.6% of total electricity use and 2.97% of non-electricity energy use.

14 Desinvestment effects occur where falling energy prices reduce profitability in domestic energy supply sectors, leading to a contraction in capital stock in these sectors, which may in turn lead to rebound effects that are smaller in the long run than in the short run.
Study the factors influencing the impacts of energy efficiency as a form of technological change on absolute levels of CO2 emissions, on the carbon intensity of the economy (CO2 emissions relative to real GDP), and the per capita EKC relationship in Scotland. These factors include the elasticity of substitution between energy and non-energy inputs, responses in the labour market and the structure of the economy.

Improvements in energy efficiency produce a range of general equilibrium effects: a pure efficiency change, which reduces emissions and energy use, and substitution, competitiveness and structural change effects, which tend to increase energy use and a “rebound” in emissions. If countervailing effects outweigh the pure efficiency effect then the rebound effect becomes a backfire effect.

Lecca et al. (2014) Study of the economic impact of a 5% improvement in the UK household energy efficiency, focusing specifically on total energy rebound effects. Focus on comparing results from partial and general equilibrium.

Improving energy efficiency of using electricity has the largest positive impact on GDP among the five energy types. Inter-fuel substitutability does not affect the macroeconomic results significantly.

In general, macro-level rebound is larger than production-level rebound.

Primary energy goods show larger rebound effect than secondary energy goods.

Rebound effect should be considered through the long rather than the short-run.

### Table A.1 – Complete the previous page

<table>
<thead>
<tr>
<th>Study</th>
<th>Household’s utility</th>
<th>(KL)(EM)</th>
<th>0.4-1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turner and Hanley [2011]</td>
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<tr>
<td>Study the factors influencing the impacts of energy efficiency as a form of technological change on absolute levels of CO2 emissions, on the carbon intensity of the economy (CO2 emissions relative to real GDP), and the per capita EKC relationship in Scotland. These factors include the elasticity of substitution between energy and non-energy inputs, responses in the labour market and the structure of the economy.</td>
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<td>Lecca et al. [2014]</td>
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<tr>
<td>Lu et al. [2017]</td>
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<tr>
<td>Study economy-wide rebound effect of China in three dimensions: (i) different energy types, including coal, crude oil and gas, refined petroleum, electricity and steam supply and gas supply, (ii) long-run versus short-run closure, and (iii) inter-fuel substitutability. A one-off 5% efficiency improvement is imposed in all sectors in each of the scenarios.</td>
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</tbody>
</table>

15: E: energy and M: materials.

16: Authors consider an increase of household efficiency in the use of all sources of energy: coal, oil, gas and electricity.

17: This parameter represents the elasticity of substitution between energy and non-energy commodities in consumption.

18: Authors highlight how the value of the elasticity of substitution between energy and non-energy commodities in household consumption is important in determining the size of the rebound effect.


20: Authors set the elasticity of substitution among the energy inputs, i.e. coal, crude oil and natural gas, refined oil, electricity supply and gas supply, in the five energy sectors corresponding to the energy goods to be zero. However, in the other 130 sectors, they are set to be 0.5.

21: When inter-fuel substitutability is allowed, total rebound varies from 23.1 to 51.2% in the short-run and from -0.1 to 42% in the long-run. When inter-fuel substitutability is not allowed, it varies from -9.5 to 23.6% in the short-run and from -28.2% to 31.1% in the long-run.
<table>
<thead>
<tr>
<th>Lemoine (2020)</th>
<th>Study the rebound effect from energy efficiency innovation in the U.S.</th>
<th>Household’s utility</th>
<th>0.09-1.27%</th>
</tr>
</thead>
</table>

- General equilibrium channels reduce rebound in consumption good sectors from 39 to 28% but increase rebound in the energy supply sector from 42 to 80%.

\[22\] Author considers the elasticity of substitution between energy (E) and labor (L).
B Numerical analysis: sensitivity analysis

Figure B.1. (a) Deviation from the initial steady-state % with respect to $\phi$, (b) Deviation from the initial steady-state % with respect to $\beta$, (c) Deviation from the initial steady-state % with respect to $\sigma_{k,l}$, (d) Deviation from the initial steady-state % with respect to $\sigma_{k,l}$. 
Figure B.2: Deviation from the initial steady-state % with respect to $a$
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