TOTAL RETURN ECONOMICS
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abstract

Total return economics (TRE) is a new look at economic basics. It takes prediction of behavior as its objective, and reasons toward this objective from definitions in principle to definitions and inferences in practice. It finds for example that output, if defined in principle as creation of economic value, is implicitly total return or the sum of growth and yield. Likewise capital, if defined in principle as a store enabling future taste satisfactions, is implicitly the integral of the present value of those expected satisfactions time-discounted at the output rate (rate of return) at which it is expected to produce them. Although these algorithms are well-established for physical capital (“property” in TRE), TRE reasons that they are also tautologies or logical certitudes which apply equally to human capital (here called “self-value”).

This unusual approach yields surprises. Economists will be interested in its findings, for example, that:

- output is not consumption plus investment;
- the value of a worker’s output sold to his employer is less than half his pay;
- rate of return is not zero but several percent per year in the stationary state; and
- belt-tightening has little effect on overall growth.

extended abstract

Consumption is of three kinds. “Maintenance consumption,” or “maint,” is an expense recovered dollar for dollar in the value of work services and thence in product prices. “Invested consumption,” or “insumption,” is recovered dollar for dollar in self-value. “Waste consumption,” or simply “waste,” is recovered in taste satisfaction alone. Meanwhile yield either is “transfer yield” which conserves capital by reinvestment or by gift to other “owners,” or is “final yield” which exhausts capital in satisfying “final tastes.” Thus insumption is an example of transfer yield, while waste is an example of final yield.

Output is defined in principle as value added, meaning creation of new capital of either factor by old capital of either factor. Although it may also be possible to create means of immediate taste satisfaction without passing through a capital phase, this satisfaction would be a fortuitous free good, rather than final yield, and its creation would therefore not count as output. The “exhaust rule” concludes that all final yield, in order to be such and not free goods, must cost an equal “exhaust,” meaning annihilation, of the capital affording the yield.

We see at once that the consumption-plus-investment formula for output, which is taught as the foundation of macroeconomics, double-counts maint by including it in consumption while its whole value is recaptured in product prices as part of investment. Likewise the worker’s output or value added cannot be measured as the whole of his pay, which typically includes his maint, without the same double-counting. Rather his “sold work,” meaning sold output, is his pay less his “work expense,” meaning maint plus any “non-yield expense” such as bus fare.
which supports his pay without satisfying his tastes. This is the “sold work rule.” Non-yield expense is not part of consumption, and is not double-counted in the consumption-plus-investment formula.

The idea that pay equilibrates to sold work, which seems to be unquestioned in the literature, is easily refuted in a thought experiment. As self-value (human capital) declines toward zero at the end of productive life, so must its output. But pay can never be less than work expense unless the worker’s needs are bought from other sources. Thus pay declines toward the work expense level rather than toward zero as a downward limit at the end. Although this “last day parable” does not prove the sold work rule in general, it refutes tradition vividly. Data confirm that pay, on statistically significant scales, is never less than work expense at any age.

We saw that insumption is transfer yield, while waste is final yield. We now see that maint has a double character, and is both at once. As the worker’s expense, it is transfer yield converting consumption value to product value with no value added and at no net cost to the worker, since that cost is recovered from his employer and then from the market. But maint is also defined as the part of work expense which satisfies final tastes, and as such must be exhausted dollar for dollar from the capital affording it. This capital is not the consumption goods, since their value was converted into products rather than exhausted. We will see that it is uniquely the self-value which accomplished the conversion.

Insumption carries no such duality, but is transfer yield alone. To begin with, much insumption is the work of learning which satisfies no one’s final tastes. Much other insumption is “parental gift” which does not satisfy final tastes of the donors whose capital afforded it. Any taste satisfaction to donees, and any from self-afforded insumption, is a free good since its cost is recovered separately in new self-value. Therefore final yield is not the whole of consumption, but is limited to waste and to the taste satisfaction aspect of maint.

The heart of TRE is something called the “duplication rule.” The starting idea is that since capital is evaluated at the integral of time-discounted future yields it is expected to afford, and since the time discount on current yield is zero, expected yield equals expected loss or outflow of capital at each future moment. Assuming reasonably good prediction, then, meaning prediction which is imperfect but neither prevailingly optimistic nor prevailingly pessimistic, current yield should converge toward current loss of capital over large samples. Loss of capital can be understood as a kind of ex-ante yield, measured at value drained from the asset rather than value realized by the owner or his donees in return. Thus the yield of an ice cream cone in taste satisfaction will fall short of value drain if the cone slips to the floor, but will exceed outflow if such misadventures are less than the actuarial discount for them in the price of the cone. The difference between yield and value loss, which equilibrates to zero under reasonably good prediction, is a positive or negative “leakage” component in the “free growth” of the yielding asset.

Capital can also be evaluated at present value of expected final yields alone, if we include final yields which will eventuate through other assets as a consequence of transfer yields received from the first one. Although this evaluation from direct and indirect final yields is generally impractical, since it requires more knowledge of the future than we are likely to have,
it gives the same value as the first algorithm in principle. And since final yield is afforded from 
exhaust alone, parallel logic shows that final yield converges toward exhaust under the same 
good prediction.

This argument breaks no new ground in describing property, where we know that 
depreciation is recovered in cash flow. But TRE reasons that it is a tautology which must 
somehow hold for any capital, including self-value, given that capital is quantified at present 
value of expected yields. TRE now defines “senescence” as the expected or normative exhaust 
of self-value as age depletes the pool of expected future “work yield” (pay less non-yield 
expense) on which it is capitalized, and defines “self-liquidation” as the final yield somehow 
brought about by senescence and convergent to it.

Then self-liquidation is the exhaust of self-value recovered in final yield, and is 
convergent to senescence. But since self-liquidation does not satisfy tastes in itself, like the 
exhaust of an ice cream cone in the mouth, it must be cash liquidation which buys final tastes 
satisfaction in a separate form. What the cash buys is the taste satisfaction value of maint, even 
though the expense value was paid separately by buyers of products into which it was converted. 
This is clear because self-liquidation is the only available source of the capital exhaust implicit in 
this final yield, while maint is the only available disposition of this exhaust in satisfaction of 
final tastes.

The duplication rule might therefore be expressed in the relations

\[
\text{maint} = \text{self-liquidation} \quad \text{and} \quad \text{self-liquidation} \sim \text{senescence},
\]

where the operator \(\sim\) means “converges to.” Since self-liquidation is a concept much less easy to 
intuit than senescence, which simply means normal loss of self-value with age, we will prefer 
formulas which avoid the former. Then the duplication rule will usually be shown as

\[
\text{senescence} \sim \text{maint}.
\]

The duplication rule, which first seems nonsensical, is explained by the dual nature of 
maint. As a part of work expense, as we saw, that flow is an input contributing to product value 
and recovered in pay and product prices. From this objective viewpoint, maint costs the worker 
nothing. But subjectively it satisfies his final tastes, and hence must be taken into final yield. 
Final yield must be liquidated from the capital enabling it, which in this case is his self-value. 
Then the duplication rule is simply the rule that exhaust equilibrates to final yield, and is the 
specific application of that rule to the yield and outflow of self-value. Self-liquidation, or the 
recovery of senescence in yield, is effectively the nominal and subjective source of payment for 
the worker’s equally nominal and subjective duplicate purchase of maint, which was objectively 
bought by his employer and recovered in product prices.

The duplication rule is useful to theory and to practice. One of its practical uses is in 
prediction of the “social rate of return,” or ratio of output to capital at the scale of the closed 
economy. To see how, write
rate of return \[= \frac{\text{output}}{\text{capital}} = \frac{\text{growth}}{\text{capital}} + \frac{\text{yield}}{\text{capital}} \]
\[= \text{growth rate} + \text{yield rate},\]

where “capital” means total capital or the sum of self-value and property. At the scale of the closed economy, where the reinvestment and gift components in yield cancel out, this becomes

rate of return \[= \text{growth rate} + \text{final yield rate},\]

where

\[\text{final yield rate} = \frac{\text{final yield}}{\text{capital}} = \frac{\text{maint}}{\text{capital}} + \frac{\text{waste}}{\text{capital}}.\]

Now define

\[\text{waste rate} = \frac{\text{waste}}{\text{capital}} \quad \text{and} \]
\[\text{senescence rate} = \frac{\text{senescence}}{\text{capital}},\]

and substitute the duplication rule to get

\[\text{rate of return} \sim \text{growth rate} + \text{senescence rate} + \text{waste rate}.\]

This shows that rate of return does not equal zero in the stationary or growthless state, as Joseph Schumpeter taught that it does, but rather equals the sum of the senescence and waste rates. This seems obvious in hindsight. While property depreciation is recovered in cash flow, senescence is deadweight loss recovered in nothing. Then the rate of return just sufficient to hold capital constant, as leakage converges to zero, is the flow offsetting the loss of the factors to senescence and waste respectively.

The senescence rate can be modelled from vital statistics, with simplifying assumptions, and looks to be several percent yearly. A crude proxy is the generation turnover rate, or reciprocal of R.A. Fisher’s generation length. If this span is 29 years, say, then its reciprocal is 3.4% per year. Whatever the real value, the senescence rate is large enough to matter. Since lower rates of return are implicitly unstable, for the economy overall or for average-risk assets, it follows that price / earnings higher than its reciprocal, after correction for inflation and real growth and waste and risk, are also suspect. That is the sense in which the duplication rule, and the overturning of Schumpeter’s doctrine, count among the most fruitful innovations of TRE.

TRE’s analysis fits the data better than the Schumpeterian one, which is usually interpreted to predict that the social or economy-wide rate of return equilibrates to the growth rate. If it did, the real social rate of return would have approximated three percent yearly over
the middle and late twentieth century. Mark-to-market real rate of return to corporations, cap-
weighting equity and debt claims, can be estimated at six or seven percent yearly over that period
from data published by Ibbotson Associates. Although the senescence and waste rates remain
vague, this corporate rate seems broadly consistent with what TRE would expect for the
economy overall. Then TRE sees return in the corporate part of the business sector as roughly
typical of all sectors, whereas the Schumpeterian view would expect return in other sectors to be
far lower.

Return in the non-corporate business sector is less clear, since it is difficult to separate
profit from the owner’s self-paid wage in proprietor’s income or partnership income. But return
in these businesses should logically be about the same as in corporations. The remaining sectors
are the government and household ones. Return in the household sector is cryptic, since the
yield of owner-occupied houses is paid in direct taste satisfactions rather than in dollars with
which to buy them. Rental value of such houses is an unreliable guide, since lessors must accept
the best bid from the small pool of lessees willing to rent for exactly the period when lessors will
be away. A better guide is interest rates on home mortgages, which represent about half the
value of homes on average. If return to the equity remainder is assumed higher, because riskier,
then cap-weighted return to the household sector cannot be so low as to support the
Schumpeterian doctrine.

But TRE is not all iconoclasm and debunking. Although gross and net national and
domestic product are poor measures of the nation’s output, since they are founded on the
consumption-plus-investment formula, we will see for example that gross national or domestic
product is a good proxy for the nation’s “total cash flow” available for budgeting. That is
probably what Keynes and Kuznets meant it to be.

“Free growth” is defined in principle as growth due to productivity gain rather than to
belt-tightening. This definition in principle, unlike most in TRE, allows latitude in deciding the
diagnostic definition implied. TRE prefers to define “free growth rate” as increase in rate of
return (output / capital ratio) since a chosen past moment of growthlessness or stasis, and then to
define “free growth” as current capital times the free growth rate. The rest of growth is “thrift
growth.” TRE reasons that the only possible source of thrift growth at the scale of economic
closure is “waste conversion,” meaning reduction in the waste / capital ratio or “waste rate,”
since the rest of consumption is indispensable as insumption or maint. I will argue that the scope
for waste conversion is small under realistic assumptions. Since the waste rate is likely to be
lower in stasis (recession) than in growth, moreover, thrift growth is likelier to be negative than
positive. It follows that growth at the scale of closure is probably all free growth attributable to
productivity gain and not to tighter belts. This conclusion, from different arguments, is familiar
in the writings of Robert Solow. It suggests that policymakers should look less to thrift
inducement and more to imagination as the engine of growth.

Thus TRE does offer some immediate guidance. But it is essentially a new description of
fundamentals whose practical benefits are left to speculation. Its aim is understanding for the
sake of understanding. Like more conventional surveys of economics, it has no story line or
punch line. It is worth the effort of reading if it explains and predicts better than alternatives.
The last day parable, and the data on macroeconomic rate of return, suggest that it may meet this test.
I. TRE AND TRADITION

Economics goes beyond accounting when it predicts as well as describes market behavior. Typically it seeks this end by defining maximands, such as capital or output, for which individuals are expected to compete. One old obstacle to this plan is that much capital, meaning self-value (human capital), is opaque. How should economists best define this factor and its flows, either in principle or in practice, toward the pragmatic end of explaining behavior?

The problem is more than the fact that workers are effectively rented rather than sold, and hence leave no direct record of market value. Even under slavery, only a fraction of a worker’s value would command a market price. Maint (maintenance consumption) is defined as part of the worker’s expense in the sense that a dollar’s worth less would have meant a dollar’s worth less service and pay. It follows that the worker’s maint, which is yield to him because it satisfies his tastes, is simply expense to the slaver. Then if we count his value on his own books when he is free, and thereafter on the slaver’s, the present value of his maint over the rest of his life is value lost to the economy when he is enslaved.

TRE, like other treatments of the field, chooses its terms and strategies toward the common hard-headed end of market prediction. This concurrence of aim leads TRE to agree with tradition on some definitions in principle. Output for example means creation of value in TRE, as it does elsewhere, and yield means decapitalization for reinvestment or for satisfaction of tastes. But what is the yield as distinct from the output of self-value (human capital)? And again, how is self-value quantified?

TRE begins to solve these problems by choosing certain definitions in principle as uniquely apt to the end goal of prediction. Capital, for example, is defined not as the means of output but as the means of satisfying future tastes through yield. This definition draws a shorter logical line between behavior, meaning taste satisfaction, and pursuit of capital. TRE then identifies certain traditional diagnostic or practical definitions into which these definitions in principle are translated, and reasons that these translations are logical certitudes or tautologies which must hold for self-value as well as physical capital (called “property” in this text). These tautologies, all commonly accepted for property (physical capital), include:

1) the “present value rule” finding that the worth of an asset to its owner is the integral of present values of its expected yields time-discounted at its average expected rate of return over the delay period before each yield;

2) the “total return rule” finding that output (current creation of value) is capital growth plus yield; and

3) the “realized output rule” finding that realized (sold) output is sales less expense.

A popular variant of the present value rule is a valid inference from that rule, and thus a tautology as well. The variant notes that the present value of an asset does not depend on the amount of cash flow plowed back in the same asset to leave net positive yield as a residue. Thus
the present value of a bank account is the same whether its owner expects to spend it all, or spend all interest, or to leave all interest to compound. Discounted cash flow (DCF) therefore gives present value as the integral of all future cash flows, discounted likewise at the expected rate of return, expected as if all that flow were to be yielded and none plowed back. This imagined lack of plowback would make yields larger in the short term than actual ones net of plowback, assuming that there will in fact be plowback, but smaller later when the plowback would have been returned with interest.

The advantage of TRE’s unusual strategy of argument is the safety of tautology which it extends from the well-known realm of property (physical capital) to the vaguer ones of self-value and total capital. The disadvantage is the unfamiliarity and abstractness of the terms in which these rules are first expressed. Thus TRE tends at first to produce statements which are easy to trust but hard to interpret.

Gradually the mists will clear, and the statements will be judged against data. Strictly speaking, it is not the equations or “rules” of TRE which will be tested by experiment. Tautologies (logic) cannot be. This fact has led a gallant group called Popperians to condemn them as unsporting. What logic tests, rather, is the accuracy of imposed assumptions.

Under mild and lifelike ones, they enable surprising predictions which hold up in observation. I already argued, for example, that a worker’s pay exceeds his realized output (sold work) by the amount of his work expense. This prediction fits the fact that pay, at the end of economic life, tends toward work expense as a final minimum. If all pay compensated the worker’s output (work), as implied in conventional doctrines, we saw that it would necessarily tend toward zero at the end as the self-value that generates it must. One inference is that the National Income, defined in principle as aggregate rights to output but in practice as pay plus profit, overstates macroeconomic output by the amount of work expense. I will argue however that National Income and other NIPA measures of income and output are also understated by most of “free growth,” meaning growth due to rising productivity as distinct from deferment of yield.

Thus TRE will enable prediction. But I repeat that TRE is not any particular doctrine or theory. Like Keynesian economics, it is a framework to accommodate many such. It is simply economics with a philosophy of definition designed to clarify both factors equally, and to demand and reward patience in interpreting the novel concepts which it must therefore propose.

a traditional view of output

An old consensus, ratified in NIPA since the second quarter of the twentieth century, holds that output means investment plus consumption. This argument appeals intuitively. It is valid if the sum of consumption and investment has no source but new creation of value, and if new creation of value has no disposition but in the same sum. All of that is easy to believe. We cannot easily imagine any other source of investment. We also know that consumption comes partly or even mostly from property disinvestment, as in eating a can of peaches, and we sense that new creation of consumer services must account for all the rest. And one would think that new creation can take no form other than consumption or investment. Then we might have sworn that:
part of new creation is all of investment; \hspace{1cm} (1.1)

the rest is part of consumption; and \hspace{1cm} (1.2)

the rest of consumption is all of disinvestment. \hspace{1cm} (1.3)

The inference would be

\[
\text{creation} + \text{disinvestment} = \text{investment} + \text{consumption},
\]

or

\[
\text{creation} = \text{net investment} + \text{consumption},
\]

where “net investment” means investment less disinvestment.

Consumption plus net investment gives “net product,” exemplified in NIPA by *Net National Product* (NNP) and *Net Domestic Product* (NDP). Therefore net product seems to give a better measure of output in the sense of creation than *Gross National Product* (GNP) and *Gross Domestic Product* (GDP), which are measured gross of disinvestment (i.e. of *extinguishments*, defined as the sum of depreciation, amortization and depletion). If these measures of “gross product” were intended as measures of creation, they would neglect disinvestment. I will argue however that they are good measures of “total cash flow” available for national budgeting, just as Keynes and Kuznets probably intended.

“National” and “domestic,” in these usages, respectively mean “by nationals” and “within the nation.” Thus National Product of Americans, gross or net, would mean product by American workers and capital wherever located, while Domestic Product in America would mean product of workers and capital in America regardless of nationality of workers and investors. Since this text will have little to do with the domestic / national dichotomy, I will tend to say “net product” or “gross product” without specifying further.

NIPA elaborates the consensus view by distinguishing government consumption and investment from the private-sector part. The simplified version of the consensus is “output equals net investment plus consumption,” where the last two terms lump private and government components. Since this version is conventionally noted \( Y = C + I \), let us call it either the “CI equation” or “net product” equation. “CI” will mean the doctrine that this equation gives a good description of output as creation of value.

*Income* is sometimes understood as claims on output, and therefore equal to output at the scale of closure. A consensus as old as the net product equation, and likewise adopted in NIPA, holds that aggregate income is the sum of *factor payments* to workers and investors producing the output. These factor payments can be summarized as pay and profit respectively, where the word “profit” is generalized to include such flows as net rent and interest. Then the consensus view on income may be put “income equals pay plus profit.” Let this be the “PP equation,” and let “PP” mean the doctrine that the equation is valid. “CIPP” will mean both doctrines combined. Then CIPP is a prime rationale and philosophy of NIPA.
The CIPP argument appeals in its simplicity and its exclusive reliance on terms easily grasped. Everyone knows, more or less, what consumption and investment mean. In particular, the equations manage to allow for investment in human capital through consumption without explicit mention of human capital. The argument therefore seems to have arrived at a description of the output of both factors combined, meaning human as well as physical capital, with no need to clarify the stocks and flows of the former.

How could so simple a train of reasoning be mistaken? (1.1) overlooks the recapture of maintain expense in pay and then in product prices, including prices of both consumption goods and investment goods. (1.2) overlooks the “free growth” of self-value, meaning its growth from increased productivity as distinct from consumption invested in it. (1.3) repeats the error of (1.1) by missing the conversion of maintain into consumption goods. Thus the argument is wrong at every step.

We just saw where the net product equation went wrong. Now comes the tougher challenge of showing how to fix it and make it right. Our starting point is the tautology that net output, defined in principle as value-added or creation of capital, is implicitly equal to total return or the sum of growth and yield. This is the “equivalence principle”

\[
\text{output} = \text{value-added} = \text{total return}.
\]

Let us first evaluate output as total return. Note

\[
\text{growth} = \text{self-growth} + \text{prop growth},
\]

where the right-hand terms mean net growth of self-value and property respectively. Then

\[
\text{total return} = \text{self-growth} + \text{prop growth} + \text{transfer yield} + \text{final yield}
\]

\[
= \text{self-growth} + \text{prop growth} + \text{transfer yield} + \text{maint} + \text{waste},
\]

for any asset or set of assets.

Next we need a closer look at growth of the factors. Tradition defines the growth of property as net investment, meaning investment less disinvestment. Since investment includes retained output, including the free growth component in output, we will not need an additional term for free growth of property. Define

\[
\text{TRE investment} = \text{CIPP investment} - \text{maint} \quad \text{and}
\]

\[
\text{TRE disinvestment} = \text{CIPP disinvestment} - \text{maint}
\]

in order to see
net investment = TRE investment − TRE disinvestment
= CIPP investment − CIPP disinvestment,
showing that net investment means the same in both languages. TRE, by eliminating the double-
counting of maint in investment, implicitly eliminates it from disinvestment as well. Then

prop growth = net investment

in TRE and CIPP alike.

The idea of insumption naturally embraces some self-invested output, for example the
work of learning, and might be defined to include all of it. Since the actual rather than normative
outflow of self-value in yield was defined as self-liquidation, self-growth in that case would
come to insumption less self-liquidation. But this usage would seem strained. The growth in a
land lawyer’s self-value because gold is discovered nearby, enhancing his fee prospects, is not
naturally described as invested consumption. I therefore prefer

self-growth = insumption + free self-growth − self-liquidation,

where free self-growth is free growth in self-value. Note that this leaves insumption as a
residual, meaning self-growth less the difference between the other terms.

Now

total return = net investment + insumption + free self-growth − self-liquidation + transfer yield + maint + waste
= net investment + consumption + free self-growth − self-liquidation + transfer yield.

This last equation is a version of the “second output rule.”

Next we must evaluate output as value-added, or creation of capital of both factors. That is

value-added = creation of prop + creation of self-value.

Creation of property is TRE investment if retained within the asset creating it, and is otherwise
transfer yield. Since self-value cannot be given away or otherwise alienated, also, all transfer
yield from output is transfer yield from creation of property. Consequently

creation of prop = TRE investment + transfer yield from output.

Meanwhile
creation of self-value = insumption + free self-growth
                     − insumption from TRE disinvestment.

Further note

TRE disinvestment = transfer yield from TRE disinvestment + waste
                   + insumption from TRE disinvestment

and

                   − maint + transfer yield,

which becomes an alternate statement of the second output rule. Both versions plus the equivalence principle combine to give the duplication rule in the form

maint = self-liquidation.

The second output rule, in either version, shows how the net product equation must be corrected. The chief misstep is the double-counting of maint. The omission of free self-growth is not necessarily a fault, as we just saw, since one might have defined insumption to include this flow. Likewise the omission of transfer yield is correct if the net product equation is understood to hold at the scale of economic closure only. The second output rule holds at all scales, in any case, and is a more rigorous and reliable account.

an imaginary debate

Tom works all day fixing Dick’s truck for $100. Tom buys $60 worth of consumption from Harry and banks the rest. Suppose that all of Tom’s consumption is maint, and that free growth is zero. What is Tom’s output or work?

A CIPP economist finds that it equals the sum of his net investment, meaning the amount banked, and his consumption bought from Harry. Thus he says that it is his whole pay of $100. A TRE economist finds it at sales less expense, meaning the pay less the cost of consumption. Thus Tom’s output is the $40 banked, according to the TRE economist, and nothing more.
Suppose that both agree that output is implicitly growth plus yield. The CIPP economist measures the growth as the $100 added to the value of the truck less the cost of the consumption, and counts the latter as yield. Thus the CIPP economist still adds Tom’s output up to $100. The TRE economist agrees that the yield to Tom is the value of the consumption, in this particular case where there is no insumption, but notes that his net growth should also reflect his loss of self-value to senescence. The TRE economist adds that since Tom’s pay is wholly cash flow, meaning the sum available for plowback insumption or positive yield, and since Tom’s output plus liquidation (recovery of senescence) gives his cash flow while his output plus expense gives his pay, his senescence and consumption converge. Thus the TRE economist holds to his view that Tom’s output equals the amount banked and no more.

Suppose that both then both further agree that capital is present value of expected yield, or of expected cash flow assuming zero plowback. Now the TRE economist begins to gain ground. He points out that Tom must recover all his liquidation (senescence) in his sales (pay) before finding his realized output (sold work) as a residue. If the CIPP economist agrees, he cannot hold that all of Tom’s pay recompenses output unless he argues either that Tom has no self-value or that it does not depreciate. But the present value rule makes it clear that Tom’s self-value equals the integral of his discounted future expected cash flow (roughly pay), and that this sum declines inexorably with age.

If that is not enough, the TRE economist proposes a thought experiment called the “last day parable.” What if this day will be Tom’s last productive one? His present value then is one day’s pay discounted for one day. If the appropriate discount rate is perhaps 7% per year, or even 15%, or even 30%, the discount for one day will be 1/365 times that or almost nothing. Thus Tom’s present value is very slightly less than $100. Then if his whole $100 pay were output from that capital, the implied rate of return would be a little more than 100% per day even if there were a way to get around the requirement that the last day’s senescence (a little less than $100) must be subtracted first. In fact, the last day’s pay must be nearly all recovery of senescence and nearly no output.

If more convincing is needed, the TRE economist proposes a second thought experiment. Tom rents a one-cow dairy from Dick for one dollar a day. The cow gives five gallons of milk per day. One gallon nourishes a calf which will replace the cow. The cow and dairy require no maintenance from outside. Tom maintains himself on one gallon per day, and sells the remaining three for one dollar each. He pays one dollar to Dick in rent, and banks the final two. What is the output of the dairy?

The output to Dick is his clear profit of one dollar per day. Both economists agree that the gallon of milk drunk by the calf is an expense of maintaining value of the dairy including its livestock, and both agree that the two dollars banked measure output (work) by Tom. Both also agree that the dollar’s worth of milk drunk by Tom is yield to him. They disagree in that the CIPP economist asserts, and the TRE one denies, that this final dollar’s worth should be included in Tom’s output as well. The TRE economist reminds his colleague that output is growth plus yield, and that Tom’s self-value declines steadily by the amount of his senescence. (We continue to assume zero insumption and zero free growth.) Then it is impossible for the entire dollar to add to Tom’s output, since that would imply zero loss in self-value. The TRE economist adds
that realized output is sales less expense, whether the expense satisfies tastes or not. If less milk drunk by Tom would have meant so much less job performance and hence less pay (sales), the whole dollar’s worth is expense which cannot be counted in his output without being counted twice.

Then the dairy’s daily output is one dollar in profit to Dick plus two dollars in work by Tom. Tom’s imputed pay is three dollars, of which one is expense recaptured in the price of the three gallons sold. This thought experiment is the “dairy parable.”

The value-added chain is described in many textbooks. A favorite illustration is the evolution of wheat into flour into bread. The chain is described as ending with the bread or other final goods. Net product (NNP or NDP), by subtracting capital depreciation which will be recaptured in consumer goods later, correctly recognizes final consumer goods such as the bread, but not final producer goods such as the bakery, as the only true outflow of the traditional chain. The depreciation of the bakery, when converted eventually into the value of bread, would otherwise be counted as output twice.

TRE adopts the net product version of the chain, but extends it past the consumption phase to include self-value as the final product. We saw earlier that leakage is the sole avenue of outflow from the extended chain, meaning exhaust from the closed economy, before the consumption stage. Exhaust at that stage is waste, meaning consumption recaptured neither in products through work expense nor in self-value through insumption. Exhaust from self-value is senescence. This process is shown schematically in Figure 1.1:

**Figure 1.1: The Value-Added Chain**

```
  waste consumption
    /    \
senescence  insumption
       /     \
  maint     
    /     \\
 workers (self-value)
       /      \\
bread                   
       /       \\
flour
       /     \\
wheat
       /     \\
  leakage     \
    /  \      \
leakage     (consumer goods)
     /    \    
leakage   (producer goods)
     /       \
  leakage     \
    /     \\
  (raw materials)
```
Then leakage plus waste consumption plus senescence make up the exhaust of the closed economy. But a problem remains. Senescence is capital extinguishment, and all capital extinguishment is expected to be recovered in yield. There can be no exception, even for extinguishment of self-value. Capital is priced at the integral of future yields it is expected to enable, discounted at the rate at which it is expected to produce them. Actual yield is maturation of these expected ones, plus or minus differences between expectation and reality, in current outflow available for reinvestment or taste satisfaction. Extinguishment without expected yield in return, even in the case of senescence, would be a contradiction in terms.

What then is the final yield expected to be afforded from the exhaust of self-capital in senescence? All of the yield value of maint is clearly enabled by self-value, and hence must be decapitalized from that source. And no other final yield can be. Although the worker contributed to the value of waste consumption, which is the rest of final yield, he was already paid in full for his contribution. While that is also true of his maint as an expense contributing to product value, it is not true of his maint’s yield value. This argument confirms the duplication rule holding that maint equilibrates to senescence. We see also that leakage equilibrates to zero, so that prop exhaust, or the sum of leakage and waste, equilibrates to waste.

TRE holds nothing more flummoxing and flabbergasting than the duplication rule. It seems to lay paradox on paradox. How can it claim that maint is final yield when maint is defined as an expense recovered in product prices? And since senescence does not satisfy tastes in itself, like the extinguishment of an ice-cream cone in the mouth, how can it satisfy them unless recovered in cash spent on yield goods? And if so recovered, how again can it count as exhaust?

The key to the riddle, once again, is the insight that maint has two distinct economic values. As a part of work expense it is an input recovered in sales, and therefore no net cost to the worker. As work expense its cost is thus not decapitalized from the worker’s self-value or other wealth, since it is effectively paid for by his employer and then the market. As a part of final yield, however, its cost must be liquidated from the capital enabling it. That capital is his self-value. Then the yield value of maint, but not its expense value, is decapitalized in the form of self-liquidation. This flow is not recovered in tangible cash spent on tangible maintenance goods, which are separately paid for by the market, but rather is an accounting entry to meet the requirement that final tastes are satisfied in the liquidation component of yield.

None of this lessens the reality of self-value and self-liquidation, or the predictive power of the duplication rule. These concepts are subjective, not fictive. All capital, and not only self-value, is means of expected time-discounted taste satisfactions. Taste is subjective. We cannot measure both factors by the same ruler, and predict behavior, unless we capitalize all expected taste satisfactions comprehensively. Maint satisfies final tastes. And although objective analysis might find that this satisfaction is a free good whose cost was paid by the market and not by the worker, it is not so from the perspective of the worker. He did not convert its cost into pay effortlessly, as if selling goods from inventory, but rather worked as hard for the maint component in his pay as for the non-yield expense and sold work parts. He defines his self-value, and invests in it, as if the taste satisfaction drawn from his maint were an economic good bought from his own resources. Since our aim is behavior prediction, TRE capitalizes expected maint into self-value and decapitalizes current maint in self-liquidation.
The duplication rule, like much of TRE, speaks in areas where tradition is silent. Maintain and senescence and self-liquidation are not old terms relabeled, like self-value, but rather are concepts scarcely addressed in the textbooks. The exhaust idea is a more surprising omission. What would we think of a medical curriculum which gave no clue whether or how people might die? The traditional counterpart to exhaust is not property extinguishment, which is generally recaptured in later goods or in consumption, and it is not simply consumption, since part of that flow is recaptured in human capital (insumed into self-value). It seems to me that tradition sees some consumption as an endpoint, and probably intuits that worker mortality is also an endpoint, but leaves the connection between these termini to the imagination.

The chief value of the duplication rule lies in its clarification of exhaust at the end of the value-added chain. This clarity is essential to a grasp of output and growth at the scale of closure. Thus TRE is not only a more accurate version of economics, which allows better understanding of such things as pay/age profiles and average-risk rate of return, but a more complete one as well.

What would the market value of a slave be, if slavery were legal and practiced? Since the slaver must now pay the slave’s maintenance, and gets no taste satisfaction from doing so, the yield available to the slaver is the remainder that would have gone to gift plus waste plus insumption plus property investment if the slave were free. This remainder was termed sold work. Then the market value of the slave is the present value of future sold work alone. It follows that self-value exceeds the value of a slave doing the same work by the present value of his expected maintenance over his remaining lifetime.

Assuming Ricardian equilibrium, under which pay is just sufficient to maintain a worker and replace him from the next generation, diversion of his sold work to satisfy the tastes of a slaver would mean incomplete replenishment of the slave population except through new capture of other slaves. This cruel logic fits the observed low reproduction rates and high import rates of slaves in Rome and in the Sugar Islands.

Why new words?

TRE will adapt familiar terms where practical. It must coin some, like “exhaust” or “insumption” or “senescence,” where traditional words do not seem to exist. In at least two cases, also, it will use new synonyms for reasons of convenience and taste alone. Physical capital, for example, has twice too many syllables for easy repetition within a sentence. And it is not particularly apt. We are just as physical as our belongings. That is why this study renames it “property.” Human capital likewise becomes “self-value” or “worker value,” for brevity and for potential generalization to other species. Terms coined here will usually be introduced in quotes, and more conventional ones in italics.

The description of certain usages here as traditional comes with a caveat. No traditional economic term, as far as this writer knows, has a settled meaning independent of context. Those
who doubt it may look at any entry in the *New Palgrave Dictionary of Economics*, or in any other reputable lexicon. While cash flow in corporate accounting tends to mean gross cash flow before plowback, for example, the same term as meant in discounted cash flow analysis tends to specify net cash flow after plowback. TRE calls the first or gross kind “cash flow,” and the net kind “yield.” I also argue that present value can be found by discounting either flow, among others, under assumptions appropriate to each.

**other conventions**

“We” in this writing is collective, not editorial or royal, and means reader and writer working together. “He” means he or she. Terms in simple algebra will tend to be written out rather than notated. All values mean values as of the current instant unless otherwise noted. Percent growth rates are instantaneous, meaning that they are compounded continuously rather than over discrete periods. Growth functions are thus natural logarithms.

Economic quantities are expressed in the two dimensions of value and time. *Stocks* are quantities dimensioned in value alone. The only stock is capital, which is further divided into self-value (human capital) and property (physical capital). *Flows or streams* are dimensioned in value over time. Examples are yield, output and growth. *Rates* are pure numbers over time, or equivalently flows divided by the capital generating them. Examples include growth rate and rate of return, which mean capital growth and output respectively divided by capital. An *index*, in this text, will mean a fraction which normally varies between zero and unity (the number one).

The measure of value or *numeraire* in TRE is the constant dollar. I must leave the exact definition of this numeraire to others. I have no solution to the old *numeraire problem*, which asks what happens to money value when apples become dearer and oranges cheaper. The best I can do is to assume that inflation can be defined, and to adopt that definition sight unseen. Then the real or constant dollar is the actual dollar adjusted to offset inflation.

**rate focus**

Stocks and flows are less informative, as a rule, than the rates which draw their logical connection. If a million-dollar portfolio grows at ten percent per year while a hundred-thousand-dollar one of similar risk grows at fifty percent per year, we are likely to be more interested in the strategies of the latter even though its growth expressed as a flow, not a rate, is only half as high. I will argue that the economic maximand which all individuals can be expected to maximize from moment to moment is not yield, which we restrain for the sake of tomorrow, nor capital, which we routinely waste and give away, but the risk-adjusted rate of return which gives each individual the best of both from the productive assets actually available to him. It is therefore rate of return, not capital or yield or output, which equilibrates to a competitive norm from one asset of a given risk to another. Similarly it is the rate of productivity growth, and not the flow, which tends to be uniform among assets of varying size. Thus TRE will tend to treat rates as primal.

**scale**

The rules of TRE are designed to hold at the scale of the individual owner, as well as at all smaller and larger scales. Only a study of individuals, as distinct from households or
corporations or populations, can isolate the flows of maintain and senescence whose equilibration explains the stasis rate of return. Thus TRE will focus closely, though not exclusively, on the economies of biological individuals.

CIPP, conversely, is designed to describe at large scales first. The CI equation illustrates this focus by excluding a transfer yield term. Output may be given away as easily as consumed or invested. Thus NIPA is not CIPP writ large. CIPP, it seems to me, is meant to describe at the scale of the whole world. NIPA shrinks rather than expands its reach to the scale of the individual nation. Certainly Keynesian and other systems study individual behavior in microeconomics, but it seems to me that CIPP output as such is defined primarily for larger scales.

summary

Economic tradition shows a Machian bent in its relative indifference to quantities not directly measurable. One would suppose that this litmus-mindedness, expressed in CIPP for example, would make mainstream economics more solid and reliable than TRE. Yet we now see that TRE is at least as pragmatic in its goals and strategies, and makes better predictions.

Its prize exhibits include the sold work rule, the duplication rule and the exhaust model of economy-wide rate of return. The sold work rule, which is clearly heretical and clearly supported by data and reflection, is TRE’s natural attention-getter or foot in the door. The duplication rule, which clarifies exhaust, is special. No economist would be surprised by the claim that the final drain of capital from the economy is senescence plus waste, or by the claim that it is yield consumption. The shock of the duplication rule comes with demonstration that these natural speculations come to the same thing.
II. BASIC IDEAS AND ASSUMPTIONS

TRE can begin with the idea of a reproducing population of “owners” who cooperate and compete in markets to satisfy current and anticipated tastes. “Owners” means biological individuals of all ages from birth to death, whether legally entitled to own things or not. Owners are typically in “near-agreement,” but not in perfect agreement, as to these tastes. Trades imply that the buyer has more taste at the moment for the product than for the cash it costs, and the seller the opposite. But some degree of consensus in tastes is a prerequisite for markets to form. Restaurants and clothing stores which serve many customers are practical because people agree more or less on what to eat and wear, and on how much can prudently be given up in return. Only under near-agreement can buyers know where to go to get what they want and can afford.

It can be assumed that these reproducing owners in near-agreement are members of the same species. Others are unlikely to meet in the market. The ants and the picnickers may compete for the lunch, but cannot bargain for it.

Some degree of “efficiency” is also necessary for markets to form and hold. Efficiency can be understood as good prediction, though never perfect prediction, of eventual costs and realizations likely to follow from a given investment. In efficiency, that is to say, predictions of gain or loss are never exactly right but are neither optimistic nor pessimistic on balance. Efficiency in the TRE sense means no more than that. It is not a separate assumption, but an inference from the assumed existence and stability of markets. The restauranteur or clothier would soon fail if he were not a reasonably good predictor of which products would sell at a profit.

The twin conditions of efficiency and near-agreement lead to a very useful third one of “cost-value parity.” This means that goods on average should fetch what they cost, including the cost of hiring capital. Since prediction is never perfect, however, this parity is approached over large aggregates of sales rather than realized in individual sales. These conditions collectively will be called the “prime assumptions.”

One might have supposed that imperfect prediction would imply some excess of cost over market value regardless of scale. That is true in a way. With hindsight, anything might have been done better. But raw materials and labor inputs are priced to allow profitable products when managed with normal competitive skill rather than perfect skill. Were it not so, materials and labor would find few buyers. Thus cost-value parity, over large-scale averages, is a realistic assumption rather than an idealistic one. It is really no more than an inference from the fact that markets form and endure.

All these assumptions boil down to three, namely reproductive owners, tastes, and persistent markets. Near-agreement, good prediction and cost-value parity are already implicit in those three. TRE needs no more assumptions, and no hypotheses as to the nature or object of tastes, in deriving its general rules.
Yield is defined in principle as disinvestment flow to pay costs of current reinvestment by the owner in his other assets or of satisfaction of his tastes. This definition is more or less the same in TRE as elsewhere. Unless otherwise stated, yield means net yield after plowback in the asset generating it. An owner’s “total yield” is defined as his aggregate yield from all capital of both factors, meaning from all his property plus his self-value, net of plowback investment in either. The point here is that he might invest yield from property in his self-value, say by paying tuition, and that he might invest yield from self-value (meaning from pay) in property. His total yield is whatever yield remains after all such reinvestments. Implicitly this residue is the cost flow of his taste satisfactions alone. The “total yield” concept may be unique to TRE. Note again that an owner’s insumption, or consumption invested in his own self-value (human capital), is explicitly excluded from his total yield, since it is not net decapitalization from his total wealth.

TRE also distinguishes “transfer yield,” meaning reinvestment in another asset of the same or another owner, from “final yield,” or evaporation of value in satisfying tastes. Transfer yields satisfy “transfer tastes” for gift or asset redeployment, while final yields satisfy “final tastes.” The present value of expected final yield quantifies transfer yield. If Tom gives Dick a hamburger, for example, and Dick eats it in final yield, then the present value of the final yield to Dick reveals the value of the gift to Tom. If Tom gave Dick a percent equity in a car repair shop instead, we might find no practical means to project the train of assets and owners into which the depreciation and output of the repair shop would be converted enroute to disappearance in final yield. But it would clarify the value of the percent equity if we could.

Total yield, or net decapitalization from the owner’s wealth in satisfaction of his tastes, is implicitly gift plus final yield. Final yield is never negative. Gift however is “outgift” less “ingift,” and may be either positive or negative on balance. Thus total yield too may be either positive or negative. Since negative yield is investment received from outside, the only possible source of negative total yield is ingift received from another owner. The most interesting kind of gift from a TRE perspective is “parental gift,” gift from adults to non-adults regardless of relatedness. The young grow from this ingift, which is negative total yield to them, and from their own self-invested work in exercise and learning.

All ingift in the TRE sense is capitalized by the receiver. Although gift in the colloquial sense may be exhausted as soon as received, this result is seen in TRE as final yield to the donor who paid for it, and not to the donee. We saw that final taste satisfaction is a free good, and not yield, unless its value is liquidated from capital stocks of the owner whose tastes are satisfied. Then these immediately-exhausted transfers are free goods to the donee. To the donor they are final yield, and not maint, and are therefore waste consumption in satisfying his tastes for making the donee happy.

At a scale of “gift closure,” referring to a set of owners who may give as they like but who collectively show no net balance of ingift or outgift vis-à-vis the collective remainder of owners outside the set, gift cancels to zero. Then yield in gift closure is final yield alone. Economic closure always shows gift closure, although not conversely.
Recapitulating, where “pos” and “neg” are contractions for positive and negative,

\[
\begin{align*}
\text{yield} & = \text{reinvestment in owner’s other assets} + \text{total yield}, \\
\text{yield} & = \text{pos yield} - \text{neg yield}, \\
\text{yield} & = \text{transfer yield} + \text{final yield}, \\
\text{pos yield} & = \text{yielded output} + \text{liquidation}, \\
\text{neg yield} & = \text{investment from outside}, \\
\text{transfer yield} & = \text{reinvestment in owner’s other assets} + \text{gift}, \\
\text{total yield} & = \text{gift} + \text{final yield}, \\
\text{gift} & = \text{outgift} - \text{ingift}, \\
\text{neg total yield} & = \text{ingift}, \quad \text{and} \\
\text{yield in gift closure} & = \text{final yield}.
\end{align*}
\]

capital

In the nineteenth century, capital was typically understood as the means of production. It meant physical capital only, as a rule, and sometimes only man-made physical capital. Land including minerals might be distinguished as a separate factor. Labor was seldom viewed as a stock of capital, since it is easier to measure in man-hours or in the flow of pay. The twentieth century took more interest in the human capital concept, despite difficulties in measurement, if only because factors cannot be added unless they are written in the same dimensions.

TRE, like other modern approaches, prefers a two-factor viewpoint in which land is merged into physical capital and in which labor is a stock of human capital. Government property and workers are divided into the two. Thus land and government are not overlooked, or simplified away, but included within the two categories. It seems to me, as to others, that limitation to two factors costs nothing in sophistication and generality while making math and logic easier.

The sum of the two factors is simply capital in TRE, although the more traditional total capital may be substituted for emphasis. And once again, TRE prefers the shorter and apter “self-value” and “property” over the more familiar “human capital” and “physical capital.” Thinking becomes easier as key words become shorter. “Property” is apt in that nothing has value until it is owned, whether individually or through a commonwealth. “Self-value” is apt in that it implicitly excludes slave value, which misses the present value of future maintenance consumption. It also leaves room for application of TRE to other species, in case this proves possible, and dodges the pomp and piety that mention of ours can intone.

TRE defines capital in principle as the sole stock from which to pay future (positive) yields, meaning reinvestments and taste satisfactions. This definition in principle is already implicit in capitalization by present value methods, which have been mainstream for half a
century. It says that capital is valued at the integral (i.e. aggregate) of time-discounted expected future yields over the remaining life of the asset, where the discount rate for each increment of yield is the expected average rate of return to the asset over the period ending with the yield. Since rate of return equals potential growth rate if all output is plowed back, this is the unique discount rate at which a buyer should be indifferent between owning the right to the yields as they arise or the right to the asset now. TRE stretches tradition by asserting that this “present value rule” follows from the given definition in principle as a logical certitude, or tautology, which must therefore hold for self-value as well. TRE therefore quantifies self-value at present value of expected work yields to its owner. The equations for the present value rule will appear later in this chapter when work yield has been clarified.

What then is the difference in principle between the two factors? TRE’s answer, which may be original, is that self-value is the factor whose maintenance (maint) satisfies final tastes and must consequently be paid from final yield. Other distinctions, such as the normal untradability of self-value, follow from that most basic one. Although an owner’s car may be worth about as much to someone else as to him, his self-value is worth far more to himself than to any other bidder because he alone values its maintenance per se. This fact shows the aptness of the term “self-value.”

Capital includes public and collective property such as parks and streets and endowment funds. But all capital in TRE is imputed to be owned by mortal individuals. That means that ownership of public and collective property is imagined to be passed through to the mortal constituents that make up their groups. Capital is reckoned at the value either of underlying assets or of claims upon them, but not both. Thus financial capital may be included so long as no assets are counted twice.

Since owners are mortal, workers are reproducible depreciating assets like buildings which begin existence with negative yield (investment from outside) through a period of parental gift and which then pass into a productive phase of positive yield and negative growth in value as they become sources rather than beneficiaries of this investment. We saw that “parental gift” in this survey means any investment in the young by adults, whether literally by parents or not. Humans may also experience a final “redependent” phase in which liquidation of savings plus family support and social transfer payments such as social security or Medicare may supply new external sources of upkeep. These negative components in the retiree’s work yield are complemented and offset by a positive one shown in his continuing self-maintenance effort. Then overall work yield and self-value might fluctuate at levels near zero during redependence.

Consumption is an idea central to CIPP, but peripheral to TRE. TRE prefers to focus on its component parts rather than to lump all kinds together. It is worth defining in that it is a term often met in the literature. Let us simply try a consensus definition. Consumption can be understood in principle as the flow of services and property liquidations not recaptured within the property segment of the value-added chain, but rather applied to owners directly in replenishing self-value or in satisfying tastes. TRE first divides consumption into “insumption,” meaning “invested consumption,” and final yield. Insumption is dollar-for-dollar transfer of consumption cost into self-value. Final yield, or annihilation of consumption value in satisfaction of final tastes, is further divided into “maint” whose cost is recovered dollar-for-
dollar in pay and thence in product value, and “waste” which satisfies final tastes but whose cost is recovered in nothing. Then the “consumption rule” is

\[
\text{consumption} = \text{insumption} + \text{final yield},
\]

where

\[
\text{final yield} = \text{maint} + \text{waste}.
\]

Substituting,

\[
\text{consumption} = \text{insumption} + \text{maint} + \text{waste}.
\]

Note that non-yield expense is excluded from consumption. It seems to me that non-yield expense is a kind of “agent expense” paid by the employer, although routed through the worker as his agent, and falls outside of consumption because it neither builds self-value nor satisfies final tastes. This distinction prompts a question as to whether non-yield expense should be recognized as a part of pay. TRE does so recognize it, but qualifies by also defining

\[
\text{pure pay} = \text{pay} - \text{non-yield expense},
\]

so that the sold work rule may also be written

\[
\text{pure pay} = \text{sold work} + \text{maint}.
\]

Meanwhile it is interesting to see that economists sense the distinction between true consumption and what TRE calls non-yield expense, and have questioned whether non-yield expense ought to be included in net product under the CI equation. The entry for “National Income” in the *New Palgrave Dictionary of Economics* (1991 reprint) by Wilfred Beckerman, for example, includes:

“… it is often claimed that some—or even most—public consumption really constitutes undesirable necessities rather than additions to final output, and hence ought to be treated as intermediate goods (or sources) that are used up in the course of production in the same way that the wheat was used up in the production of bread. This might apply, for example, to the services of the police…. Similarly, it is often argued that various services, such as passenger transport, are really inputs into the system since they … are merely part of the costs that people incur in order to go to work.”

the duplication rule

Final yields, as distinct from free goods, are drawn from exhaust of the assets enabling them. Suppose for example that Harry takes a dollar from his bank account to buy an ice cream cone, and then consumes the cone in waste. A dollar has been reinvested from the account into
the cone in transfer yield, and then exhausted from the cone in waste. In this and every case, waste is exhausted from the consumption good used up.

Next suppose that Dick follows the same procedure, but consumes the cone in maint rather than waste. Maint is both expense and final yield. As expense, it is a kind of transfer yield in converting the dollar’s worth of cone into a dollar’s worth of pay. Dick comes out even on the exchange, and incurs no cost or outflow of capital so far. But the final yield value of the cone, in satisfying his final tastes, imposes a separate cost. Final taste satisfaction must be paid for, penny for penny, in exhaust of the capital enabling it. The exhaust cannot come from the cone, whose whole value has already been transferred to his pay. It cannot come from the dollar earned, which was a consequence of his work services rather than a condition precedent. It is enabled by neither of these, but uniquely by his self-value which earned the pay. Then the exhaust of self-value in senescence is the source of the whole cost of the final taste satisfaction value of maint, and of nothing else.

This seems to be the most compact and straightforward proof of the duplication rule. The rule is ubiquitous in TRE, and will be confirmed in several ways. Its reliance on subjective values, meaning the satisfaction and the self-value itself, may test our patience. But its predictive power, for example in modeling the stasis rate, will convince in the end.

The value of any depreciating asset is implicitly equal to the integral of its expected depreciations, so long as that integral excludes depreciation of capital to be added later. For self-value, that added depreciation equals expected future insumption. Then

\[
\text{self-value} = \text{expected lifetime senescence} - \text{expected lifetime insumption},
\]

meaning aggregate senescence and insumption expected over lifetime remaining. Now the “second duplication rule” appears as

\[
\text{self-value} \sim \text{expected lifetime maint} - \text{expected lifetime insumption}.
\]

This rule will prove helpful in modeling the stasis rate from pay / age profiles.

By the definitions of capital and yield, all extinguishment of capital of either factor is implicitly expected to be recovered in yield. Then define “liquidations” as the part of yield realized from extinguishments, so that the “extinguishments rule” is

\[
\text{extinguishments} \sim \text{liquidations}.
\]

One might have supposed that liquidations can never exceed extinguishments in any instance, but will fall short in some instances, implying that extinguishments should be the larger flow overall. But that is not so. Either flow may be larger in individual cases. If the bakery is destroyed by fire or flood, or its inventory depleted by pests or looters, these extinguishments will not be recaptured in the value of the bread sold. But the market value of the bakery is discounted to allow for a normal incidence of such losses. Extinction exceeds liquidation...
when actual losses exceed the norm, and conversely. Thus “leakage,” or excess of extinguishments over liquidations, may be either positive or negative.

Define “outflow” as yielded output plus extinguishments. Then the extinguishments rule, plus the definition of pos yield as yielded output plus liquidation, combine for

\[ \text{pos yield} \sim \text{outflow}. \]

Now let “exhaust” mean unrecaptured extinguishments or final drain from the economy in satisfying final tastes. Here

\[ \text{final yield} \sim \text{exhaust}. \]

It is axiomatic that senescence, or the erosion of individual self-value as age depletes the pool of future work yields on which it is capitalized, is exhaust recaptured in nothing. The rest of exhaust is “prop exhaust,” meaning waste consumption plus any leakage at earlier links in the value-added chain. That is

\[ \text{exhaust} = \text{senescence} + \text{prop exhaust}, \quad \text{and} \]

\[ \text{prop exhaust} = \text{waste} + \text{leakage}. \]

Under reasonably good foresight, positive and negative leakage will converge to zero. Then leakage \( \sim 0 \), implying

\[ \text{prop exhaust} \sim \text{waste}. \]

These inferences, and the definition of final yield as maint plus waste, give the duplication rule

\[ \text{senescence} \sim \text{maint}. \]

These are the “convergence rules.”

Work expense is divided into its yield part, which satisfies tastes, and its non-yield part which doesn’t. How can these fractions be told apart? Non-yield expense means whatever must be paid for in order to realize current pay, but which is not wanted in itself. Might taxes count? They are the cost of peace and order, which is part of the cost of job performance and pay. This fact adds them to the category of work expense. The key question is whether or not we would pay these costs for what they bring, assuming that we have the means, even if they were not needed to realize current pay. If people did not pay a government a tax for what social peace it brings, they would end up paying the Mafia more for less. Then taxes on pay probably count as maint.

What about medical bills? These are insumption insofar as their cost will be recovered in future pay, and work expense insofar as needed to enable pay right now. Yet we pay the cost of
health for its own sake, and not only for the sake of the pay it secures, even though visits to the doctor bring no pleasure in themselves. Then the cost of current health is maint.

These examples make it clear that taste satisfaction, for purposes of diagnosing maint, does not imply creature pleasures. The criterion does not even imply avoidance of pain, which is part of the price of health when surgery is needed. The test is whether we would have paid the cost, assuming the means to pay it, whether necessary for job performance or not. Non-yield expense, again, is any residue which must be spent to secure current pay and would not have been spent otherwise. If payments as onerous as taxes and doctor’s bills are still maint, what might be left?

Transportation to work and back seems to fit the description of non-yield expense, as we saw in the quote from Dr. Beckerman. We must spend for it or go without pay, and generally would not have spent for it if that were not so. Likewise an executive may subscribe to Forbes or the Wall Street Journal at his own expense in order to sustain current job performance, and might not have subscribed otherwise. Other examples do not come so easily to mind. It seems to me that non-yield expense is a real but minor flow for which TRE allows in order to stay on the safe side.

capital is measured at present value of expected yields or cash flows, discounted at expected rate of return over the period of deferment. With each minute the period and discount factor grow less, and the present value more. Meanwhile value is lost as expected yields are finally realized and decapitalized. The gain in value due to time alone, before the loss in decapitalization of yield, is “accretion.” It is a positive element in output in all capital, meaning whatever has a dollar value, regardless how vain or useless or inert the capital may seem. Thus a diamond ring like an ice cream cone confers “yield in kind,” rather than cash yield, in the pride and pleasure of wearing the ring. This yield is continuously decapitalized, and value is restored as accretion brings future satisfactions nearer. Even money in the pocket, once called a model of infertility, yields a very practical current satisfaction as a safeguard for meeting unexpected cash costs or opportunities. Its value accretes as its expected use draws closer.

The rule is general. Just as retail prices exceed wholesale ones because goods are more conveniently located and arranged for customers, values to those customers rise again as goods are taken home. They continue to rise, from accretion alone, as final use comes nearer. Thus bread in the breadbox accretes, as wine in the wine cellar, so long as neither is kept too long. There is no unproductive property.

government and taxes

I said that TRE, like other two-factor approaches to economics, does not implicitly overlook government. Rather it lumps government property with other property, and government workers with other workers. Tax, in a TRE view, is one of many kinds of payments for what owners want. Tax is implicitly part of the taxpayer’s investment insofar as it benefits his property, part of his gift insofar as it benefits others, and part of his consumption insofar as it is wasted in taste satisfaction or benefits his work capacity or self-value.
None of this denies that government is special, or that economists should treat it as such for many purposes. Government sells its product to voters collectively, not to shoppers individually, and charges its opponents no less than its partisans. Thus TRE’s assumption of cost-value parity does not hold for government costs at the scale of the individual owner. TRE assumes only that it holds at the social scale on average over time.

Final yield is the dissipation of consumption from positive yield into nothing but taste satisfaction since its recapture into worker value is barred by the definition of yield as net disinvestment. Exactly when is the threshold crossed? Is it at the moment when the worker pays for the meal? At the moment of ingestion? The answer is neither. Final yield with respect to each bite, so to speak, comes when the bite loses all value to its owner. Admittedly it loses its value to others as he eats it, but it will still confer value to him until its nutritional worth is spent. The meal exhausts its value not with the eating, but with the hunger for the next. In the interim it is still his property, sustaining him just as his house and clothes sustain him.

A similar argument should hold for insumption. The meal is converted into self-value as it is digested into soma and energy, and the lesson may sink in with later ones and with experience. Meanwhile it too is internal property of no potential value to others.

Consumer services whose effect lasts beyond their application are also internalized property. It seems plausible that most if not all consumer services fit this criterion. Initial propertization of consumer goods and services in this sense is an awkward concept, but one that seems compelled by definitions.

Each owner, again, owns capital of both factors. We can say that each has a “worker” aspect owning his self-value alone, and an “investor” or “property owner” aspect owning his property alone. Thus the worker-investor dichotomy in TRE is not a class partition, but rather a distinction of specialties within the individual. Neither workers nor investors exist as separate beings.

Output is defined in principle as creation of value. Since capital was just defined as the only quantity measured in value alone, why not say “capital” here instead of “value”? The reason is that it is mathematically possible for this flow to include final yield which is concurrently created rather than liquidated from capital in place. But any final yield which costs no capital liquidation would count as a free good, and is disregarded in TRE as in other versions of economics. Then creation of value, even though its definition in principle is broader, can be understood to mean creation of capital and nothing else.

We saw that the architects of CIPP, on grounds that must have seemed airtight, inferred that creation of value means consumption plus net investment. I argued that this definition double-counts maintenance while overlooking free growth of self-value. Where then does certainty lie?
What diagnostic definition, meaning one suitable for recognition and measurement, is sure to give all creation of value and nothing else?

“Tautology,” in these pages, means logic of broad generality. What it means to Popperians or others is not our concern. Mathematics, reasoning from the assumed properties of numbers, is tautology in this sense. Examples can be as inane as the truism x = x, or “a rose is a rose is a rose,” or they can be as subtle as the proof of Fermat’s last theorem. The total return rule, which finds that creation implies net growth plus net withdrawal, comes somewhere in between. It finds that the creation of rabbits generated in a meadow, or pleadings in a law office, infallibly means the net growth within plus the quantity transferred out less the quantity transferred in. The rule applies universally so long as all three flows are measured by the same yardstick. Then does the “quantity” of rabbits here mean aggregate weight, or dollar value, or number, or volume, or something else? The rule works for any such measure applied consistently. If the measure is number of live rabbits, for example, then creation (birthrate) equals population growth within the meadow plus net exodus of live rabbits from it. A key point is that rabbit mortality is counted as a negative component in the population growth, but not counted again as a component in the exodus.

Then an asset’s output, defined in principle as its creation of value, is implicitly equal to its growth in value plus its net transfer of value to its owner or his donees. The last flow was defined as yield. This “total return rule” may be expressed as

\[ \text{output} = \text{growth} + \text{yield}. \]

TRE adopts the usual term \textit{profit} to mean the output of property and income from it to owners. TRE also adopts the common definition of profit as total return. Thus the “profit rule,” or total return rule for property, states that profit is property growth plus property yield. TRE tends to contract the word “property” to “prop” in equations. The profit rule is therefore written

\[ \text{profit} = \text{prop growth} + \text{prop yield} \]

for any property asset or any set of them. The word profit, in TRE, will include all interest and rent received.

Although this definition of profit as total return is now common, we recall that other definitions are common too. \textit{Accounting profit} or \textit{GAAP profit} or \textit{earnings} is generally the kind shown in NIPA. \textit{Economic profit} meanwhile means earnings less an imputed charge for costs of hiring capital owned by the producer. An economist might mean any of these by “profit.” Only total return, however, reliably gives the creation of new capital by old.

\textit{work, wage and pay}

TRE also adopts the usual term \textit{work} to mean the output of self value. It is customary to speak of \textit{wage} as the income from self-value, and the measure by which work is priced. TRE usually prefers to avoid the word wage, since it has gathered a connotation of payment received for work sold to the market. This is not the case with the word profit, which is generally
understood to include gains unrealized as well as realized. Much work, including the effort of self-development through study and exercise, is what TRE calls unsold work. We will also see that a worker’s output, meaning growth plus yield, may include “free growth” which cannot be sold. On these two grounds, work may exceed payment received. On another the reverse may be true. A worker’s receipts will include recovery of his work expense as well as the value of his output, just as the baker’s will include recovery of the cost of the flour. TRE prefers the word “pay” to mean the market price of the worker’s “work sales,” which exclude unsold work but include recovery of expense. The application of the total return rule to self-value gives the “work rule”

\[
\text{work} = \text{self-growth} + \text{work yield}
\]

for any worker or set of them. Self-growth is growth in self-value. Although the right-hand terms might baffle a CIPP economist, we will find that they can be given diagnostic definitions.

The work rule is easily intuited when applied to the dependent young, whose growth is positive and whose yield is negative. Here the rule can be arranged to read “growth in self-value is negative work yield (net insuffusion afforded from ingift or sometimes from the worker’s own property yield) plus the work of the young invested in themselves in such forms as eating, learning and exercise.” That simply says that growth equals the sum of its external and internal components.

But the work rule reveals a surprise when applied to adult workers whose growth in self-value is negative and whose yield is positive. Here the rule shows that the worker’s work yield exceeds his work or new creation of value by the amount of his decline in self-value. Our first intuition is that this is absurd. Workers sell services, rather than themselves. But in fact the rule is accurate and our intuition is wrong. The trick in understanding this is to remember that self-value does not mean flesh and blood and soul. It is a mathematical construct which integrates time-discounted future yields. The duplication rule shows that its extinguishment or “senescence” is recovered in the worker’s maint.

\[
\text{Rate of return}
\]

\textit{Rate of return} to property assets conventionally means current profit divided by property asset value. TRE will generalize that definition to both factors. Thus rate of return to workers is “work rate,” meaning work divided by self-value, and rate of return in general is output divided by capital.

The equilibrium time discount rate by which a future yield is discounted to present value is the expected average rate of return to the yielding asset over the intervening period. Thus for example a note expected to be redeemed a year from now for eleven dollars, and costing ten dollars now, shows continuously compounded discount rate and expected rate of return to be equal at a little less than ten percent yearly. (The actual rate comes to about 9.53%). This is the unique discount rate such that a buyer should be indifferent between the dearer future cost or the cheaper present one which will confer the same yield.
Theory and evidence agree that assets of greater \textit{risk}, meaning standard deviation in expected output rate, are less desirable and accordingly fetch lower prices than lower-risk ones whose \textit{expectation} or average output rate is judged the same. This means that riskier assets earn higher rates of return. Then we expect that a worker whose work rate is more volatile will realize a higher discount rate and lower value to himself than one whose work rate is equal on average and also steadier. Average-risk rate of return over a closed economy is implicitly the economy’s overall rate of return.

Now define

\[
growth rate = \frac{\text{growth}}{\text{capital}},
\]

\[
yield rate = \frac{\text{yield}}{\text{capital}},
\]

\[
\text{prop growth rate} = \frac{\text{prop growth}}{\text{property}} \quad \text{and}
\]

\[
\text{self-growth rate} = \frac{\text{self-growth}}{\text{self-value}},
\]

from which

\[
\text{rate of return} = \frac{\text{output}}{\text{capital}} = \text{growth rate} + \text{yield rate}.
\]

Rate of return to the individual factors is

\[
\text{work rate} = \frac{\text{work}}{\text{self-value}} = \text{self-growth rate} + \text{work yield rate}
\]

and

\[
\text{profit rate} = \frac{\text{profit}}{\text{property}} = \text{prop growth rate} + \text{prop yield rate}
\]

where

\[
\text{work yield rate} = \frac{\text{work yield}}{\text{self-value}} \quad \text{and} \quad \text{prop yield rate} = \frac{\text{prop yield}}{\text{prop}}.
\]

All these equations are versions of the total return rule.
Since yield at the scale of gift closure is final yield, meaning maint plus waste consumption, define

\[
\text{final yield rate} = \text{maint rate} + \text{waste rate}
\]

where

\[
\text{yield maint rate} = \frac{\text{maint}}{\text{capital}} \quad \text{and} \quad \text{waste rate} = \frac{\text{waste}}{\text{capital}}
\]

to express the “closure rule” as

\[
\text{rate of return} - \text{growth rate} = \text{final yield rate} = \text{maint rate} + \text{waste rate}
\]
at gift closure. Further define

\[
\text{pure senescence rate} = \frac{\text{senscence}}{\text{self-value}}, \quad \text{factor index} = \frac{\text{self-value}}{\text{capital}}
\]

and

\[
\text{senescence rate} = \text{pure senescence rate} \times \text{factor index} = \frac{\text{senscence}}{\text{capital}}.
\]

Now the duplication rule allows

\[
\text{senescence rate} \sim \text{maint rate}.
\]

The value of these terms and relations lies in their potential use in modeling the social rate of return, meaning the average rate over the economy as a whole. We see for example that rate of return in the stationary state is not zero, as implied in several influential doctrines, but rather equals the sum of the maint rate and waste rate. We see now that the maint rate, or ratio of yield maint to capital, gives the social rate of return when waste consumption is zero. But we also note that it gives the social or average-risk rate, and not the risk-free rate.

A limit for the waste rate, for example, is suggested in Ricardo’s insight that workers are commodities whose cost adapts supply to demand. It would follow that waste cannot be afforded from pay, and must therefore be drawn from property yield alone. The pure senescence rate is also convenient to model by analogy to aggregate depreciation of a “population” of buildings. The factor index, however, proves surprisingly elusive. Tradition, accepting pay as a measure of workers’ output, takes the factor index at the pay / income ratio. Two thirds is a typical reading. The sold work rule sadly invalidates that algorithm, and TRE finds nothing as simple and transparent to replace it.
We saw that TRE, like other views of economics, is a pragmatic study whose end is prediction of behavior. We also saw that economists sometimes approach this purpose by identifying maximands, or specific quantities that rational owners will always bargain or strategize to increase. Some teach that this maximand is consumption, which tradition regards as the flow of taste satisfaction. TRE would substitute the concept “total yield,” which excludes insumption but adds gift, as the true measure of current satisfaction. But neither consumption nor yield is in fact an absolute maximand. If it were, all would liquidate their holdings immediately and splurge from the proceeds.

Most economists teach that the real maximand is capital, which embodies the present value of future taste satisfactions. But this is still not realistic. Adults routinely decapitalize to provide parental gift, and probably to waste as well. Clearly we want more total yield and more capital too, with an age-dependant shift between those priorities. But we cannot say that we want the largest sum of both, because they cannot be summed. Capital is a stock and total yield is a flow.

Growth of capital is a flow, however, and wanting more capital implies wanting growth of capital. And the sum of total yield and growth of capital of both factors is the owner’s total return or output from all his assets. Then we would be nearer the truth in saying that output is the absolute maximand. But we can do a little better. If the owner had a magic wand, he could command output of a trillion dollars a second to distribute between growth and yield as he liked. What he has is not a magic wand, but a current stock of capital of both factors. What he actually can maximize, by good management, is the ratio of output to the value of that capital alone. This ratio is rate of return. Although output and rate of return are both maximized by the same decisions, that is, rate of return is the more informative measure for interpreting and predicting behavior.

Let us illustrate. Suppose that Tom has twice the wealth of Dick and four times the wealth of Harry. Each is trying to maximize output, or equivalently rate of return. But Tom’s output is very likely to be greatest, and Harry’s least, however carefully each manages. Rate of return is the better common yardstick of the maximization efforts of each. If Dick’s rate of return is greatest, for example, then Dick shows the most effective combination of intention and proficiency in maximizing output from what he has. Thus rate of return is the more informative maximand, not necessarily the more accurate one, for purposes of predicting and measuring behavior.

But we are not quite finished. Tom, Dick and Harry do not in fact all maximize rate of return. This rate tends to vary as risk, meaning standard deviation in the same rate. A few bold spirits are risk-indifferent, and buy bumpiest but most rewarding assets. Others are risk-averse, in varying degrees, and buy assets with slower but steadier returns. The final and absolute maximand which rational owners actually can and actually do maximize is risk-adjusted rate of return.

This inference should hold for all owners at all ages. The young generally maximize growth of self-value. Highest growth means highest return with least yield. The young grow by parental gift as well, but cannot maximize this source because they do not control it.
adults continue to prioritize capital growth, and then prioritize yield to bring the new generation into place. Note that the rational actor of TRE is not implicitly the selfish *homo economicus* of other systems. By including gift in total yield, in place of insumption, TRE accommodates the realities of parental altruism more easily and explicitly than its CIPP counterpart does.

Risk-adjusted rate of return is the economic quantity which all rational actors can be entrusted to maximize right now. “Rational,” in this context, simply means behaving consistently with tastes. This maximand allows most splurging to a wastrel, most saving to a miser, most gift to a philanthropist, and most destruction to a Bin Laden or Doctor No. Then the “maximand rule” infers from definitions and the base assumptions that all owners maximize current risk-adjusted rate of return.

 parity in risk and return

Modeling is easiest if we assume that both factors, on average, are equal in rate of return. This would be expected, under efficiency, if the two factors on average are also equal in risk. These two conditions are “return parity” and “risk parity.” Note incidentally that correlation between risk and return should prove less predictable among individual workers than among property assets, since a worker cannot sell himself to others when volatility of his work rate does not suit his degree of risk aversion. The best he can do is predict which skill developments will lead to which jobs, and trust outcomes to fit predictions.

With that qualification, we may be confident that risk parity should imply return parity. Risk-adjusted rate of return is the absolute maximand. Then an owner will invest in whatever asset of either factor will expect highest return at his chosen degree of risk. Arbitrage will develop equal return for equal volatility regardless of factor, insofar as outcomes match predictions.

I will argue nonetheless that owners will typically invest mostly in self-value when young, and more in property as they age. The young learn easiest, and have most time left to pay out investment in skills. As these skills accumulate, owners become more adept in acquiring and managing property. And property, unlike self-value, can be left to descendants when there is more than enough. Thus insumption will tend to maximize return at the start of a worker’s career, and property investment will maximize it later on.

 sales and expense

Money economies, as distinct from barter economies, realize some market exchanges in *sales*. Yet many trades, even in the most modern economies, are still effectively bartered. The unpaid services of housewives are a favorite textbook example. Sometimes the distinction between barter and sales is fuzzy. If a ten-dollar steak is traded for five two-dollar tacos, one could interpret the trade as reciprocal ten-dollar sales. Economists may well so interpret it for the sake of logical consistency. NIPA includes many such imputations. TRE, for the same reason, prefers to treat all transfers of value between owners, or even between an owner’s worker and investor alter-egos, as sales at market price for money or money’s worth. Gift, under this imputation, is forgiveness of payment for the sale.
“Sales” will mean sales of services as well of corporeal things. Thus sales includes rent receipts and performance fees and interest received. TRE calls the sales of a worker his “work sales” measured in his “pay” received in return. TRE also imputes that all worker services are sold for pay except for those directly self-invested, as in the effort of learning. The reason for this exception is that other worker services, but not this one, might have been hired from outside. There is no market substitute for study and exercise.

The value of what is sold, less the value of the part not concurrently created by the seller, is the part concurrently created by the seller. This tautology is the “realized output rule,” shown as

\[
\text{sales} - \text{expense} = \text{realized output.}
\]

Note that expense is defined here as value actually recovered in sales, regardless of cost to the seller. Otherwise the logic would not be airtight. GAAP, on the other hand, is constrained for practical reasons to measure expense at cash or accrued cost. Cash expense is value of inputs currently got from others, regardless who created them or when, and liquidated expense is the remainder contributed from assets already on hand. Liquidated expense is recovery of extinguishments, or what TRE calls liquidation. Sales from inventory, or even sales of plant and equipment, are included in liquidation in the TRE sense. Since the extinguishment of self-value is senescence, which is exhaust recovered in no other capital, the expense of self-value (work expense) is wholly cash expense got from others or from the worker’s property-owning alter-ego.

The realized output rule, in its GAAP form, is the foundation of profit-and-loss accounting. GAAP profit, or earnings, is realized output in that sense plus change in inventories, or other accounts designed to capture the part of output unsold. TRE recognizes that as a tautology, like the total return rule and present value rule, it must hold for both factors equally. For self-value, it is the “sold work rule”

\[
\text{pay} - \text{work expense} = \text{sold work, where}
\]

\[
\text{work expense} = \text{maint} + \text{non-yield expense.}
\]

Since pay and sold work are contingent on work expense, we can also think of sold work as “discretionary pay.”

The realized output rule for property is

\[
\text{prop sales} - \text{prop expense} = \text{realized profit.}
\]

Realized and sold mean the same. “Sold” is shorter and plainer. “Realized” is used in TRE where it is traditional, as in realized output or realized profit, but “sold” is preferred where no tradition exists.
Costs will exceed sales at times. A restaurant closed for the day, or a plant down for repairs, may still experience costs of interest or tax or depreciation. Might realized output be negative under these circumstances? Not by the definitions chosen. Expense was defined as cost recovered in sales, and hence cannot exceed sales. Cost not recovered in sales is typically investment converted into plant or inventory value. It is not expense in the TRE usage. The lowest possible value of realized output is zero.

**maintenance and the definition of self-value**

Investment and maintenance, in their familiar senses, are inputs into property value which are told apart by the lastingness of their effects. Inputs which wear off quickly and must be repeated often, like vacuuming carpets or oiling machines or watering plants, are usually expensed against sales rather than capitalized. The dividing line is more or less arbitrary, and may be set by convention for book purposes and by law for tax purposes. In strict TRE analysis, which takes flows as continuous rather than as collected over discrete periods, the vacuuming and oiling and watering would appear as short-term investments and not maintenance. Only costs converted into product value at the moment of outlay, of which the only sure example seems to be the recovery of extinguishments, would meet this “passthrough” criterion of expense.

Thus maintenance in the instantaneous mark-to-market TRE sense may not exist for property. For self-value, and for any capital in principle, it is expense which restrains or allays extinguishments in the sense that a dollar less maintenance, at the margin, would have meant a dollar more extinguishments. It is distinguished diagnostically from investment in that it forestalls extinguishments while investment offsets extinguishments. It is effectively a substitute good for extinguishments, and is charged against sales just as extinguishments would have been. The worker’s maint meets the passthrough criterion of expense under the consumption lag principle, which treats consumption as continuous rather than intermittent, and meets the test of maintenance in that it also restrains senescence.

Meanwhile the worker’s non-maint expense is defined as his costs which support and enable his pay, but which do not satisfy his final tastes or add self-value and are therefore excluded from consumption. Since they do not restrain his senescence, neither do they qualify as maintenance. Maint does satisfy final tastes, as we saw in the example of visits to the doctor, since workers would spend to allay senescence even if the expense were not recovered in pay, and do so even when the expense brings discomfort rather than enjoyment. These distinctions justify TRE’s definition of self-value as the factor whose maint satisfies final tastes, and hence must cost liquidation of value in final yield.

**cash flow**

Yield, in TRE, means net outward transfer of value after plowback in the producing asset. Gross positive transfer before this plowback is cash flow. Then the “first cash flow rule” is

\[
\text{cash flow} = \text{plowback} + \text{pos yield},
\]
where “plowback” means plowback from sales. Negative cash flow is investment from outside, and as such is identical to negative yield. This is written

\[
\text{neg cash flow} = \text{neg yield}, \quad \text{allowing}
\]

\[
\text{net cash flow} = \text{cash flow} - \text{neg cash flow} = \text{plowback} + \text{yield}.
\]

The first cash flow rule analyzes cash flow by its possible dispositions. Tradition also analyzes it by source as sales less cash expense. This equation must be adjusted in order to hold for self-value, since most expense of self-value is maint which satisfies final tastes and consequently must be paid from pos yield rather than deducted before. Then the general form of the “second cash flow rule” is

\[
\text{cash flow} = \text{sales} - \text{non-yield cash expense}.
\]

Another common formula for property cash flow, also analyzed by source rather than disposition, is the “third cash flow rule”

\[
\text{cash flow} = \text{realized output} + \text{liquidation}.
\]

For self-value, the first cash flow rule is

\[
\text{work cash flow} = \text{plowback insumption} + \text{pos work yield},
\]

where plowback insumption is insumption bought from pay. Since all work expense is cash expense, meaning maint plus non-yield expense, the second is

\[
\text{work cash flow} = \text{pay} - \text{non-yield expense} = \text{pure pay}.
\]

The third is

\[
\text{work cash flow} = \text{sold work} + \text{self-liquidation},
\]

where the last term is recovery of senescence in yield. These are the “work cash flow rules.”

The three rules, applied to either factor, are mutually implicit in that each of the second two flows gives the whole source of the first and nothing further. Now combine the second two work cash flow rules to read

\[
\text{pure pay} - \text{sold work} = \text{self-liquidation}.
\]

By the sold work rule, pure pay less sold work equals maint. Then

\[
\text{maint} = \text{self-liquidation},
\]

so that the extinguishments rule now leads to the duplication rule.
Note that cash flow, particularly in its application to self-value, is not always paid in literal cash. It is simply the sum of plowback and pos yield. Pos work yield includes maint, whose taste satisfaction value is paid in subjective cash realized from the subjective recovery of value from the worker’s senescence.

**work yield**

The total return rule for growth shows that the output of self-value, called work, is growth in self-value plus work yield. It is now possible to clarify the last two flows. The first cash flow rule, applied to self-value alone, can be written as

\[
\text{work cash flow} = \text{plowback insumption} + \text{work yield} + \text{neg work yield},
\]

which can be arranged as

\[
\text{work yield} = \text{work cash flow} - \text{plowback insumption} - \text{neg work yield}.
\]

Since work cash flow equals pure pay, meaning pay less non-yield expense, the “work yield rule” is

\[
\text{work yield} = \text{pure pay} - \text{plowback insumption} - \text{neg work yield}.
\]

Here neg work yield means insumption got from current ingift or from current prop yield from the worker’s own property.

This definition helps clarify self-value and its growth flow. In general, the present value rule is

\[
J = \int_{0}^{\infty} \eta_{t} e^{-r_{t} t} dt
\]

where \(J\) is capital value, \(\eta_{t}\) (\(\eta\) is *eta*) is yield flow expected at time \(t\), and \(r_{t}\) is average rate of return expected over the period ending at time \(t\). For self-value, then,

\[
H = \int_{0}^{\infty} \eta_{H,t} e^{-r_{H,t} t} dt
\]

where \(H\) is self-value, \(\eta_{H}\) is work yield and \(r_{H}\) is work rate.

**alternative capital pricing**

The present value rule shows that capital is market-priced at the integral of yields expected to be gained from it, time-discounted to present value at its risk-adjusted rate of return. We saw that several variants of this rule work equally well, and give the same present value under consistent information. One popular variant is called discounted cash flow or DCF. What
this method discounts is not actual expected cash flows over the life of the asset, but what those flows would be if no part were plowed back. Thus the imaginary flows discounted would tend to be higher at the start than the real ones reflecting plowback, but smaller later. The discount rate would be the expected rate of return in each case.

DCF methods may sometimes be more practical than discounted yield ones for evaluating individual worker value. The worker’s cash flow is his pay less his non-yield expense. This last flow is probably small, and pay is convenient to measure. Absent plowback insumption, pay should hold steady until skills deteriorate or obsolesce toward the end. We might assume neutrally that the rate of return to self-value, or work rate, approximates the social rate of return. If this and the worker’s pay and age are known, his self-value can be estimated by DCF methods.

consumption over the life cycle

TRE’s tripartite division of consumption into insumption, maint and waste expresses economic rather than biological concepts. Analysis of consumption over the life cycle is easier if we begin with a less technical and more intuitive description. Let a worker’s “sustenance” mean whatever consumption he needs, at any age, to carry out his expected physical development and maintain health. Then sustenance will probably mean about the same thing as maint over the adult years, or an expense recovered in pay, and will mean a part of insumption in childhood.

Since capital is present value of future cash flow, and since work cash flow is pure pay, insumption can be understood as any consumption which is expected to raise future pure pay, but not present pay, when other things are equal. It seems natural to model most consumption as insumption through some age in the teen years at least. For what the argument ad auctoritatem is worth, that is what economists assume when they evaluate emerging adults at the sum of past inputs by parents and others. Biologists add that creatures tend to begin life in a somatic phase where resources are applied to maximize bodily development, and to end it in a reproductive phase maximizing investment in the young. Then suppose that maint and waste are largely deferred until maturity. That would mean that essentially all sustenance is insumption at the start.

Insumption begins as parental gift alone, and adds an increasing component of self-invested work as self-value accumulates to produce it. Insumption may be analyzed into costs of somatic development, teaching and learning. Somatic development is paramount at the start, and typically ends at an age in the early twenties in humans. Teaching and learning, particularly the latter, continue much longer. There is learning without teaching at every age, and the ratio of learning to teaching rises steeply once formal education is completed. All learning which raises future expectations of pure pay, whether the learning is deliberate or subconscious, is a gross input into self-value by the learner and is therefore self-insumed work. Some somatic development is also self-insumed work, for example in exercise. Most costs of somatic development and all costs of teaching, however, are met from parental gift in the young, and from other cash sources in adults.

Cost of teaching is tuition plus cost of books and other didactic materials such as videos or CD ROMs. Tuition is the dominant component until formal schooling ends. Since this
terminus coincides more or less with physical maturity and with the end of parental gift, adult insumption is essentially learning (self-insused work) plus costs of books and other teaching materials. These cash costs are likely to prove minor overall for adults, mainly because much adult learning is incidental experience independent of them.

It is hard to say when sustenance converts from insumption to maint, and whether smoothly or abruptly. It is not clear, for example, that sustenance before maturity is wholly somatic development. Sustenance in the late teen years, when bodies are nearly complete, is more easily taken into insumption if interpreted as a subsidiary cost of learning. That same interpretation could allow sustenance into insumption even after maturity. We saw earlier that taste satisfactions given by sustenance would not prove the presence of maint, in that case, but would rather be accounted for as free goods concomitant with the insumption. Then it is hard to analyze consumption on a priori principles in the late teens and early twenties. But it should be possible in principle to infer the nature of consumption from its effects. Maint supports pay now. Insumption does not, but will raise pure pay later. Waste does neither. These definitions, and clues from the data, make some models more plausible than others.

Let us take what clues we can. Intuition suggests that sustenance, whatever its economic disposition, does not change greatly, say by more than a factor of two, from late teen years to the end. Pay / age profiles meanwhile show that pay rises sharply with age to a peak in the forties or fifties, and declines only mildly by retirement. On the neutral assumption that this is also true of pure pay, these profiles are strong evidence of adult insumption until the peak pay age or later. Adult insumption is largely learning, and learning is unsold work. Then it may be best to model all thrift work as unsold at entry into the job market, so that all of pay at entry is work expense. This assumption fits the idea that entry-level workers are essentially apprentices who maximize skill improvement while earning nothing beyond their keep. The share of thrift work sold can be allowed to rise gradually thereafter, and to reach 100% at some age which leaves too little time for recovery of any further investment in self-value.

This model would establish entry-level pay as a proxy for work expense, within a factor of two or so, over the rest of adulthood. It would reveal thrift work most clearly in older working cohorts, where all thrift work is assumed to be sold, as observed pay less work expense extrapolated from this proxy. It is the view of lifetime consumption which this text will favor in estimating the senescence and pure senescence rates.

We will suppose then that sustenance is wholly somatic development at the start, and that it converts gradually into a subsidiary cost of learning as maturity nears and bodies are nearly complete. Thus it is insumption in both cases. At maturity and entry into the job market, however, we will interpret sustenance as maint supporting pay rather than as insumption supporting learning.

modeling the pure senescence rate

Imagine a “population” of buildings of all ages, with “birthdays” equally spaced at interval dx. Each is constructed with constant investment over period N, the “non-adult span.” Its value at completion, $B_{max}$, thereafter depreciates linearly to zero over its “adult span” A. Then
the depreciation of each “adult” building is \( B_{\text{max}} / A \). The number of adult buildings is \( A / dx \).

Then aggregate depreciation of the population is \( B_{\text{max}} / dx \). Average value of all adult and non-adult buildings is \( B_{\text{max}} / 2 \), by linearity of both investment and depreciation, and total number of all these buildings is \( (N + A) / dx \). Then aggregate value of the population is \( B_{\text{max}} (N + A) / 2 dx \). Now define

\[
\text{population dep rate} = \frac{\text{aggregate depreciation}}{\text{aggregate value}} = \frac{B_{\text{max}} / dx}{B_{\text{max}} (N + A) / 2 dx} = \frac{2}{N + A},
\]

showing that population dep rate is simply twice the reciprocal of overall building lifespan from first construction to the end.

The pure senescence rate is like population dep rate in that it compares aggregate senescence of workers actually showing that exhaust, typically meaning adults, to aggregate self-capital of adults and non-adults together. Then if we define “economic lifespan” or “lifespan” \( \lambda \) (\( \text{lambda} \)) by \( \lambda = N + A \), the pure senescence rate would be given by \( 2 / \lambda \) insofar as assumptions for the buildings model applied realistically to owners. Two conspicuously do not. The model neglects adult insumption, or plowback investment after building completion, and it assumes the maximum possible depreciation period \( A \) for all investment. On both counts, the model underestimates the pure senescence rate. Adult insumption will senesce faster because of shorter remaining lifespan alone. And it does not seem realistic that all self-value,endures to the end. Some learning and skill development hold their usefulness better, and some worse. Each transfer or promotion means some old knowledge and technique obsoleted. Then it seems best to take the population dep rate analogy as a minimum or downside limit for the pure senescence rate. If worker lifespan is measured at 76 years, that is, we could estimate the minimum limit for the pure senescence rate at \( 2 / (76 \text{ years}) \) or 2.6% per year.

The senescence rate, which is what we really want to know, is the pure rate times the factor index. And the factor index, which CIPP estimates at the pay / income ratio, is shown to be no such thing by the sold work rule. Then the factor index is cryptic, and senescence rate remains a little beyond reach for the moment. Later chapters will estimate it by different approaches.

A population of mortal and reproducing “owners” competes and cooperates to gain means of satisfying current and anticipated economic tastes, meaning tastes not satisfied effortlessly. The means of satisfying current tastes is positive yield, and the means of satisfying future ones is capital. Capital is self-value plus property. It is valued at the integral of present values of final yields it is expected to confer to the owner or his donees, directly or through reinvestment in other capital of either factor, time-discounted at the risk-adjusted rate of return at which it is expected to produce them. Negative yield is investment received from outside. Yield is positive yield less negative yield. Any final taste satisfaction whose cost is not liquidated from the capital of the owner satisfied is a “free good” outside the purview of economics.
Positive yield is either “transfer yield” reinvested in another asset, becoming negative yield with respect to that receiving asset, or is final yield exhausted from the economy in satisfaction of the owner’s tastes. Positive yield invested in assets belonging to another owner is “outgift” from the viewpoint of the donor. Gift received and invested is “ingift” from the aspect of the receiver. A transfer used up in taste satisfaction as received is a free good to the receiver, and a part of final yield to the giver. “Gift” is outgift less ingift.

An owner’s aggregate yield from both factors, net of reinvestment in other assets of his own including his self-value, is his “total yield.” The negative component in total yield is ingift. The positive one is outgift plus final yield. Then total yield equals gift, meaning outgift less ingift, plus final yield. Although final yield is never negative, total yield is negative whenever ingift exceeds the sum of outgift and final yield. Owners begin life in a period of negative total yield featuring ingift received from adults, called “parental gift,” and continue in a phase of positive total yield as this ingift ends.

At the scale of the closed economy, or at any smaller scale of “gift closure” where ingift and outgift are in balance, total yield is reduced to final yield alone. An advantage of the total yield concept is that it enables these inferences, and more, without need to clarify the yield of self-value first.

An asset’s output is its creation of value, measured as the sum of its growth and yield. That sum can be divided by the asset’s value to give its rate of return. These definitions are the “total return rule.” The economic quantity which each owner is expected to maximize is the risk-adjusted rate of return to his overall capital of both factors, since this maximizes his choice between yield now and capital growth for yield later.

Consumption is invested consumption or “insumption” converted into self-value, plus “maint” converted into pay and then into product value, plus “waste” which satisfies tastes but is converted into neither. Final yield may therefore be understood as “yield consumption,” meaning consumption drawn from positive total yield of the owner consuming it. His insumption by contrast is consumption but not net total yield, since it is reinvested in his self-value, while his outgift is part of total yield but not of his own consumption. Maint is expense such that a dollar less would have meant a dollar less in current job performance and pay. Maint satisfies tastes while allaying senescence. Examples of non-yield expense might include bus fare to work and back, if not reimbursed by the employer, and any other cost necessary to enable current job performance which does not concomitantly satisfy final tastes. Its extent is probably small. In any case it follows that final yield, or consumption bought from total yield, is maint plus waste.

An economist from another planet might sensibly object that the worker’s satisfaction from his maint is a free good, since its cost is reimbursed by the market. An economist from this planet, whether an adherent of CIPP or of TRE, would answer that the aim of economics is prediction, and that the worker behaves as if that satisfaction were an economic good for which he must invest. Thus earthly economists, including TRE ones, count maint twice. The
difference is that TRE counts it once each as expense and as final yield, while CIPP counts it twice as input into product value alone.

Meanwhile an owner’s “exhaust” can be defined as any extinguishments from value of his capital of either factor not recaptured in value of other capital, whether belonging to him or another owner. What extinguishments might that be? The depreciation of the farm buildings is normally recaptured in the price of the wheat, which is normally recaptured with the depreciation of the mill in the price of the flour, which is normally recaptured with the depreciation of the bakery in the price of the bread. The only extinguishments not normally recaptured are waste consumption and the senescence which drains value from the final link of the extended chain (Figure 1.1). But we see by the definition of capital as present value of expected yields that all extinguishment is expected to be recovered in yield. This reasoning, and the clarification of final yield as yield maint plus waste consumption, give the duplication rule.

“Sales” is broadly construed to mean all transfers and all services other than self-invested work. Sales less expense gives “realized output.” The worker’s sales is pay, his expense is work expense, and his realized output is sold work. Then the “sold work rule” finds “pay less work expense equals sold work.” “Pure pay” is pay less non-yield expense. Thus the rule may also be stated “pure pay less maint equals sold work.” A discussion of cash flow, defined in principle as plowback plus pos yield, led to a new derivation of the duplication rule and to a diagnostic definition of work yield.
III. THE GROWTH DICHOTOMY

All economists want a better understanding of the roles of thrift and productivity in growth. But it is not so easy to pin down what that means. Productivity is the ratio of output to capital employed, or simply rate of return. Since yield is generally positive, the return rule shows that rate of return generally exceeds growth rate. Then the share of growth due to productivity gain cannot well be measured as their ratio, which tends to exceed unity. What we might do instead is compare the change in each rate since some past starting point. Meanwhile thrift is yield restraint, but again by a benchmark or starting point which seems open to choice. Then let us begin by considering what the origin or benchmark standard of each might be.

For any asset or set of assets continuously identifiable over time, say the self-value of Joe Smith or a share of XYZ Corporation or the aggregate wealth of the world, it is possible in principle to compare rate of return today with rate of return at a given moment in the past. It seems reasonable to say that productivity gain in that capital over that period is increase in its rate of return. Thrift gain would mean any decrease in yield rate over the same period. Let us illustrate. If \( g, r \) and \( \phi \) (\( \phi \)) are growth rate, rate of return and yield rate, the total return rule can be arranged as

\[
gr, \text{ implying } \Delta g = \Delta r - \Delta \phi
\]

over the chosen interval. Here \( \Delta r \) and \( -\Delta \phi \) give productivity and thrift gain as just defined.

We are most interested in the contributions of productivity and thrift to growth rate \( g \) as a whole, rather than to a recent increment \( \Delta g \) only. Then let the chosen time interval over which changes are measured begin with a moment of zero growth or stasis. Such moments are not rare, either for individual assets or for whole economies, even in an age of strong secular growth. The United States seems to be passing through one, more or less, as I write. If such a starting moment is chosen, then \( g \Delta g = g \) and

\[
g = \Delta r - \Delta \phi, \text{ since } g_0 = 0.
\]

Now \( \Delta r \) and \( -\Delta \phi \) may be called the “free growth rate” and “thrift rate” components in current growth rate \( g \). For a more intuitively natural notation, let us show these rates as \( \Pi \) and \( \Theta \). Then

\[
\Pi = r - r_0 = \text{free growth rate} \quad \text{and} \quad \Theta = \phi_0 - \phi = \text{thrift growth rate},
\]

where \( r_0 \) and \( \phi_0 \) are rate of return and yield rate at a chosen past moment of stasis. Implicitly \( r_0 = \phi_0 \). \( r_0 \) is the “stasis rate” for that particular instance of stasis. Next define “free growth” \( \Pi \) (upper-case \( pi \)) and “thrift growth” \( \Theta \) (upper-case \( theta \)) by
\[ \Pi = g_{\Pi} J \quad \text{and} \quad \Theta = g_{\Theta} J. \]

Also define the “prod index” \( C_{\Pi} \) and “thrift index” \( C_{\Theta} \) by

\[ C_{\Pi} = \frac{g_{\Pi}}{g} \quad \text{and} \quad C_{\Theta} = \frac{g_{\Theta}}{g}, \quad \text{and note} \]

\[ g_{\Pi} + g_{\Theta} = g, \quad \Pi + \Theta = \dot{J} \quad \text{and} \quad C_{\Pi} + C_{\Theta} = 1. \]

Since the flow \( r_0 J \) gives current output \( rJ \) net of its free growth component \( \Delta r J \), call \( r_0 J \) “thrift output.” Now substitute \( r_0 \) for \( \phi_0 \) in the definition \( \Theta = \phi_0 J - \phi J \) and arrange as

\[ r_0 J = \Theta + \phi J, \quad \text{reading} \]

thrift output = thrift growth + yield.

Also divide by \( J \) for

\[ r_0 = g_{\Theta} + \phi, \quad \text{reading} \]

stasis rate = thrift growth rate + yield rate.

These are thrift versions of the total return rule. Free growth can be added to both sides of the first, and free growth rate to both sides of the second, to get the original versions.

Now see

\[ (\Delta r)_0 = r_0 - r_0 = 0 \quad \text{and} \quad -(\Delta \phi)_0 = \phi_0 - \phi_0 = 0, \]

showing that free growth rate and thrift growth rate were each zero at the moment of the origin in stasis. This result is counterintuitive. Since overall growth rate \( g \) is the sum of the two, one would think that they might easily have both been nonzero but opposite in sign. But each was defined as change since that moment, and each therefore began at zero.

\( J \), if a set of assets such as the aggregate capital of a closed economy, may include both self-value \( H \) and property \( K \). Then define

\[ g_{\Pi} = \frac{\dot{H}}{H} \quad \text{and} \quad g_{\Theta} = \frac{\dot{K}}{K}, \]

noting that overall growth rate \( g \), defined as \( \dot{J}/J \), is a cap-weighted average of \( g_{\Pi} \) and \( g_{\Theta} \). This time it is possible for one to be positive, and the other negative, while their sum comes to stasis.
It may therefore be inconvenient to find a single past moment when all three rates were zero. With that reservation, let us define free and thrift growth of the factors just as for capital as a whole. Free growth rate in self-value, notated $g_{H_{11}}$, is thus gain in its rate of return (work rate $r_H$) since a chosen past moment when $g_H$ was zero. Free growth in self-value is notated $\Pi_H$ and given by the flow $g_{H_{11}} H$, and so forth. In general,

$$\Pi_H = g_{H_{11}} H, \quad \Theta_H = g_{H_{11}} H, \quad \Pi_K = g_{K_{11}} K \quad \text{and} \quad \Theta_K = g_{K_{11}} K.$$ 

Also $\phi_H$ is work yield rate and $\phi_K$ is prop yield rate. $r_{H,0}$ and $r_{K,0}$ are the “stasis work rate” and “stasis profit rate,” meaning work rate and profit rate at the chosen past moments when $g_H$ and $g_K$ respectively were zero. Now define “thrift work” $w_\theta$ and “thrift profit” $p_\theta$ by

$$w_\theta = \Theta_H + \phi_H H = r_{H,0} H \quad \text{and} \quad p_\theta = \Theta_K + \phi_K K = r_{K,0} K.$$ 

Since self-value is liquidated only in senescence paid subjectively for the yield value of maint, and since free growth is implicitly retained in capital, sold work is drawn from current thrift work and no other source. The rest of thrift output is “unsold thrift work” of self-investment through learning and exercise. That is,

$$\text{thrift work} = \text{sold work} + \text{unsold thrift work}.$$ 

the anatomy of thrift growth

Our aim was to define the proportionate influences of thrift and innovation in growth. The best I could do was to define the share of current growth due to gain in rate of return since a given past moment of stasis. That may be enough to satisfy the question. Growth rate in an investor’s portfolio of securities may well change signs several times a day, and even the growth rate of the U.S. economy returns to zero with each recession. Then the information carried in the growth dichotomy may seem reasonably fresh.

With due caution about the adequacy of these definitions, the difference between thrift and technology growth is well worth marking. Although we understand that rate of return varies from asset to asset according to risk, we know less about what makes it vary for the economy as a whole over time. We think it has something to do with the accumulation of ideas and a climate for nurturing them. But ideas of all sizes simply come as they come. Therefore this text will treat free growth as largely exogenous and unaccountable. It is random “leakage” splitting outcomes from expectations, for good and bad, plus the secular magic of innovation. Thrift growth, to the contrary, is all common sense and intuitive justice. It is the ethic of the Practical Pig. Though less interesting and not a bit more important than free growth, it is far more tractable. Thus this text will often prefer the thrift versions of the total return rule, which cancel free growth from both sides, for their gain in compactness and clarity.
Then let us take a closer look at thrift growth as defined. The growth rules show it as thrift output less yield. Thrift output, in turn, is current capital times the stasis rate. Since negative capital is undefined, this shows that thrift output cannot be negative unless rate of return was negative at the last stasis point. Since rate of return equals yield rate in stasis, and since negative yield (negative yield) is investment from outside, negative thrift output is impossible at the scale of economic closure. Although even national economies fall short of closure, we may expect that thrift output is positive at large scales.

Positive thrift output must be either retained as investment or surrendered in yield. Then thrift growth, defined as thrift output less yield, may be expressed as invested thrift output plus yielded output less yield. Meanwhile yield is pos yield less neg yield, and positive yield is yielded output plus liquidation. After substitution and cancellation of terms, these equations give the “thrift rule”

\[
\text{thrift growth } = \text{ invested thrift output } + \text{ neg yield } - \text{ liquidation.}
\]

For property assets, the sum of invested thrift output and neg yield, meaning investment from outside, may be naturally described as “thrift investment.” Equivalently it is the whole of investment less free growth. Then let us state the thrift rule for property as

\[
\text{net thrift investment } = \text{ thrift investment } - \text{ disinvestment},
\]

where disinvestment is measured as liquidation value actually recovered rather than as ex-ante extinguishments.

This last point comes as a mild surprise. One would think that thrift growth, or growth at static productivity, would simply mean investment less depreciation. But thrift growth rate means decline in yield rate, so that the only negative component in thrift growth is pos yield. Liquidation is pos yield, and depreciation per se is not.

TRE is particularly interested in the application of the thrift rule to self-value. We will take it as axiomatic that all neg work yield, or investment in self-value from outside, comes only in the form of consumer goods and services. Insofar as their value is transferred dollar for dollar into self-value, they are insumption. The worker’s insumed thrift work adds his plowback from pay plus his unsold thrift work in learning and exercise. Somewhat arbitrarily, we will define this unsold thrift work as the remainder of his insumption. Then the whole positive component in his thrift growth is his insumption. The negative one, which one might have thought to be senescence, is rather his “self-liquidation” or the recovery of that senescence in his maint. This flow alone meets the pos yield criterion for negative thrift growth. Then the thrift rule for self-value is

\[
\text{net insumption } = \text{ insumption } - \text{ self-liquidation},
\]

or equivalently

\[
\text{net insumption } = \text{ insumption } - \text{ maint.}
\]
Tom, Dick and Harry buy ice cream cones for a dollar each. Tom gets the dollar’s worth of satisfaction he expected. Dick was extra-hungry, and gets two dollar’s worth. Harry takes his cone outside, where it melts over his shirtfront. What is the economic analysis?

Yield is outflow of economic value from an asset captured by its owner or his intended donees. Outflow of value was one dollar in all three cases. Tom’s yield matched his outflow. Dick’s, however, seems ambiguous. It appears that we might interpret the extra dollar’s worth of satisfaction either as a free good outside economic measurement, or as free growth in the cone’s value between the buying and eating. Harry’s case is complex, but clearer. Since there are no economic “free bads,” the dollar’s worth lost must be accounted for in economic terms. And since negative yield would mean transfer of value into the cone from outside, which is certainly not what happened, the loss of the cone’s cost is negative free growth. His shirtfront suffered additional negative free growth to be rectified later by cleaning.

Dick’s case becomes clearer when we remember objectives. The idea is behavior prediction. If Dick had been surprised at the tastiness of his cone, the extra delectation should count as a free good. Since he foresaw it, being extra-hungry, the gain is more usefully described as extra yield explained by his acumen in spotting the cone as a bargain. Then the cone realized free growth of one dollar as he bought it. Harry’s misadventure is offset by Dick’s good luck in meeting an opportunity and his good sense in taking advantage.

Harry’s loss was negative free growth, and not yield, because it was unintended. If he had meant his cone to melt over his shirtfront, the outflow would have counted as yield in satisfaction in tastes. That is why there are no free bads. Dick’s gain was positive free growth converted to yield, and not a free good, because it was chance exploited by design. Since these pratfalls and windfalls tend to offset over scale, under reasonably good prediction, and since they represent the fortuitous components of positive and negative free growth, the residual component of innovation emerges and dominates.

The term “leakage,” which was met in presentation of the convergence rules and of the value-added chain, means these equilibrating fortuitous components in positive and negative free growth. The convergence rules show

\[
\text{pos yield} + \text{leakage} = \text{outflow}, \quad \text{and}
\]

\[
\text{leakage} \sim 0.
\]

the second output rule

The total return rule shows that output is growth plus yield. At the scale of the owner or any set of them, yield is total yield. Total yield is gift plus final yield, and final yield is maint plus waste. Then output is growth plus gift plus maint plus waste at the scale of any set of owners. Free growth may be subtracted from both sides to show
thrift output = thrift growth + gift + maint + waste.

Also

thrift growth = net thrift investment + net insumption

= net thrift investment + insumption - maint.

These combine for the “second output rule”

thrift output = net thrift investment + insumption + gift + waste.

Now the fact

consumption = insumption + maint + waste

allows the rule to be stated alternatively as

thrift output = net thrift investment + consumption + gift - maint.

Finally the duplication rule allows

thrift output ~ net thrift investment + consumption + gift - senescence.

These last two statements of the second output rule correct the net product equation.

total cash flow

Cash flow from property, which can be called “prop cash flow,” is the only kind known to economic tradition. The third cash flow reveals it as

prop cash flow = realized profit + disinvestment,

since “disinvestment” means property liquidation. Meanwhile work cash flow may be expressed as sold work plus recovery of senescence, meaning sold work plus maint. Define “total cash flow” as cash flow from the two factors combined. Then

total cash flow = realized output + disinvestment + yield maint.

Let us compare total cash flow to CIPP’s “gross product.” Recall

gross product = net product + disinvestment, where

net product = net investment + consumption

= net thrift investment + free prop growth + consumption.
By the second output rule, net thrift investment plus consumption equals thrift output plus maintain less gift. Then

\[
\text{net product} = \text{thrift output} + \text{free prop growth} + \text{maint} - \text{gift},
\]

and

\[
\text{gross product} = \text{thrift output} + \text{free prop growth} + \text{maint} - \text{gift} + \text{disinvestment}.
\]

Now combine equations and cancel terms to arrive at

\[
\text{gross product} = \text{total cash flow} + \text{thrift output} - \text{realized output} + \text{free prop growth} - \text{gift}
\]

\[
= \text{total cash flow} + \text{unrealized thrift output} + \text{free prop growth} - \text{gift}.
\]

Prop cash flow gives a firm’s budget for plowback or dividends. Total cash flow is the stream available for plowback in both factors plus final yield, with the proviso that maintain is a non-discretionary obligation. Thus gross product appears to be a useful proxy for the flow available for overall budgeting, with that proviso, subject to correction for the three rightmost terms in the last equation. This inference helps support the judgment of Keynes and Kuznets, and of macroeconomists today, in treating gross product as an important measure of the economy.

**an insumption fund model**

Imagine a constant-valued fund of capital of both factors whose output over the non-adult span is just sufficient to provide enough insumption to raise one worker to maturity and independence. Fund value includes the increasing self-capital of the worker himself, and consequently a decreasing remainder of capital supplied by parents and other adults. Since fund value is constant, output equals yield converted into insumption. Neglect free growth, and assume a constant thrift rate of return. Then

\[
\text{F} r_0 \theta = \iota \quad \text{and} \quad \text{F} r_0 N = H(M)
\]

where \( F \) is fund value, \( r_0 \) is thrift rate of return, \( \iota \) (\emph{iota}) is insumption, \( N \) is duration of the non-adult span and \( M \) is age at maturity. We will generally take \( N \) as one year greater than \( M \) to allow for investment \emph{in utero}. Now solve for \( r_0 \) to see

\[
r_0 = \frac{1}{N} \frac{H(M)}{F}.
\]

The age of maturity and independence will be gauged at 20 years, so that the non-adult span or period of parental gift becomes 21 years. Then the model shows
\[ r_0 = \frac{1}{21 \text{ years}} \frac{H(M)}{F} = \frac{4.8\% \ H(M)}{\text{year} \ F}. \]  

(2.1)

But what is known of the ratio \( H(M) / F \)? Let us assume pure Ricardian pay equilibrium, so that each worker inherits nothing and replaces himself from his own thrift work and interim profit with nothing left over. Then the insumption fund represents his self-value at maturity, meaning \( H(M) \), plus any additional capital of both factors which he can earn before parenting begins. Let us estimate this capital buildup period. The mean age of reproduction, or R.A. Fisher’s generation length, looks to be about 28 years in America today. That means the average age of parenting at all births, not only first births, where maternal and paternal ages are weighted equally. Suppose then that the capital accumulation period is the eight years between maturity and the mean age of reproduction, called the “bachelor period,” and that these eight years are just enough to grow the worker’s capital of both factors from \( H(M) \) to \( F \). Again assume age-independent thrift rate of return \( r_\theta \) to both factors. Then \( F \) would represent exponential growth of \( H(M) \) at rate \( r_0 \) over eight years if there were no drain of capital during that period. But pay in the twenties should be mostly work expense, as we just saw, and most work expense is maint which equilibrates to senescence. Consequently

\[ F = H(M) e^{\delta B} - \bar{\delta} B, \]

where \( B \) is the bachelor period and \( \bar{\delta} \) (\( \delta \) is *delta*) is average senescence over that span. Rearrange for

\[ \frac{H(M)}{F} = e^{-\delta B} \left( 1 + \frac{\bar{\delta} B}{F} \right), \quad \text{or} \quad \frac{H(M)}{F} \geq e^{-\delta B}. \]

Now take \( B \) at eight years, and substitute in (2.1) to get

\[ r_0 \geq \frac{4.8\%}{\text{year}} e^{-\delta(8 \text{ years})}. \]

This relation can be solved implicitly to show

\[ r_0 \geq 3.6\% \text{ per year.} \]

Then 3.6\% gives a lower bound for the rate of return necessary to replace losses to senescence under the assumptions. 4.8\% per year may be taken as an upper bound, since we have assumed that \( H(M) \) is less than \( F \). Note also that the rate of return offsetting senescence is implicitly the senescence rate. The insumption fund model therefore places the senescence rate somewhere in the range of 3.6\% to 4.8\% per year.

Both extrema probably err on the high side. The model assumed pure Ricardian pay equilibrium without external subsidy. Subsidy probably exists, and lowers the equilibrium rate of return by adding property other than interim property to the insumption fund. If subsidy from
wealthy investors provides 10% of insumption, say, then the extrema are each lowered by 10% to 3.3% and 4.4% per year respectively.

Later we will meet something called the “maturity model” which will reconstruct most of this argument with somewhat different assumptions. Until more is known, under either version, it is enough to say that the senescence rate looks to be several percent yearly.

which starting point?

Free and thrift growth rates are defined as changes in rate of return and yield rate since a chosen past moment of stasis, or zero growth. If more than one such moment has occurred, measurements might not agree from one to the next. Thus each measurement should specify the moment chosen.

The growth dichotomy will prove particularly useful at large scales, including the scale of closure. I will argue in Chapter Four that thrift growth at that scale is impractical, so that growth is substantially free growth alone. Positive thrift growth of younger cohorts, and of assets under construction, is balanced by negative thrift growth of parental cohorts and mature assets in this hypothesis of “thrift offset.” If the hypothesis is right, net growth at the scale of closure equals free growth regardless of the starting point chosen. The prod index, or ratio of change in rate of return to change in growth rate, should not be particularly hard to measure. If it is consistently measured near unity at large scales, the hypothesis gathers support.

summary

We sought measures of the proportionate influences of productivity and thrift on current growth. Our choices were the “free growth rate” and “thrift growth rate,” measured respectively as increase in rate of return and decrease in yield rate since a chosen past moment when growth was zero. The “prod index” and “thrift index,” whose sum is unity, are the ratios of these rates to current growth rate. The same rates are multiplied by current capital to give the flows of “free growth” and “thrift growth.” Rate of return at the chosen past moment of stasis is the “stasis rate,” implicit equal to the yield rate at that moment. Since thrift growth is more predictable and manipulable than free growth, though less interesting, TRE will often prefer equations where free growth is cancelled out. Free growth can be subtracted from both sides of the total return rule, for example, to show “thrift output equals thrift growth plus yield.”

The negative component in thrift growth is recovery in liquidation, not ex-ante extinguishments. Thus thrift growth in self-value is insumption less maint, rather than the more natural-sounding insumption less senescence. This fact allows cancellation of maint in the thrift version of the total return rule to show that thrift output equals net thrift investment plus consumption plus gift less maint. This reasoning illustrates the double-counting of maint in the net product equation.
IV. THE THRIFT LEVERAGE RULE

Individual workers and property assets can grow by neg yield, meaning investment from outside. But growth at the scale of the closed economy is limited to productivity gain and restraint of final yield. Final yield is maint plus waste. Since less maint means less pay and sales and output, the only avenue of growth through yield reduction at closure is waste reduction. But waste reduction alone does not bring growth. There must also be “waste conversion” in modifying plant and retraining workers to make products which are less wasteful or distribute products less wastefully. Without conversion, less waste is simply less output. And when yield and output drop equally, the total return rule shows that growth is zero.

Strictly speaking, an equal drop in both output and yield since the stasis origin would mean positive thrift growth and negative free growth, each by the amount of the drop. Belt-tightening in this case has idled plant and workers, lowering productivity. Thus thrift growth without waste conversion is mathematically possible. But it is unlikely, since it brings no growth to compensate for the loss in taste satisfaction. Anyhow let us define “thrift effect” to mean thrift growth rate less the drag it imposes on free growth rate by dislocation of the old productive capacity. That is quantified as

\[ \text{thrift effect} = \text{waste rate reduction} \times \text{conversion efficiency}. \]

We can show these dynamics with the help of new terms and notation. Let \( \phi_{\text{final}} \), \( \phi_{\text{maint}} \), and \( \phi_{\text{waste}} \) mean final yield rate, maint rate, and waste rate respectively. Then \( \phi_{\text{final}} = \phi_{\text{maint}} + \phi_{\text{waste}} \). Thrift effect can be notated \( g_{\text{eff}} \). Define

\[ C_{\text{waste}} = \text{waste index} = \frac{\phi_{\text{waste}}}{\phi_{\text{final}}} = \frac{\text{waste rate}}{\text{final yield rate}}, \]

\[ C_{\text{waste},0} = \text{stasis waste index} = \frac{\phi_{\text{waste},0}}{\phi_{\text{final},0}} = \frac{\text{stasis waste rate}}{\text{stasis final yield rate}}, \]

\[ C_{\text{reduc}} = \% \text{ reduction} = \frac{\phi_{\text{waste},0} - \phi_{\text{waste}}}{\phi_{\text{waste},0}} = \frac{\text{waste rate reduction}}{\text{stasis waste rate}}, \]

and

\[ C_{\text{conv}} = \% \text{ conversion} = \frac{\frac{g_{\text{eff}}}{\phi_{\text{waste},0}}}{\phi_{\text{waste}} - \phi_{\text{waste}}} = \frac{\text{thrift effect}}{\text{waste rate reduction}}. \]

This allows definition of thrift effect by
Since rate of return equals yield rate in stasis, and since yield is final yield in economic closure, we may substitute stasis rate \( r_0 \) for stasis final yield rate \( \phi_{\text{final},0} \). This gives the “thrift leverage rule”

\[
g_{\Theta^{\text{eff}}} = C_{\text{waste},0} C_{\text{reduc}} C_{\text{conv}} \phi_{\text{final},0}.
\]

By the inferences that waste rate reduction is the whole of yield rate reduction at closure, meaning the whole of thrift growth rate, and that rate of return equals final yield rate at closure. This allows

\[
g_{\Theta} = C_{\text{waste},0} C_{\text{reduc}} r_0 = \frac{g_{\Theta^{\text{eff}}}}{C_{\text{conv}}},
\]

\[
\text{thrift growth rate} = \frac{\text{stasis waste rate} \times \% \text{ reduction} \times \% \text{ conversion} \times \text{stasis rate}}{\% \text{ conversion}}.
\]

The advantage of the thrift leverage rule lies in its convenience for modeling. We see that the maximum possible value for each of the three indices is unity, with some room for debate as to the third. Then let us consider their likely ranges.

The stasis waste index

We see

\[
\text{stasis waste index} = \frac{\text{stasis waste rate}}{\text{stasis final yield rate}} = \frac{\text{stasis waste rate}}{\text{stasis rate}}
\]

by the equality of output and yield in stasis. The two ratios are instructive in different ways. Since final yield is maint plus waste, and since any of these rates can be multiplied by stasis capital \( J_0 \) to give flows instead, the first ratio can be written as either

\[
\frac{\text{stasis waste rate}}{\text{stasis maint rate} + \text{stasis waste rate}} \quad \text{or} \quad \frac{\text{stasis waste}}{\text{stasis maint} + \text{stasis waste}}.
\]
Several lines of thought suggest that this ratio is small. The strongest argument may come from measurements. Cap-weighted real rate of return to corporations has averaged six or seven percent per year over the past century, and real growth rate has averaged something like three percent. Then if return in the corporate subsector is typical of return in all sectors, the stasis rate looks to be something in the range of three to four percent per year. Meanwhile the stasis rate converges to the sum of the senescence and waste rates, and the senescence rate alone was modeled at several percent per year. Thus there seems to be little room for waste in the figures, even assuming that corporate return is typical of all sectors. If overall return is actually less, as many teach, then there is still less room for waste.

A parallel argument comes from the equilibrium pay idea of David Ricardo, who saw workers as commodities whose price (pay) holds supply (the workforce) equal to demand (the job market). It would follow that pay does not enable waste, so that the only source of waste is profit from inherited capital. I will treat this idea more fully in Chapter Six. Meanwhile it seems true that scope for waste is limited in a world of competition.

The next two indices should be considered as a pair, as well as individually, since they will tend to vary inversely. The more ambitious the percent reduction of waste, the lower the likely gain in growth per dollar of waste reduction. Conversion efficiency is limited by niche resistance as growth goods (consumption and investment goods) are inserted into markets already at equilibrium, by market impact as such goods are bid up and waste goods down, and by reengineering and retraining costs to expand production and distribution of growth goods at the expense of waste goods. Each of these costs will expect steeply declining returns to scale.

The niche resistance idea is that the environment enables only so much final yield at a given technological level. Some economists teach that capital accumulation might rise toward infinity under a static technology, with average rate of return converging toward zero as successively leaner investment opportunities are exploited (the golden rule of accumulations). This doctrine would illustrate declining returns to waste reduction very nicely, if valid, but it misses the fact that the stasis rate cannot fall below the senescence rate. TRE’s analysis pictures the curve of declining returns as even steeper, since it does not allow capital/output ratio to vary toward infinity.

No man can dig with two shovels. Shovel-piling is misallocation of resources, and is likelier to lower the wealth of the economy than to raise it. It is not capital growth, in TRE’s view. It is not even waste, since shovels do not satisfy final tastes. It is simply leakage, or a failure of efficiency. The same would be true more or less of digger-piling, or training of diggers beyond a need for them. Since the cost of training will not be recovered in pay, it is not consumption. Since it does not satisfy tastes, it is leakage.

Only innovation can widen niches. If a way is found to double the production of potatoes per acre, or the food value per potato, perhaps the carrying capacity of the niche will rise. This innovation is productivity gain or free growth.

The mechanical and retraining costs of waste conversion should also worsen with scale, since it is possible to effect easiest conversions first. Some conversion needs only redistribution.
The fat gourmand may be able to convert waste into growth by pushing his second helping of pot roast across the table to his famished son. But he cannot convert his cognac and cigar so easily. If these are inherently waste goods, regardless who uses them, conversion will mean modification or closing of plant plus retraining of workers. This will prove easier in some cases than others. If a vodka distillery is converted into an orange juice plant, or a pogo-stick factory into a shovel factory, some equipment and inventory and worker skills may carry over from the old use to the new. But relative demand for orange juice and shovels may not be proportionate to the relative supply of vodka distilleries and pogo-stick plants. What are the costs when the latter must be adapted to make orange juice, and the former to make pogo sticks?

The cost of waste reduction in itself is political. It is the cost of suasion and sanction imposed by moral and temporal authority. This cost too should show diminishing returns to scale insofar as waste reduction is involuntary. Individuals who happily give up waste will need no policing to do so, but the rest will make work for the constabulary and the courts. The farther they are constrained from what they would have done voluntarily, the more work for them they will make.

I do not argue that social costs are least, and economies healthiest, where there are no sumptuary laws. Most people seem to expect and prefer such laws up to a degree. The social contract balancing freedom and order allows some indulgence and some boundaries. This text cannot advise where to poise the balance. States hold together when laws gauge it wisely, and policies are costlier to enforce as they move farther from it in either direction.

In view of these political costs, Table 4.1 below will model percent waste reduction at no more than 50%. But the most natural expectation is that it should be zero or negative. The waste index should normally be lower, and not higher, in stasis. Recessions are not binges of waste consumption, but rather phases of tightened belts as owners struggle to recover.

Table 4.1 shows values for thrift effect under a range of assumptions.

<table>
<thead>
<tr>
<th>stasis waste index</th>
<th>% reduction</th>
<th>% conversion</th>
<th>if stasis rate is 3%/yr.</th>
<th>if stasis rate is 4%/yr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.75</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.375</td>
<td>0.14</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.5</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
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<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
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<td>0.5</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>0.375</td>
<td>0.06</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4.1: Thrift Effect at Closure under Various Assumptions
These results suggest that little growth can be expected from waste reduction. The greatest amount of thrift effect projected is half of one percent per year, which is about one sixth of the growth which the industrial world has experienced over the past century or so. And the estimated data supporting that result were unrealistically high. Nations and individuals compete, and wasters fall behind. It would be unreal to suppose that waste is half of final yield, meaning that it is equal to maint. And even Stalin might have been daunted by a challenge to reduce waste by half while realizing a 50% conversion efficiency. Data and results in the lower half of the table are more plausible, and show attainable thrift growth limits to approximate something in the range of one fifteenth to one hundredth of the growth we commonly experience. In the likely event that yield rate is lower in stasis, rather than higher, thrift effect is actually negative.

We sought to evaluate the scope for economic growth through thrift at the scale of closure. Strictly speaking, growth through thrift is not the same as thrift growth as defined. The point is that waste reduction per se contributes no growth unless the waste goods forsaken are replaced with “growth goods.” “Thrift effect” is growth rate due to thrift after this “waste conversion.” The “thrift leverage rule” showed thrift effect to equal the product of the stasis waste index, the percent reduction, the percent conversion, and the stasis rate. I argued that the stasis waste index, or ratio of waste rate to final yield rate (or stasis rate) at the origin in stasis, must be small since the senescence rate alone looks to account for the whole 3% or 4% yearly that was modeled for the stasis rate. I argued that the percent reduction should be zero or negative, since the waste index ought to be lower in stasis than in growth, and that compulsory belt-tightening should bring political costs. Finally I argued that percent waste conversion should face steeply diminishing returns to scale because of niche resistance, market impact and mechanical and retraining costs. Table 4.1 projected thrift effect at half of one percent per year under unrealistically favorable assumptions, and at much less under more likely ones.
V. A GROWTH MODEL

So far, TRE has invoked no assumptions of fact beyond those needed for markets to form and endure. The time has come to risk some new ones. Chapter Four argued that thrift effect is unlikely to account for significant growth at the scale of closure under any plausible estimates, and in fact is rather more likely to be negative than positive. Then let us model it at zero. Since thrift growth at the scale of closure is thrift effect divided by percent conversion, it is implicitly modeled at zero as well.

Each of the factors may show positive or negative thrift growth through transfer yield, even at the scale of closure. Thus one factor might thrive-grow at the expense of the other, even while combined thrift growth of the two factors holds at zero. Self-value and property tend to find an optimum balance appropriate to the current state of technology, and the balance might change as technology does. The change, although essentially free growth, might involve transfer yield as prop yield is insumed or work yield is invested in property. Some textbooks picture growth as evolution from an origin in a propertyless Garden of Eden to progressively higher ratios of property to self-value. I personally see no reason to think that the ratio has changed, or that there has been more inter-factor transfer yield in one direction or the other. Therefore I will suppose for simplicity and neutrality that thrift growth of each factor overall is zero at the scale of closure, noting however that this assumption is not as firm as that of zero thrift growth for their sum.

These assumptions collectively are a model of “thrift offset,” meaning a state where interpersonal and inter-factor transfers offset to zero at the scale of closure. The equations defining thrift offset are

\[
\begin{align*}
thrift \text{ effect} & = thrift \text{ growth} = 0, \\
\text{insumption} & = \text{yield maint}, \quad \text{and} \\
\text{thrift investment} & = \text{disinvestment}
\end{align*}
\]

at closure.

Many an economist has hypothesized that secular growth is a level tide which lifts all boats. One would think at first that this could not be true of ostensibly little-evolving things like buildings and furniture and food. But appearances can mislead. A schoolroom which has seen little investment in fifty years will still become more productive as better doctrines are taught there. This logic extends to the desks and to the teachers’ clothes and homes. A new truck is more productive if it hauls more profitable cargo, even if it is of the same design as the truck it replaces. Likewise shovels in a hardware store may jump in price if gold is discovered nearby. The shovels can now produce gold instead of potatoes, for one thing, and even the potatoes will
produce gold by helping to feed the hungry argonauts. Lawyers who have acquired no new skills become more productive because their old skills can produce richer verdicts for their clients. All these effects can be reversed just as abruptly if the gold strike proves to have been salted. It is in that sense that discovery and innovation raise values of nearby property and self-value which are inherently unchanged.

This evenly-spreading kind of growth is wholly TRE’s “free growth” of innovation and discovery. Thrift growth is not uniform from one asset or worker to the next, but varies widely as some receive investment and others return it. Non-adults, and buildings under construction, grow much faster than the economy because they accrue neg yield and self-invest most output. Adults and buildings in service tend to grow slower than the economy because they must yield up their stored values over their finite lifespans. Thus if free growth or productivity is like the tide that lifts or lowers all boats more or less the same, thrift growth or yield management is like the crisscross of individual incentives that sends some boats toward some ports and others to different ones.

The model will assume that free growth is uniform in certain key respects. For one, we will suppose that the two factors are equal in free growth rate at the scale of closure. That assumption is notated

\[
\frac{\Pi}{H} = \frac{\Pi}{K} = g_{\Pi} = g_{H,\Pi} = g_{K,\Pi}
\]

at closure. Since thrift offset also gives \( g = g_{\Pi} \), the assumptions combine to show

\[
g = g_{\Pi} = g_{H} = g_{H,\Pi} = g_{K} = g_{K,\Pi}
\]

at closure. The uniform free growth rate \( g_{\Pi} \) is free to change over time, but affects both factors alike at each moment.

The model will further suppose that all cohorts free-grow at this same uniform rate. Cohort, a term borrowed from population genetics, means all members of the population born at the same moment. Thus the cohort \( H(x, t) \) would mean aggregate self-value of all individuals aged \( x \) at time \( t \), and hence born at time \( t - x \). Since cohort birth spacing is conventionally the differential interval \( \Delta t \), the number and aggregate value of individuals in the cohort are themselves differentials (i.e. infinitesimals).

Let us notate the free and thrift growth rates of a cohort aged \( x \) by \( g_{\Pi}(x, t) \) and \( g_{\Theta}(x, t) \), so that its overall growth rate \( g_x \) is given by \( g_{\Pi}(x, t) + g_{\Theta}(x, t) \). Then the model’s assumption of uniform free growth for all cohorts can be expressed as

\[
D_x g_{\Pi}(x, t) = 0.
\]
Cohorts add up to a biological population, and populations of humans or other species develop from ovulation to maturity according to a more or less stereotyped timetable. The development can be analyzed in economic terms as thrift growth, as we have seen. Parental gift is neg yield to the donee, and the rest of cohort growth with age is yield deferment through self-investment of work. Under our assumption of free-growth uniformity, differences in growth from one cohort to the next are explained by thrift growth alone. Let us also suppose that the constancy of the pattern of ontogeny from one generation to the next implies that the growth rate of each cohort is a function of age alone, and not of time. In notation, that would be written

\[ \dot{D}_t g_0(x, t) = 0. \]

This condition plus the earlier one \( D_x g_1(x, t) = 0 \) combine for a picture of “age-time independence.” This feature of the growth model means that the difference in self-value between a cohort of a given age and a cohort of the same age in a previous generation is explained by free growth alone, but that the proportionate difference in self-value between two cohorts of different ages at the same time is explained by thrift growth alone. Thus for example if aggregate self-value of the ten-year-old cohort is double that of the four-year-old cohort today, then the ten-year-old cohort held twice as much self-value as the four-year-old one in each past generation, and will continue to hold twice as much in each future generation. Meanwhile if the ten-year-old cohort has twice the self-value held by the ten-year-old cohort of fifteen years ago, the same will be true of the four-year-old cohort and every other cohort.

It is tempting to suppose that productivity growth in self-value is rate of gain in output per worker, and that growth in the work force itself is thrift growth. Let us show otherwise.

Let \( n \) equal number of workers, meaning population size, and let \( q \) equal average value per worker. Then

\[ H = nq \quad \text{and} \quad \frac{\dot{H}}{H} = \frac{\dot{n}}{n} + \frac{\dot{q}}{q}. \]

At a given past time \( t = 0 \), growth in self-value was zero. Suppose that all growth since that moment was free growth, so that

\[ \frac{\dot{H}}{H} \frac{H_0}{H_0} = \frac{\dot{H}}{H} = \frac{w}{H} - \frac{w_0}{H_0}. \]

Also suppose that output per worker is unchanged since time 0. Then \( q = q_0, \dot{q} = 0 \) and
\[
\frac{\dot{H}}{H} = \frac{\dot{n}}{n} = \frac{w}{H} - \frac{w_0}{H_0},
\]

showing that free growth rate since time 0 has equaled population growth rate under those assumptions.

The meaning is that free growth does not automatically mean more output per worker. Rather it means more output per unit of self-value. The presence of population growth is not evidence that some growth is thrift growth.

the adult second output rule

The second output rule finds that thrift output is net thrift investment plus insumption plus gift plus waste. The rule holds at the scale of any owner or set of owners. For the set of all adults, it is the “adult second output rule”

\[
\text{adult thrift output} = \text{adult net thrift investment} + \text{adult insumption} + \text{parental gift} + \text{adult waste},
\]

since parental gift is gift from adults to non-adults regardless of relatedness. Now suppose that adults own virtually all property, so that adult net thrift investment is virtually all of net thrift investment. Next suppose thrift offset, so that net thrift investment overall is zero. Then

\[
\text{adult thrift output} \sim \text{adult insumption} + \text{parental gift} + \text{adult waste}.
\]

Under thrift offset, then, the whole of adult output not wasted is adult insumption plus parental gift. This inference shows the centrality and magnitude of these two flows.

the growth dichotomy and tradition

Economists have long understood the importance of innovation to growth. Certainly mainstream economics includes concepts akin to TRE’s free growth. The innovation of TRE in growth theory, I think, lies in its clarification of outflows. Tradition tends to interpret saving as less consumption, and to treat consumption as the drain of capital. TRE agrees that the drain of property becomes consumption, but identifies the drain of total capital as total and final yield at the scales of the individual and the closed economy respectively. Thus TRE’s thrift index is probably more novel than its productivity index.

My personal assessment, again, is that the former is likely to approach zero and the latter unity over broad scales and periods. I have not claimed that this is an inference to be drawn automatically from the insights of TRE. I do feel that TRE’s clarification of terms allows such views to be stated more clearly and better tested.
Doubtless Puritans, or the Practical Pig, would disdain the idea of growth without belt-tightening. If this writer’s intuition is right, this serendipitous and morally neutral growth is more or less the only kind at the scale of closure.

The relative importance of the two kinds of growth can be disputed. But the reality of free growth cannot. Data leave no doubt that rate of return tends to be higher, at the scale of closure, when growth rate is higher. This alone makes it clear that the prod index is greater than zero.

Any Popperians who have read this far should have cut their losses long since. But they will be relieved to have found falsifiable hypotheses at last. The thrift offset principle claims only that significant thrift growth at closure is impractical. It is not a logical impossibility. Measure the prod index and prove me wrong. My claim that each factor should tend to show zero thrift growth at closure was riskier, since growth of one factor at the expense of the other requires no waste conversion. I rejected this factor shift scenario as intuitively unreal, not as impossible or even as technically difficult.

The growth model offered in this chapter first proposes “thrift offset,” meaning a condition where thrift effect and thrift growth are zero at the scale of closure. The reasoning behind this assumption was the “thrift leverage rule.” Table 4.1 showed that thrift growth is unlikely to account for more than one sixth of typical overall growth under realistic assumptions, and probably much less. We saw in fact that thrift effect is negative if the waste index is lower in stasis than in growth, which it probably ought to be.

The model further supposes zero thrift growth of the two factors separately at closure, although with less confidence. It also assumes uniformity of free growth in the sense that the two factors each free-grow at the same rate at closure, so that thrift offset and uniformity together require the overall ratio of self-value to property to hold unchanged over time. The uniform free growth idea was extended to individual cohorts in that free growth rate, but not thrift growth rate, was expected to affect each cohort the same. The assumption of “age-time independence” completed the model by supposing that thrift growth rate is a function of cohort age alone, and not of time.

This growth model is not proposed as exact, but as a useful simplification and approximation. It is not the only model that might adapt to the basic tautologies of TRE. It is not so much the “TRE model” of growth as my personal idea of what makes sense.
VI. EQUILIBRIUM PAY

David Ricardo, in 1817, reasoned that labor is a commodity whose price (pay) is determined by supply and demand. He inferred that the pay level at which supply and demand intersect is the level just sufficient to hold the work force intact over the generations. Workers would be expected to bid pay down from this equilibrium level when supply of workers exceed demand (jobs), thus reducing the work force until equilibrium was restored. The minimum level to which bids might fall would be the work expense level, which would enable work and life but would leave nothing else for parental gift and generational replenishment. When jobs exceed worker supply, however, employers would be expected to bid pay up to meet the employers’ alternative costs of carrying idle machinery. This pay rise would restore the worker supply to equilibrium with demand.

What rings true is that workers, like commodities, are subject to the laws of supply and demand. These laws say that market price is that at which supply and demand intersect. Each of the three adapts to the others in finding this equilibrium. And the price of worker services (pay) at which supply stably equals demand (the job market) is the price just sufficient to maintain the work force.

The work force is not static under equilibrium, although Ricardo may have thought it is, but rather varies to keep pace with the job market (economy). Assume thrift offset for each factor, so that growth of each at closure is wholly free growth. Then insumption equals senescence at closure. The pure Ricardian idea is that pay is just sufficient to enable this match.

We should not conclude that equilibrating forces are weak because the generation span is long. When jobs are in short supply and the alternative is hunger, some will work for work expense. When workers are short and the alternative is idle plant and machinery, some employers will pay up to the cost of that alternative. Thus a small elasticity in worker supply might be enough to restore equilibrium pay quickly, and so nip potential supply/demand imbalance in the bud.

There are social interventions, in our society or Ricardo’s or any other, to soften this arbitrage. Unemployment insurance, social security, minimum wage and fair wage laws, graduated taxes and make-work programs are examples of subsidies imposed on the Ricardian mechanics. But a typical effect of subsidy is to lower prices of the commodity whose production is subsidized. Welfare and leveling, however necessary, provide a non-market source for maintenance of worker supply. The larger this alternate source of income, the less pay is needed to keep the work force in balance with the job market. Only makework programs are likely to bid pay up by creating artificial demand. Minimum wage laws, which have no clear effect but to eliminate some jobs by illegalizing them, raise average pay of those employed while reducing their number.
Overall pay levels therefore probably fall below the level for Ricardian equilibrium, not above it, and probably more so as social intervention rises. Exceptions might occur in times and places where most subsidy takes the form of makework programs.

Let’s try to model the equilibrium pay idea. Most workers have entered the work force with little property, and leave little to heirs at the end, although an exception has been noted with the “inheritors” currently. Thus workers typically begin maturity with no source of cash income but pay. Pay is work expense plus sold work (the sold work rule). The young adult invests his unsold thrift work in adult insumption for the sake of higher future and overall pay. Compensation for his sold work is spent on parental gift as needed, and otherwise on property investment. As he ages, he disinvests his property and adds this property yield to his parental gift. Meanwhile all his self-value, including the part gained by adult insumption, is also disinvested as senescence and recovered in pay to be applied to the same parental gift. His children, or other donees of that gift, self-invest the whole of their thrift work in study and exercise in order to maximize the worker replenishment rate.

This picture of total ultimate focus on population replenishment sounds as Darwinian as Ricardian. It is simply the modeled condition for work force equilibrium under competition for jobs. It shows all thrift work of both young and old as applied to maximize current or later insumption by the young. Although some is diverted to adult insumption and invested in “interim property,” meaning property which workers buy from thrift work and then liquidate during their adult spans, all this is returned with interest to the young in the end.

At any given moment, anyhow, the condition for pure Ricardian equilibrium without subsidy from profit would be

\[
\text{thrift work} + \text{interim profit} = \text{senescence}
\]

at closure, under the assumptions, where “interim profit” is profit on interim property. Any subsidy from thrift profit would be added to the left-hand side. That is,

\[
\text{thrift work} + \text{interim profit} + \text{subsidy} = \text{senescence}
\]

if subsidy from profit is present. Now divide this result by aggregate self-value to get the “Ricardo model”

\[
\frac{\text{thrift work rate} + \frac{\text{interim profit} + \text{subsidy}}{\text{self-value}}}{\text{self-value}} = \text{pure senescence rate}
\]

or

\[
\text{thrift work rate} \leq \text{pure senescence rate}
\]

since the sum of interim profit and worker subsidy is assumed not to be negative.
Now suppose “risk parity,” under which the factors are equal on the whole in risk and rate of return. That would give

thrift work rate = stasis rate

and

stasis rate ≤ pure senescence rate

under the assumptions. Consequently the Ricardo model, under assumptions of risk parity and thrift offset, shows the pure senescence rate to be an upward limit for the stasis rate.

mutual limits for the waste and factor indices

One of the disappointments of TRE has been its lack of an algorithm for measuring the “factor index,” or ratio of self-value to total capital at the scale of closure. The equilibrium pay model helps us say something about this ratio, however, and about waste as well.

The pure senescence rate is \( \delta / H \), where \( \delta \) (delta) is senescence and H is self-value. The senescence rate is \( \delta / J \). The equilibrium pay model gives \( r_{t,0} \leq \delta / H \), and more generally \( r_0 \leq \delta / H \) in risk parity. By the duplication rule, the senescence rate converges to the maint rate \( \phi_{\text{maint}} \). Then the model expects

\[
r_0 \leq \phi_{\text{maint}} \frac{J}{H}, \quad \text{or} \quad \frac{H}{J} \leq \frac{\phi_{\text{maint}}}{r_0}.
\]

The second relation shows that the factor index, in the model, is bound on the upside by the ratio of the maint rate to the stasis rate.

Since we have assumed thrift offset throughout the pay equilibrium model, we will continue to do so in supposing that the final yield rate \( \phi_{\text{final}} \) is unchanged since the origin in stasis. Then \( \phi_{\text{final}} = \phi_{\text{final,0}} \). Now the facts \( \phi_{\text{final}} = \phi_{\text{maint}} + \phi_{\text{waste}} \) and \( \phi_{\text{final,0}} = r_0 \) allow

\[
r_0 = \phi_{\text{maint}} + \phi_{\text{waste}}.
\]

This and the fact \( J = H + K \), where K is property, allow

\[
\frac{H}{J} = 1 - \frac{K}{J} \quad \text{and} \quad \frac{\phi_{\text{maint}}}{r_0} = 1 - \frac{\phi_{\text{waste}}}{\phi_{\text{final}}},
\]

and consequently
\[ 1 - \frac{K}{J} \leq 1 - \frac{\phi_{\text{waste}}}{\phi_{\text{final}}} \]

Subtract unity from both sides for

\[ \frac{-K}{J} \leq \frac{-\phi_{\text{waste}}}{\phi_{\text{final}}}, \quad \text{or} \quad \frac{\phi_{\text{waste}}}{\phi_{\text{final}}} \leq \frac{K}{J} \]

This shows that the “waste index” \( \phi_{\text{waste}} / \phi_{\text{final}} \) is less than the property / capital ratio under the pay equilibrium model.

The work expense and maint indices

The closure rule shows that thrift output at closure is thrift growth plus maint plus waste. Assume thrift offset, so that thrift growth at closure is zero. Then thrift output is maint plus waste at closure. Also thrift output is thrift work plus thrift profit. Then

\[ \text{maint} + \text{waste} = \text{thrift work} + \text{thrift profit} \]

under thrift offset at closure. By the Ricardo model, also,

\[ \text{thrift profit} = \text{interim profit} + \text{subsidy} + \text{waste} . \]

These equations combine for

\[ \text{maint} = \text{thrift work} + \text{interim profit} + \text{subsidy} . \]

Also thrift work is sold work plus unsold thrift work, and work expense is maint plus non-yield expense. Then the “maint equations” are

\[ \text{maint} = \text{sold work} + \text{unsold thrift work} + \text{interim profit} + \text{subsidy}, \]

and

\[ \text{work expense} = \text{sold work} + \text{unsold thrift work} + \text{interim profit} + \text{subsidy} + \text{non-yield expense} . \]

Since all right-hand terms in both equations are assumed positive, it is clear that maint is larger than sold work, and work expense even more so. Meanwhile pay is sold work plus work expense by the sold work rule. Then if the “work expense index” and “maint index” give the respective ratios of work expense and maint to pay, it appears that each index exceeds 50% under thrift offset and pay equilibrium.
the biological imperative

Many economists today have taken up the suggestion of William Hamilton and others that natural selection has programmed all species, including humans, to maximize investment in progeny and collateral descendants. This text does not invoke an assumption of Darwinian fitness, if only because the modified Ricardian model does as well for most practical purposes. Equilibrium pay, by allowing population replenishment with nothing left over for waste or thrift investment, is simply an expression of the laws of supply and demand. The Ricardian model differs from the Darwinian or Hamiltonian, however, in its indifference to relatedness of donors to donees. Although science and common experience make it plain that the young are maintained predominantly by parents and other close kin, in our species and others, TRE has not yet found need to model this general rule of nature.

review

Workers are like commodities in that they are more or less interchangeable within their guilds or trades. This fact makes it convenient to model the interplay of their supply (the work force), demand (the job market) and price (pay). In stasis, pay should be just sufficient to meet demand by making up losses to senescence. Under the growth model, where growth of each factor overall is free growth alone, this remains true. In pure Ricardian pay equilibrium without social subsidy of workers, thrift work plus “interim profit,” meaning thrift profit from property bought from pay, exactly offsets senescence. Divide this equation by self-value to see that thrift work rate is less than the pure senescence rate. This gap widens as social subsidy of workers is introduced.

The equilibrium pay model helps clarify the “work expense index” and “maint index,” meaning the ratios of these respective flows to pay. It is clear that both indices are over 50%. This inference will prove useful in modeling the life cycle in Chapter Seven. We also saw that the ratio of waste to final yield is expected to be less than the ratio of property to total capital.
VII. A LIFE CYCLE MODEL

We saw that life begins with investment of “parental gift,” meaning support by adults whether related or not, and that the young supplement this ingift by self-investment through the unsold work of exercise and learning. All these inputs count as insumption. When the phase of dependence ends, owners become sources rather than recipients of parental gift as their accumulated self-value declines.

Although TRE is meant to describe any species, modern humans are the subject at hand. Then let us try to model self-value as a function of age for Americans today. The Statistical Abstract of the United States 1999, Table No. 129, (SAUS99 (129)), shows that expectation of life at birth, weighting the sexes equally, was 76.1 years in 1996. Economic life is a little different. It begins at first investment, not at birth, and ends at return of self-value to zero rather than at death. Since gestation is investment, and since investment will sometimes begin earlier still, let us say that economic birth begins at age minus one. At the other end, self-value disappears when its output (work) does. This moment of “redependence” arrives when actual or imputed pay, meaning the value of the owner’s services to others or himself, no longer covers his work expense cost. Meanwhile retirees maintain themselves, and help with the grandchildren as well, until near the end. Let us round the final redependence phase to one year, as with the gestation phase, and therefore round economic lifespan in 1996 to 76 years (i.e. from age minus one to age 75).

SAUS99 (259) also shows that in each year in the period 1985 through 1997, school enrollment drops below 50% as age rises from the 18-19 year bracket to the 20-21 year bracket. Meanwhile SAUS98 (645) shows that participation in the Civilian Labor Force in 1997 was 52.3% and 51.0% for males and females in the 16-19 year bracket, rising to 82.5% and 72.7% respectively in the 20-24 year bracket. Then let us suppose that the period of dependence on parental gift ends at age 20, or 21 years after economic birth.

The next critical point in the model is the mean age of reproduction, or R.A. Fisher’s generation length. This is average age difference between parents and offspring, meaning all offspring and not firstborns only. Like Fisher, I prefer to weight the maternal and paternal lengths equally. Determination of this age for Americans today allows us to lump all offspring into a single imaginary offspring cohort for simplicity. SAUS99 (93) shows number of live births by age bracket of mothers. Birthrate peaked in the 25-29 year bracket, for each year shown, and declined symmetrically on both sides of that peak in 1997. Distribution was skewed increasingly toward older brackets in successively earlier years. This indicates that the maternal generation length has shortened somewhat since the earliest year shown (1980), and was something near 27.5 years in 1997. SAUS99 (158) shows meanwhile that men were roughly two years older than women at first marriage, in each year shown (1970 through 1990), and were typically about three years older at remarriage after divorce. Then let us interpret the paternal generation length at 30 years, and round the average for both lengths to 29 years. By the convention just adopted, that would mean that the cohort begins investing in the next generation one year earlier or at age 28. Then we will take 28 years as the age when the cohort must first divert resources to parental gift.
Table 7.1 below summarizes these measurements.

**Table 7.1: Life Cycle Parameters in the United States Today**

<table>
<thead>
<tr>
<th>life cycle period</th>
<th>age at beginning</th>
<th>age at ending</th>
<th>length of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-adult span</td>
<td>−1 (years)</td>
<td>20 (years)</td>
<td>21 (years)</td>
</tr>
<tr>
<td>generation length</td>
<td>−1</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td>adult span</td>
<td>20</td>
<td>75</td>
<td>55</td>
</tr>
<tr>
<td>economic lifespan</td>
<td>−1</td>
<td>75</td>
<td>76</td>
</tr>
</tbody>
</table>

Self-value of the young, again, is accumulation of parental gift supplemented by self-invested work of the young in exercise and learning. The first of these two components is measured up to a point. SAUS99 (737), reproduced as Table 7.2 below, itemizes expense per child by families.

**Table 7.2: Annual Expenditure Per Child by Husband-Wife Families in 1998 (Source: SAUS99 (737))**

[In dollars. Expenditures based on data from the 1990-92 Consumer Expenditure Survey updated to 1998 dollars using the Consumer Price Index. For more on the methodology, see report cited below.]

<table>
<thead>
<tr>
<th>Age of Child</th>
<th>Expenditure Type</th>
<th>Total</th>
<th>Housing</th>
<th>Food</th>
<th>Transportation</th>
<th>Clothing</th>
<th>Health care</th>
<th>Child care and education</th>
<th>Misc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>INCOME: LESS THAN $36,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 2 yrs. old ..........</td>
<td></td>
<td>5,950</td>
<td>2,270</td>
<td>850</td>
<td>720</td>
<td>390</td>
<td>410</td>
<td>720</td>
<td>590</td>
</tr>
<tr>
<td>3 to 5 yrs. old ..............</td>
<td></td>
<td>6,060</td>
<td>2,240</td>
<td>940</td>
<td>690</td>
<td>380</td>
<td>390</td>
<td>820</td>
<td>600</td>
</tr>
<tr>
<td>6 to 8 yrs. old ..............</td>
<td></td>
<td>6,180</td>
<td>2,170</td>
<td>1,210</td>
<td>810</td>
<td>420</td>
<td>450</td>
<td>480</td>
<td>640</td>
</tr>
<tr>
<td>9 to 11 yrs. old ..............</td>
<td></td>
<td>6,210</td>
<td>1,960</td>
<td>1,450</td>
<td>880</td>
<td>470</td>
<td>490</td>
<td>290</td>
<td>670</td>
</tr>
<tr>
<td>12 to 14 yrs. old ............</td>
<td></td>
<td>7,020</td>
<td>2,180</td>
<td>1,520</td>
<td>990</td>
<td>780</td>
<td>500</td>
<td>210</td>
<td>840</td>
</tr>
<tr>
<td>15 to 17 yrs. old ............</td>
<td></td>
<td>6,920</td>
<td>1,760</td>
<td>1,650</td>
<td>1,330</td>
<td>690</td>
<td>530</td>
<td>340</td>
<td>620</td>
</tr>
<tr>
<td>INCOME: $36,000 – $60,600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 2 yrs. old ..........</td>
<td></td>
<td>8,240</td>
<td>3,070</td>
<td>1,010</td>
<td>1,070</td>
<td>450</td>
<td>540</td>
<td>1,190</td>
<td>910</td>
</tr>
<tr>
<td>3 to 5 yrs. old ..............</td>
<td></td>
<td>8,460</td>
<td>3,040</td>
<td>1,170</td>
<td>1,040</td>
<td>440</td>
<td>520</td>
<td>1,320</td>
<td>930</td>
</tr>
<tr>
<td>6 to 8 yrs. old ..............</td>
<td></td>
<td>8,520</td>
<td>2,970</td>
<td>1,490</td>
<td>1,160</td>
<td>490</td>
<td>590</td>
<td>850</td>
<td>970</td>
</tr>
<tr>
<td>9 to 11 yrs. old .............</td>
<td></td>
<td>8,470</td>
<td>2,760</td>
<td>1,750</td>
<td>1,230</td>
<td>540</td>
<td>640</td>
<td>550</td>
<td>1,000</td>
</tr>
<tr>
<td>12 to 14 yrs. old ............</td>
<td></td>
<td>9,200</td>
<td>2,980</td>
<td>1,770</td>
<td>1,330</td>
<td>910</td>
<td>640</td>
<td>400</td>
<td>1,170</td>
</tr>
<tr>
<td>15 to 17 yrs. old ............</td>
<td></td>
<td>9,340</td>
<td>2,560</td>
<td>1,960</td>
<td>1,690</td>
<td>810</td>
<td>680</td>
<td>700</td>
<td>940</td>
</tr>
<tr>
<td>INCOME: MORE THAN $60,600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less than 2 yrs. old ..........</td>
<td></td>
<td>12,260</td>
<td>4,880</td>
<td>1,340</td>
<td>1,490</td>
<td>600</td>
<td>620</td>
<td>1,800</td>
<td>1,530</td>
</tr>
<tr>
<td>3 to 5 yrs. old ..............</td>
<td></td>
<td>12,530</td>
<td>4,850</td>
<td>1,520</td>
<td>1,470</td>
<td>580</td>
<td>600</td>
<td>1,960</td>
<td>1,550</td>
</tr>
<tr>
<td>6 to 8 yrs. old ..............</td>
<td></td>
<td>12,440</td>
<td>4,780</td>
<td>1,830</td>
<td>1,580</td>
<td>640</td>
<td>680</td>
<td>1,350</td>
<td>1,580</td>
</tr>
<tr>
<td>9 to 11 yrs. old .............</td>
<td></td>
<td>12,320</td>
<td>4,570</td>
<td>2,120</td>
<td>1,650</td>
<td>700</td>
<td>730</td>
<td>940</td>
<td>1,610</td>
</tr>
<tr>
<td>12 to 14 yrs. old ............</td>
<td></td>
<td>13,170</td>
<td>4,790</td>
<td>2,230</td>
<td>1,760</td>
<td>740</td>
<td>720</td>
<td>720</td>
<td>1,780</td>
</tr>
<tr>
<td>15 to 17 yrs. old ............</td>
<td></td>
<td>13,510</td>
<td>4,370</td>
<td>2,350</td>
<td>2,130</td>
<td>1,050</td>
<td>780</td>
<td>1,270</td>
<td>1,560</td>
</tr>
</tbody>
</table>

1 Expenses include personal care items, entertainment and reading materials.


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Parental gift in the TRE sense means all investment in the young by adults, including investment by non-parents. One important omission in Table 7.2 is the part of school costs paid by taxpayers rather than parents. Total cost per child for public elementary and secondary day schools in 1998 is reported at $6,548 in SAUS99 (286). It seems to me that this cost should be added. Another important omission, particularly in costs of pre-school children, is the imputed pay of mothers and other family members for child care. Let us estimate this imputed pay very roughly at $10,000 per child per year in the pre-school years, or $20,000 for a two-child family, and half as much thereafter. Table 7.3 below adjusts data from Table 7.2 to reflect these additions.

Table 7.3: Adjusted Expenditure per Child in Middle-Income Families through Age 17 in 1998

<table>
<thead>
<tr>
<th>age bracket</th>
<th>per SAUS99 (737)</th>
<th>school cost</th>
<th>parental care</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 2</td>
<td>$ 8,240</td>
<td>$ 0</td>
<td>$ 10,000</td>
<td>$ 18,240</td>
</tr>
<tr>
<td>3 – 5</td>
<td>8,460</td>
<td>0</td>
<td>10,000</td>
<td>18,460</td>
</tr>
<tr>
<td>6 – 8</td>
<td>8,520</td>
<td>6,548</td>
<td>5,000</td>
<td>20,068</td>
</tr>
<tr>
<td>9 – 11</td>
<td>8,470</td>
<td>6,548</td>
<td>5,000</td>
<td>20,018</td>
</tr>
<tr>
<td>12 – 14</td>
<td>9,200</td>
<td>6,548</td>
<td>5,000</td>
<td>20,748</td>
</tr>
<tr>
<td>15 – 17</td>
<td>9,340</td>
<td>6,548</td>
<td>5,000</td>
<td>20,888</td>
</tr>
<tr>
<td>TOTAL FOR 18 YEARS:</td>
<td></td>
<td></td>
<td></td>
<td>$355,266</td>
</tr>
<tr>
<td>AVERAGE PER YEAR:</td>
<td></td>
<td></td>
<td></td>
<td>$ 19,737</td>
</tr>
</tbody>
</table>

Since TRE pictures parental gift as beginning at age minus one rather than at birth, cost of this prenatal investment should be added. Suppose arbitrarily that it is $5,000. On the other hand, we would prefer to know costs in 1997 rather than 1998 for consistency with other data. We can bear these reservations in mind as we continue into the college years.

Estimates of parental gift received in ages 16 through 19 are complicated by the fact, just seen in determining the age of maturity, that about half of male and female students at those ages are employed. Anyhow SAUS99 (319) shows cost per student in higher education at $16,131 in 1997, or about $10,000 more than in primary and secondary schools after allowance for the difference between 1997 costs and 1998 ones. Suppose that other imputed plus out-of-pocket costs continue to total a little less than $15,000 per year, whether the student boards at college or continues to live at home, and that the student earns about $5,000 per year to defray these costs. Then total cost per year in ages 18 and 19 come to about $31,000 per year, of which parents and other adults must pay about $26,000. We can add this final $52,000 in parental gift to the $355,266 shown in Table 7.3, plus another $5,000 for the prenatal year, to get $412,266. Finally we can round downward to $400,000 to offset the use of 1998 data in estimating pre-college outlays. This comes to about $19,000 per year, on average, over the 21 years of parental dependency from age minus one through the twentieth birthday.

Then aggregate value of outlays by adults on the young until maturity can be modeled crudely at some $400,000 in 1997. This equates to parental gift insofar as the outlays are
actually converted into self-value. Tradition seems to assume that they are in calculating human capital (self-value) at their sum. And it seems to me that this tradition is probably sound. Outcomes should correlate with intentions under efficiency, and it seems that the intentions of parents and taxpayers focus on developing skills. Children are sent to school, not to Pleasure Island, and are disciplined to study and learn. Therefore it seems likely that aggregate parental gift, meaning outlays actually insurned, should mean the whole outlays more or less if parents assess probable outcomes efficiently.

Self-value at maturity is this aggregate parental gift plus the value of self-invested work of the young accumulated from first investment at age minus one through that age. Although this last flow eludes measurement, it can be described under simplifying assumptions. Recall first that differences among cohorts at a single snapshot in time, under the growth model, are explained by thrift growth alone. Then self-invested work at any age is thrift work rate times the fraction self-invested. If \( r_{x} \) is thrift work rate of the cohort aged \( x \), and \( f_{x} \) is the fraction of thrift work insurned at that age, self-invested work of that cohort would equal \( f_{x} r_{0,x} H(x) \), where \( H(x) \) is cohort self-value. Also

\[
H_{x} = \int_{x}^{\infty} \gamma \, e^{r_{x} r_{0,x} (x-z)} \, dz,
\]

where \( \gamma \) (gamma) is parental gift, assuming that there has been no loss of self-value through senescence through age \( x \). \( \theta \) is theta. If \( M \) is the age of maturity, and there has occurred no senescence in younger cohorts, self-value of the maturity cohort becomes

\[
H_{M} = \int_{0}^{M} \gamma \, e^{r_{x} r_{0,x} (M-x)} \, dx.
\]

Tables of Integrals show that if parental gift \( \gamma_{x} \) is age-independent through maturity, which our crude study suggested to be not far from the case, we can find

\[
H_{M} = \gamma \int_{0}^{N} e^{r_{x} r_{0,x} (M-x)} \, dx = \frac{\gamma}{\bar{f}_{x} r_{0,x}}\left(\frac{1}{e} - 1\right)
\]

where \( N = M + 1 \) (the non-adult span), and where \( \bar{f}_{x} r_{0,x} \) is average value of \( f_{x} r_{0,x} \) over that span. Again, we implicitly assume zero work-dep before age \( M \).

What might the value of \( f_{x} \) be? Note first that sold work is self-insurned if the discretionary pay received in return is spent on insurnption rather than on investment or gift or waste. If all pay to non-adults is compensation for sold work, and none is maint, evidence that many older non-adults have joined the work force does not imply that \( f_{x} \) is less than unity. But it might be less on other grounds. Although we may suppose that parental outlays are insurned as parents intend, since parents decide the nature of those outlays, the intentions of the young are not so easily interpreted. A newborn does as programmed by natural selection. Children in general do not “intend” as coherently as adults, but follow developmental patterns presumably
inherited from a simpler past. Although parents influence $f_x$ by guiding and disciplining behavior, it seems wisest to allow for the chance that less than all thrift work of non-adults is insumed. Implicitly $f_x$ could hold a value less than unity.

Thrift work rate $r_{0,x}$ cannot be measured, but can be estimated from rates of return in the corporate subsector. I said earlier that cap-weighted returns on debt and equity claims on that subsector seem to have averaged some 6.5% yearly since 1926. Although tradition pictures return in other sectors to be lower, evidence of returns to debt claims on those sectors suggests the opposite. Meanwhile growth rate since 1926 seems to have run something near 3% yearly. If all growth at large scales is free growth, as expected under thrift offset, then the implicit thrift return rate (rate of return less free growth rate) would resemble 3.5% yearly in the corporate sector. Table 7.4 below models value and thrift work of the maturity cohort $H_M$ where parental gift $\gamma_x$ is assumed age-independent at $19,000 per year, where non-adult span $M$ is 21 years, and where $\bar{r}_x r_{0,x}$ varies over a range from 2% to 4.5% yearly.

<table>
<thead>
<tr>
<th>$r_{0,x}$ (per year):</th>
<th>2%</th>
<th>2.5%</th>
<th>3%</th>
<th>3.5%</th>
<th>4%</th>
<th>4.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maturity Cohort Value:</td>
<td>$496,000$</td>
<td>$525,000$</td>
<td>$556,000$</td>
<td>$589,000$</td>
<td>$625,000$</td>
<td>$664,000$</td>
</tr>
<tr>
<td>Maturity Cohort Thrift Work$^1$:</td>
<td>9,900</td>
<td>13,000</td>
<td>16,700</td>
<td>20,600</td>
<td>25,000</td>
<td>29,900</td>
</tr>
</tbody>
</table>

$^1$ Maturity Cohort Value times $\bar{r}_x r_{0,x}$.

Again, all these values assume zero senescence before maturity. Is that realistic? Here too, economists seem to think so when they model self-capital of the young at the integral of outlays on them. Thus the young are viewed something like buildings, which normally show no depreciation or other expense until complete and ready for occupation. Although this approach strikes me as reasonably realistic, it is safest to allow for the possibility that actual value of the maturity cohort is somewhat lower than the accumulated insumption shown in Table 7.4.

Further caveats are the obvious ones. My estimates of imputed pay for parental care were arbitrary, as was my guess of the cost of the prenatal year. Finally the capitalization rate $\bar{r}_x r_{0,x}$ conceivably might not lie within the range shown.

SAUS99 (758), mentioned earlier, shows pay of full-time male and female workers by educational attainment in 1997. I prefer to weight the sexes equally in finding cohort pay, regardless of relative participation in the work force, since those not working for literal pay are assumed to be working at home or elsewhere for imputed pay. Meanwhile SAUS98 (645),
meaning Table No. 645 in *Statistical Abstract of the United States 1998*, shows this participation in the Civilian Labor Force in 1997 for the same age brackets. Table 7.5 below summarizes information from both sources.

<table>
<thead>
<tr>
<th>Age bracket</th>
<th>males</th>
<th>females</th>
<th>midpoint</th>
<th>male % of participation(^2)</th>
<th>female % of participation(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 – 24 years</td>
<td>$20,294</td>
<td>$17,510</td>
<td>$18,902</td>
<td>82.5(^3)</td>
<td>72.7(^3)</td>
</tr>
<tr>
<td>25 – 34 years</td>
<td>$34,807</td>
<td>$27,805</td>
<td>$31,306</td>
<td>93.0</td>
<td>76.0</td>
</tr>
<tr>
<td>35 – 44 years</td>
<td>$45,780</td>
<td>$31,273</td>
<td>$38,527</td>
<td>92.6</td>
<td>77.7</td>
</tr>
<tr>
<td>45 – 54 years</td>
<td>$52,429</td>
<td>$32,524</td>
<td>$42,477</td>
<td>89.5</td>
<td>76.0</td>
</tr>
<tr>
<td>55 – 64 years</td>
<td>$55,702</td>
<td>$28,876</td>
<td>$42,289</td>
<td>67.6</td>
<td>50.9</td>
</tr>
<tr>
<td>65+ years</td>
<td>$51,148</td>
<td>$27,567</td>
<td>$39,358</td>
<td>17.1</td>
<td>8.6</td>
</tr>
<tr>
<td>All workers</td>
<td>$43,709</td>
<td>$29,261</td>
<td>$36,485</td>
<td>75.0</td>
<td>59.8</td>
</tr>
</tbody>
</table>

\(^1\) of full-time workers only. Full-time workers made up 68.7% (93,578/136,297) of the Civilian Labor Force in 1997.

\(^2\) in the Civilian Labor Force.

\(^3\) for the 20 – 24 year bracket only.

Table 7.5 reports participation rates, even though I have not used them in weighting pay, because they help us gauge the representativeness of the pay levels shown. The facts that all participation rates shown are 76% or more from ages 20 through 54, and that the full-time workers for whom the pay data are shown made up 68.7% of the Civilian Labor Force, add to our confidence that averages shown are typical. We also treat data for the 55 – 64 year bracket with some confidence, since participation levels remain over 50%. Confidence is less as to the bracket over age 65, however, where the midpoint participation rate falls below 13%. There is some reason to suspect that the pay levels shown for this bracket are exceptional, and that typical levels should be lower. The point is that those actually employed were bid for by the job market, and are probably worth most to those who actually employ them. Retirees, by contrast, perform self-maintenance services for which they are less trained and market-selected, and may be worth less imputed pay to themselves in these roles than they were worth when at their specialties on the payroll. Thus I suspect that average literal and imputed pay to cohorts over age 65 may be substantially less than shown in Table 7.5.

We saw in Chapter Two that insumption can be understood as consumption which does not contribute to current pay, but which raises expected future pure pay when other things are equal. A good rule of thumb, which takes the *ceteris paribus* condition for granted, holds that any rise in expected pay per hour is evidence of rising skills, and hence of current insumption in the form of learning. The implicit assumption is that more pay means more pure pay, and that the rise *per se* does not bring a fall elsewhere. Were that not so, the economic value of the learning might be illusory.

*the adult span*
Pay/age profiles suggest that pay rises steadily to a peak in the late fifties (Table 7.5), and is never below entry-level pay at any age. Meanwhile the growth model takes all inter-cohort differences as effects of thrift growth alone. On the neutral assumption that the curve for pure pay over age is similar, we may accept these profiles as proof that adult insumption continues into the fifties at least. Rising pure pay means insumption, other things equal, even if most increase in skill is normal on-the-job experience rather than deliberate self-improvement through a course of study. Any learning, however subconscious and fortuitous, is insumption if it raises pure pay expectation in future. Is there a “cutoff age” when adult insumption dwindles to negligibility? We cannot be sure from pay data. A decline in pay does not imply lack of new investment, just as a decline in rents does not imply lack of new investment in a building. New skills learned may not be enough to offset the fading and obsolescence of old ones. But it does seem logical to expect less investment as the demolition date comes closer. Less time left means faster recovery needed in order to justify investment, and higher-yield investments may be progressively harder to find. Studies also suggest that learning comes easiest to the young, and takes more effort as we age. Therefore aging workers should grow to prefer jobs which maximize sold work, meaning pay net of work expense, in the expectation that this net can be invested more rewardingly in property, or given to the next generation, rather than to trust diminishing chances that the same work left unsold would have returned more pay by the end. It seems reasonable then to model progressively less adult insumption with age, beginning at maturity, and a cutoff point at some age in the fifties or later.

By the same reasoning, we would expect entry-level workers to be preferred for jobs, and to prefer jobs, which most reward talent for learning and least depend on applied skills at the start. We expect entry-level workers to find a niche as virtual apprentices, working for maintenance pay while learning skills that will bring pay higher and reward the patience of both worker and employer. And that is what the data in Table 7.5 seem to show happening.

As the entry-level worker’s pay rises, he will probably find no better use for the surplus over work expense than to invest it in his self-value insofar as he can. No other investment is likely to prove as ripe to yield him returns at this early stage in his career. He will probably never again be able to learn as much as fast, or to translate the learning into as steep a rise in yield (Table 7.5). But it is becoming less practical for him to find ways to convert cash into self-value. He might spend for textbooks and tuition, and convert this cost into self-value by study and course attendance at night, but he must work by day. It seems to me that most adult insumption, especially after age 30 or so, is the unsold thrift work of absorbing practical experience. Therefore I will not accord much importance to “plowback insumption” in my model. I will assume that the young adult’s “discretionary pay,” or surplus of pay after work expense, is more likely to be invested in property or social security or pension plans than to be plowed back into self-value. This first adult phase ends at the cohort’s “mean age of reproduction,” estimated earlier at age 28, when pay and property yield must be diverted to parental gift.

Pay is work expense plus sold work, thrift work is sold work plus unsold thrift work, and thrift output alone explains differences among cohorts in 1997 or any year under the growth simplifications. If the maturity cohort enters the work force earning work expense alone, then its
sold work is zero and its entire thrift work is unsold thrift work in the form of adult insumption through learning. This arithmetic yields a clue as to the scope of adult insumption. It should tend to begin at a level equal or nearly equal to the entire thrift work of the maturity cohort, or equivalently the self-value of that cohort times its thrift return rate, and to decline gradually over older cohorts to a zero point reached at some age past the mid-fifties.

**Modeling Maint and Work Expense**

We saw that the maint index for all cohorts is greater than 50% under thrift offset and pay equilibrium. How much greater? The maint equations show

\[
\text{maint} = \text{sold work} + \text{unsold thrift work} + \text{interim profit} + \text{subsidy}.
\]

Since the maint equations held only at closure, unsold thrift work here means that of all cohorts and not of pay-earning cohorts alone. Thrift work of non-adults is largely unsold. We have modeled the percent of thrift work sold at zero upon entry into the work force, then rising steadily to 100% at a cutoff age somewhere in the fifties or later. It remains at 100% until redependence. Thrift work itself, or the basis to which those percentages are applied, rises from zero at birth and then declines again to zero at redependence. The shape of the self-value / age curve, and therefore of the thrift work / age curve, is unclear between these end points. If thrift work is mostly unsold at the start, wholly sold at the end and a mix of the two in between, anyhow, one might guess neutrally that sold and unsold thrift work are not far from equal overall.

Meanwhile interim profit plus subsidy should account for most of thrift profit, since theory and evidence suggest that the waste rate is small. Since the factor index is unknown, however, TRE can say little more about their size in proportion to sold work. Suppose however that tradition is somehow right, for the wrong reasons, in estimating profit to be about half the size of work. The same ratio would hold for thrift profit and thrift work under the growth model. Subsidy and interim profit should account for most of thrift profit on the assumption that waste is slight. Maint, under these estimates, would be a little less than three times the size of sold work. Pay is the sum of the two plus non-yield expense, and the maint and work expense indices are the ratios of maint and work expense to pay for all cohorts collectively. Let us suppose that non-yield expense is minor but significant, so that work expense is moderately larger than maint. The work expense index would be projected at something near 75%, under assumptions, and the maint index would be a little less.

Clearly these indices vary with cohort age, since the work expense index for example is modeled at 100% at maturity and again at redependence. A doctrine credited to Milton Friedman, however, if translated into TRE terms, suggests that what was called sustenance in Chapter Two should be roughly age-independent over the adult span. Sustenance in adulthood equates to maint. Data and intuition agree with the Friedman idea up to a point. The problem lies in the first few years of maturity. Table 7.5 shows average pay for all cohorts in 1997 at $36,485. If the maint index is not less than two thirds, coming to $24,323 for all cohorts on average, this average would exceed pay of the youngest adult cohorts. Then let us modify the
Friedman idea to permit lower-than-average maint at the start, and higher-than-average maint later.

Chapter Three introduced the insumption fund model, which estimated thrift return, under simplifying assumptions, as a function of vital statistics alone. Let us now find a parallel argument which makes no mention of an insumption fund.

Let “base insumption” $\imath_{bs}$ ($\imath$ is iota) mean aggregate current insumption by all cohorts aged less than age M (the age of “maturity”). Since cohorts begin in an economic sense at age $-1$ years, the “non-adult span” N separating ages of the youngest cohort from the “maturity cohort” aged M equals M plus one year. The maturity cohort is the youngest to receive no parental gift. $x$ denotes cohort age. Cohort birth spacing is the differential time interval dx. Then the number of non-adult cohorts is $N / dx$, and average insumption of all non-adult cohorts is $\imath_{bs} dx / N$.

Assume the growth model, under which all inter-cohort differences at a single moment are explained by thrift growth alone. Then cumulative insumption of the maturity cohort since economic birth is equal to average insumption of all younger cohorts times the non-adult span. This cumulative insumption is $\imath_{bs} dx$. Let “base senescence” $\delta_{bs}$ (\delta is delta) mean aggregate senescence by non-adult cohorts, so that the cumulative senescence of the maturity cohort since economic birth is $\delta_{bs} dx$ by parallel reasoning. The value of the maturity cohort, notated H(M), is its cumulative insumption less its cumulative senescence. Then $H(M) = \imath_{bs} dx - \delta_{bs} dx$. Next define the “intactness index” $C_{int}$ by

$$C_{int} = \frac{\imath_{bs} dx - \delta_{bs} dx}{\imath_{bs} dx} = \frac{\imath_{bs} - \delta_{bs}}{\imath_{bs}}$$

to see

$$H(M) = C_{int} \imath_{bs} dx.$$

Define $r_o(M)$ as the thrift work rate of the maturity cohort. Thrift work by that cohort is that rate times the maturity value H(M). Meanwhile we just saw that average insumption by all non-adult cohorts is $\imath_{bs} dx / N$. Now define the “base insumption ratio” $C_{ins}$ by

$$C_{ins} = \frac{\text{thrift work of maturity cohort}}{\text{average insumption of non-adult cohorts}} = \frac{r_o(M) H(M)}{\imath_{bs} dx / N} = r_o(M) C_{int} N,$$

and solve for $r_o(M)$ to find the “maturity model”
\[
\text{ins} \Theta = \frac{\text{ins} \text{Cr}(M)}{\text{N}}.
\]

Since the greatest possible value of the intactness index \( \text{C}_{\text{int}} \) is unity, the provider model may also be expressed as

\[
r_{\Theta}(M) \geq \frac{\text{ins} \text{Cr}(M)}{\text{N}}.
\]

Now substitute 21 years for \( \text{N} \) to get
\[
r_{\Theta}(M) = \frac{\text{ins} \text{Cr}(M)}{\text{N}} \text{, or } r_{\Theta}(M) \geq \frac{\text{ins} \text{Cr}(M)}{\text{N}}.
\]

(7.1)

Let us try to evaluate the base insumption ratio by clarifying its denominator. Base insumption is parental gift \( \gamma \) (gamma) plus self-insumed work of the non-adult cohorts. Let the "base work index" \( C_{\text{work}} \) mean the ratio of thrift work of the maturity cohort to average self-insumed thrift work of all non-adult cohorts. Meanwhile average parental gift received per non-adult cohort is \( \gamma \) divided by the number of such cohorts, or \( \gamma \frac{\text{dx}}{\text{N}} \). Then average insumption by non-adult cohorts is \( C_{\text{work}} w_{0}(M) + \gamma \frac{\text{dx}}{\text{N}} \). Now the insumption index may be expressed as

\[
\text{ins} \frac{\text{work} \text{w}(M)}{\text{C}} = \frac{\text{w}(M) \gamma}{\text{work} \text{w}(M) + \gamma \frac{\text{dx}}{\text{N}}}.
\]

Next define the "base gift index" by

\[
C_{\text{gift}} = \frac{\text{thrift work of maturity cohort}}{\text{average parental gift received by non-adult cohorts}} = \frac{w_{0}(M)}{\gamma \frac{\text{dx}}{\text{N}}},
\]

and divide numerator and denominator of the right side of the previous result by \( w_{0}(M) \) to see

\[
\text{ins} \frac{\text{work} \text{w}(M)}{\text{C}} = \frac{1}{C_{\text{gift}} + C_{\text{work}}}.
\]

(7.2)

The base work index and base gift ratio are convenient to model. Note that if the self-value / age curve is linear over the non-adult span, and if all these cohorts bear the same stasis rate, average cohort value and thrift work are one half of those of the maturity cohort. They are more than one half if the curve is convex, and less if concave. Since Table 7.3 suggests that parental gift is steady or rising over the non-adult span, and since self-invested work should rise with cohort value, the curve seems likelier to be concave. Then the base work index will be modeled at something a little under one half.
The base gift ratio is a partial measure of the maturity cohort’s readiness to replicate itself through parental gift. Since parenthood begins eight years later, in the model, one might suppose that this ratio should be less than unity. But that is not so. When parenting begins, thrift output from capital of both factors must be sufficient for parental gift and for investment or adult insumption to offset losses to senescence. If there were no makeup of these losses, parental gift could not continue. The maturity cohort’s unreadiness to begin parenting does not mean that its thrift work is less than average parental gift, but rather that it is less than enough to make the gift and replenish capital too.

Capital cannot grow at maturity unless thrift work, which is modeled as the whole of the maturity cohort’s thrift output, is greater than senescence. Its senescence converges to its maint, which was modeled to equal its pay. Pay of the 20-year-old cohort can be extrapolated from Table 7.5 at about $17,000 per year. Then thrift growth over the bachelor period requires thrift work of the maturity cohort to be more than $17,000 per year. Meanwhile average parental gift was modeled at about $19,000 per year. Then the base gift ratio is probably not far from unity, and is probably greater than unity on the likelihood that growth of the maturity cohort exceeds $2,000 per year.

If the base gift ratio is somewhat more than unity, its reciprocal is somewhat less. Then if the base work index is less than one half, as expected, (7.2) shows that the base insumption ratio is greater than two thirds. Now (7.1) becomes

\[
2 \geq \frac{4.8\%}{3 \text{ year}} \quad \text{or} \quad r_0(M) \geq \frac{3.2\%}{\text{year}}.
\]

**capital growth during the bachelor period**

Suppose that there is no waste and no gift during the bachelor period, so that all thrift output is insumed or invested. Then thrift growth in total capital \( J \), regardless of the shares insumed and invested, is thrift output less senescence \( \delta \). These assumptions give the first-order linear differential equation

\[
D_xJ = r_0J - \delta, \quad M \leq x \leq P.
\]

Suppose for easier math that \( r_0 \) is constant over the interval \((M, P)\). Also suppose for the moment that maint, which converges to senescence, is likewise constant over this interval. The solution to (7.3) then becomes

\[
J = J_M e^{\delta\Delta x} - \delta \frac{e^{\delta\Delta x} - 1}{r_0},
\]

where \( \Delta x = x - M \) and \( J_M \) is total capital of the cohort aged \( M \).
Since rising pay allows rising maint, and since maint must rise eventually to its norm, let us consider the alternative possibility that maint (senescence) rises linearly over the bachelor period. In that case

\[ \delta = \delta(x) = \delta_m + \frac{x}{B} (\Delta \delta), \]

where \( \delta_m \) is maint (senescence) of the maturity cohort, B again is the length of the bachelor period, equaling \( P - M \), and \( \Delta \delta \) is overall increase in senescence over that period (i.e. \( \delta_p - \delta_m \)).

Now the solution to (7.3) becomes

\[ J = J_M e^{\delta M Ax} - \frac{\delta_m (e^{\delta M Ax} - 1)}{r_0} - \frac{\Delta \delta (e^{\delta MX} - 1 - r_0 \Delta x)}{Br_0^2}. \] (7.5)

Suppose that average maint for all adult cohorts is 70% of average pay for all adult cohorts, or about $25,500. Maint for the maturity cohort can be estimated at its pay, if non-yield expense is neglected, and we saw that pay of the 20-year-old cohort is about $17,000 by extrapolation from Table 7.5. If maint remains flat over the bachelor period, at $17,000 per year, then maint after the bachelor period must average $27,000 in order to bring the overall average to $25,500. If maint rises linearly and reaches the post-bachelor norm at age \( P \), then that norm is about $26,000 in order to meet the same condition.

Now let us find \( J_r \) from (7.4) and (7.5) respectively. \( B \) and \( Ax \) are each eight years in each equation, and \( \delta_m \) is $17,000 per year. \( \Delta \delta \) is $9,000 per year (i.e. $26,000 - $17,000) in (7.5). Table 7.6 shows resultant total capital and thrift output at the mean age of reproduction under these assumptions.

<table>
<thead>
<tr>
<th>( r_0 ) (per year)</th>
<th>( J_M ) (^1)</th>
<th>( J_P ) (^2)</th>
<th>( y_{0,P} ) (^4)</th>
<th>( J_P ) (^3)</th>
<th>( y_{0,P} ) (^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0%</td>
<td>$496,000</td>
<td>$435,000</td>
<td>$8,700</td>
<td>$397,000</td>
<td>$7,500</td>
</tr>
<tr>
<td>2.5%</td>
<td>525,000</td>
<td>491,000</td>
<td>12,300</td>
<td>452,000</td>
<td>11,300</td>
</tr>
<tr>
<td>3.0%</td>
<td>566,000</td>
<td>553,000</td>
<td>16,600</td>
<td>514,000</td>
<td>15,400</td>
</tr>
<tr>
<td>3.5%</td>
<td>589,000</td>
<td>622,000</td>
<td>21,800</td>
<td>580,000</td>
<td>20,300</td>
</tr>
<tr>
<td>4.0%</td>
<td>625,000</td>
<td>700,000</td>
<td>28,000</td>
<td>660,000</td>
<td>26,400</td>
</tr>
<tr>
<td>4.5%</td>
<td>664,000</td>
<td>788,000</td>
<td>35,500</td>
<td>747,000</td>
<td>33,600</td>
</tr>
</tbody>
</table>

\( ^1 \) from Table 7.4 \hspace{1cm} \( ^2 \) from equation 7.4 \hspace{1cm} \( ^3 \) from equation 7.5 \hspace{1cm} \( ^4 \) \( J_p \) times \( r_0 \)

Note first that growth is negative, even under flat maint, wherever \( r_0 \) is 3.0% per year or less. Growth remains negative when \( r_0 \) is 3.5% if maint rises, and is positive but modest if maint
is flat. Growth becomes significant only under the highest stasis rates modeled. If we expect
growth over the bachelor period, then, Table 7.6 supports hypotheses that the stasis rate is 3.5% per year or higher. For the same reason, it also supports hypotheses that maint is flat over the bachelor period.

Note further that the highest thrift outputs shown for age P do not much exceed maint rates, and hence senescence rates, projected for the post-bachelor period. And parenthood now implies a diversion of output to parental gift. Table 7.6 suggest, then, that total capital and thrift output are likely to peak no later than at the end of the bachelor period. This would seem particularly true of self-value, which bears the whole cost of senescence.

One might have guessed that steeply rising pay over three decades, which proves continuing insumption in the form of new skills learned, proves rising self-value as well. If maint after the post-bachelor period is more or less constant, after all, then rising pure pay must mean rising sold work. But sold work may rise, even while self-value and overall thrift work decline, if the share of thrift work sold rises at the same time.

The cutoff return rule

The duplication rule expects maint and senescence to agree more or less at every scale. The second duplication rule applies this reasoning to evaluate self-capital as the integral of expected future senescence (~ maint) less expected future insumption. Then if there exists a cutoff age at which adult insumption becomes negligible, cohort value at that or any greater age can be estimated at the integral of expected remaining maint. Since all unsold thrift work is insumption, also, absence of insumption implies that all thrift work is sold work. Therefore thrift work of that same post-cutoff cohort is found at pay less work expense by the sold work rule. The “cutoff return rule” consequently finds

\[
\text{thrift return} = \frac{\text{thrift work}}{\text{self-value}} \sim \frac{\text{pay} - \text{work expense}}{\text{integral of expected maint}}
\]

for post-cutoff cohorts only. In symbols, this is

\[
r(x) = \frac{w(x)}{H(x)} \sim \frac{\sigma(x) - \varepsilon(x)}{\int_{\lambda(x)}^{\varepsilon(x)} \varepsilon(z) \, dz}, \quad x \geq x_{\text{cf}}
\]

where \( w(x) \) is thrift work of the cohort aged \( x \), \( \sigma(x) \) is its pay (\( \sigma \) is \textit{sigma} for “sales”), \( \varepsilon(x) \) is its work expense (\( \varepsilon \) is \textit{epsilon} for “expense”), \( \varepsilon_{\text{cf}} \) is maint, \( \lambda(x) \) is expected remaining economic lifespan of the cohort at that age (\( \lambda \) is \textit{lambda}), and \( x_{\text{cf}} \) is the cutoff age.

Note that \( \lambda(x) \) is not simply 75 years less current age. 75 years gave expectation of life at birth, less one year allowed for redependence. SAUS99 (128/129) shows that although only about 80% of newborns lived to age 65 in 1990, for example, those who did expected to survive for another 17.5 years. Allowance for one year of redependence would reduce this expectation.
to 16.5 years. Then if the cutoff age for adult insumption is not greater than 65 years, for example, the value of the 65-year-old cohort is estimated by the second duplication rule at its expected remaining maint over these last 16.5 years. Table 7.7 below shows calculations for cohort ages shown, with implicit thrift rates of return.

Table 7.7: Self-Value and Thrift Return in Post-Cutoff Cohorts

<table>
<thead>
<tr>
<th>age (years)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>pay ($/yr)</td>
<td>38,527</td>
<td>42,477</td>
<td>42,289</td>
<td>39,358</td>
<td>NA</td>
</tr>
<tr>
<td>work expense ($/yr)</td>
<td>27,364</td>
<td>27,364</td>
<td>27,364</td>
<td>27,364</td>
<td>NA</td>
</tr>
<tr>
<td>sold work ($/yr)</td>
<td>11,163</td>
<td>15,113</td>
<td>14,925</td>
<td>11,994</td>
<td>NA</td>
</tr>
<tr>
<td>maint ($/yr)</td>
<td>26,000</td>
<td>26,000</td>
<td>26,000</td>
<td>26,000</td>
<td>NA</td>
</tr>
<tr>
<td>life expc (yrs)</td>
<td>37.4</td>
<td>28.5</td>
<td>20.2</td>
<td>13.1</td>
<td>10.1</td>
</tr>
<tr>
<td>maint expc ($)</td>
<td>1,009,800</td>
<td>769,500</td>
<td>525,000</td>
<td>341,000</td>
<td>NA</td>
</tr>
<tr>
<td>cohort value (if post-cutoff) ($)</td>
<td>525,000</td>
<td>341,000</td>
<td>NA</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>thrift return (per year)</td>
<td>2.8%</td>
<td>3.5%</td>
<td>NA</td>
<td>NA</td>
<td></td>
</tr>
</tbody>
</table>

1 from ranges shown in Table 7.5
2 3/4 times average pay (Table 7.5).
3 pay less work expense (sold work rule)
4 by earlier assumption
5 from SAUS99 (129)
6 maint times life expectancy
7 same as maint expectancy if age is not less than cutoff age
8 sold work / cohort value

Again, let us first highlight the caveats. The greatest uncertainties seem to lie in the estimates for work expense and maint, for the cutoff age, and for pay of the cohort aged 70, for reasons I discussed before. Note that since sold work varies inversely and cohort value directly as maint, thrift return varies inversely and sensitively to adjustments in estimates of that key flow. Thus a decrease of $1,000 each in the estimates shown for work expense and maint would have raised sold work and lowered self-value of the 60-year-old cohort to $15,925 and $505,000 respectively, and so would have raised its thrift return to 3.2% per year in place of the 2.8% shown.

We should not be surprised to find that different-aged cohorts carry different thrift returns. Return is believed to vary as risk, and some cohorts may be riskier investments than others. A 60-year-old, as I said earlier, might prefer to invest in property rather than take his chances on surviving long enough to pay out on an investment in himself. In general, then, older cohorts should prove riskier and higher in return to the few investments they justify. The maturity cohort, at the other extreme, carries uncertainty of a different kind. 20-year-olds, particularly males, are typically risk-takers who show high crime rates and not-yet-defined career potentials. Then it may be that my focus on youngest and oldest adult cohorts, in the provider model and cutoff return model respectively, leads to an upward bias in estimating thrift return.
Chapter Seven has proved the most data-intense and math-intense section of TRE, but also the most dependent on simplifying assumptions. Economic life cycle models are inherently interesting, and interest was enhanced by the prospect of estimating thrift return in the maturity model and the cutoff return rule. The latter estimate was helped by the maint equations, which show that maint substantially exceeds sold work under thrift offset, under pay equilibrium, and by pay / age profiles suggesting an insumption cutoff age somewhere in the fifties or later. Thrift return was estimated at several percent yearly by all methods, subject to uncertainties in the assumptions.
VIII. TRE AND GAAP

Assume that inflation is determinable and single-valued, so that its ratio is given by a unique inflation rate $i$ at a given moment. Inflation is spurious growth, distinguished from real growth by some unspecified test. That is,

$$i = g_{nom} - g,$$

where the right-hand terms are nominal and real growth rates. Now let $r$ and $\phi$ ($\phi$) stand for real rate of return and yield rate. Then the total return rule is

$$r = g + \phi = g_{nom} + \phi - i,$$

showing the “disinflation rule”

$$r_{nom} = g_{nom} + \phi = r + i$$

where $r_{nom}$ is the nominal rate of return.

These effects may be simplest to visualize if we take all values in constant dollars, but in constant dollars set at the present moment rather than at a given past one. This “present dollar” convention makes it easier to tell current from cumulative inflation. We see that it makes real and nominal capital value the same, and likewise for all flows which are not functions of growth. Only growth rate itself needs disinflation.

Yield rate, in particular, does not change under disinflation. Inflation impacts numerator and denominator equally, and leaves the ratio intact. But it is wrong to claim that the same is true of rate of return. That claim, which is common, implies the paradox that nominal and real growth rate are the same.

CIPP and TRE are different fundamentally. CIPP is a rationale of accounting, abstracting from a chaos of data to hypotheses of pattern and economic principle. TRE treats economics as a topic of philosophy, and approaches it from the opposite direction. It prefers the fewest and mildest factual assumptions, on the principle of Occam’s razor, and expands downward with the caution of tautology. Although TRE yields a richer and more reliable body of predictions, or so I have argued, it raises new problems for record-keepers. CIPP is designed from the start for its stocks and flows to be tallied by accountants as we know them. Meanwhile TRE, which defines all values as mark-to-market, seems to need an omniscient accountant who somehow knows how to price them without the help of a cash register currently ringing. TRE is content with that tradeoff. It allows terms to mean what they are supposed to mean, and equations to predict exactly, even if techniques of measurement are not all we would wish. And measurements are
always possible in principle, up to a point, if we replace the omniscient accountant with an imaginary market consensus. Note that even a worker’s self-value may be gauged by consensus estimates of his future cash flow, meaning pay less non-yield maint, and the rates of return at which to discount them.

The accountant’s method or GAAP (Generally Accepted Accounting Principles) tracks cumulative flows and final stocks in current dollars for stated periods. Since cash payments tell only part of the story, GAAP usually recognizes accruals as well. An accrual is a proration of a lump payment over the period of its effected effect. Thus the cost of a building is usually not expensed at once, but capitalized and depreciated over its expected remaining lifespan. Also a lessor might take a year’s prepaid rent into income month by month, rather than all at the start.

Earnings, or accounting profit, or GAAP profit, is sales less cash expense less accrued extinguishments plus internally generated investment taken at cost. All these flows are typically measured over discrete periods in current dollars. TRE profit may be analyzed as sales less cash expense less mark-to-market recovery of extinguishments plus internal investment measured at value. TRE’s sales and cash expense should measure about the same as GAAP’s, except that TRE’s are instantaneous and implicitly disinflated. These differences can be reconciled, more or less, if we take the GAAP values as representative of the midpoint of the reporting period. The “present dollars” of this midpoint need no disinflation except when measuring growth.

Comparison of liquidated expense brings a surprise. Since GAAP generally does not mark plant and equipment and inventory to market, it misses much free growth. Thus book value generally understates real value when free growth is positive. But that means that if GAAP depreciation periods are realistic, GAAP depreciation flows are unrealistically low. Thus mark-to-market depreciation is higher than GAAP depreciation, not lower, if GAAP depreciation periods are realistic. Then if extinguishments and liquidation converge, TRE’s liquidated expense will tend to exceed GAAP’s.

One would think that GAAP’s treatment should be reasonable, since unmodified structures and machines ought to be unable to grow in productivity and therefore in real value. But we saw that they do. Just as the schoolroom which receives no inputs beyond routine maintenance becomes more productive as better doctrines are taught there, or as the shovel becomes more productive when gold is discovered nearby, the plant and equipment become more productive as they house and make better processes and products. Such exogenous effects demonstrably raise the sales prices of schoolrooms and shovels and plant and equipment. That is why it is probably realistic to model free growth as more or less uniform and all-pervasive. Then disinflated GAAP earnings should tend to exceed TRE’s realized profit, even though the definitions are superficially the same. By the same token, though, earnings should fall well short of TRE profit or total return. The overlooked free growth in plant and equipment which makes GAAP depreciation artificially low has a still stronger and more direct effect in downward distortion of GAAP earnings.

Thrift growth too is often under-recognized. Additions to inventory, or to plant value, are generally capitalized at cost. Yet those additions, insofar as produced internally, reflect new output as well as cash expense and depletion of inventory. Any such output remaining after
deduction of free growth is thrift output, and is retained as unrecognized thrift growth until the plant or inventory assets are sold. Although thrift growth converges to zero under an assumption of thrift offset, unrecognized thrift growth may be significant in individual cases or sectors.

GAAP earnings therefore captures only the sold part of the growth component in profit, whether of free growth or of internally generated and retained thrift growth, and only at the moment of sale. The sold and captured part is reported cumulatively and retrospectively, rather than at the moment when each increment actually occurred, moreover, so that this reporting lag generally compounds the understatement of growth when growth is positive. Earnings thus tends to understate the magnitude of growth, whether positive or negative, by recognizing it too little and too late.

But it is only in nominal dollars that GAAP growth can be trusted to fall between zero and mark-to-market growth in magnitude. Inflation, which also may be either positive or negative, may exert an independent pull. To illustrate, let \( g_{\text{GAAP}} \) mean growth rate reported in GAAP as expressed in current dollars. Since \( g_{\text{GAAP}} \) will capture some but not all nominal mark-to-market growth in either direction, the general rule is

\[
0 \leq |g_{\text{GAAP}}| \leq |g_{\text{nom}}|.
\]

Suppose for simplicity that both \( g_{\text{GAAP}} \) and \( g_{\text{nom}} \) are positive. Then the definition \( g_{\text{nom}} = g + i \) gives

\[
-i \leq g_{\text{GAAP}} - i \leq g.
\]

This shows that disinflated \( g_{\text{GAAP}} \) will be negative when \( i \) is positive and greater than \( g_{\text{GAAP}} \), even though \( g \) is positive. If \( i \) is negative, on the other hand, disinflated \( g_{\text{GAAP}} \) may be greater than \( g \). We confirm, however, that real growth rate \( g \) may be trusted to exceed disinflated \( g_{\text{GAAP}} \) whenever growth and inflation agree in sign.

GAAP finds no problem in reporting yield, which lends itself to exact and timely measurement after allowance for the principle that flows accumulated over reporting periods suggest averages for midpoints rather than end points for those periods. Since profit in the total return sense is growth plus yield, then, differences in measurement of growth account for substantially all differences between TRE profit and GAAP earnings.

The GAAP rule

One popular GAAP counterpart to TRE’s rate of return is \( \text{earnings / price ratio} \ E/P \), typically meaning the ratio of GAAP earnings to the market price of corporate shares or other claims on those earnings. TRE’s measure \( r \) equates to \( p/K \), where profit \( p \) is currently observed growth plus yield and property value \( K \) is the same market price of corporate shares. Thus \( P = K \). Since GAAP earnings is GAAP growth plus yield, and since yield is measured the same in GAAP and TRE, we may write
\[
E/P = g_{GAAP} + \phi
\]
as the GAAP counterpart to TRE’s return rule \( r = g + \phi \).

Since \( g_{GAAP} \) can be trusted to fall between zero and \( g_{nom} \) in absolute values, it is convenient to imagine a “GAAP capture index” \( \kappa \) (\( \kappa \)appa) which ranges from zero to unity and measures the ratio of \( g_{GAAP} \) to \( g_{nom} \). That is,

\[
\kappa = \frac{g_{GAAP}}{g_{nom}}, \text{ where } 0 \leq \kappa \leq 1.
\]

This allows

\[
E/P = \kappa g_{nom} + \phi = \kappa (g + i) + \phi.
\]

Now solve for \( \kappa \) to see the “GAAP rule”

\[
\kappa = \frac{E/P - \phi}{g + i},
\]

noting that the right-side arguments are generally measurable. Those of us who aren’t embarrassed by dumb puns might think of the quantity \( 1 - \kappa \) as the “GAAP gap.”

The GAAP index becomes interesting insofar as it can be predicted. Presumably it varies with accounting conventions, for example as to book depreciation periods. It may also vary with “irrational exuberance” and other external conditions. For large-company U.S. equities over the period 1925-1928, we might get

\[
\kappa = \frac{1/(15 \text{ years}) - (4.5\% / \text{year})}{3\% / \text{year} + 3\% / \text{year}} = .36,
\]

while the same large-company equities over the period 1990-1998 would show roughly

\[
\kappa = \frac{1/(25 \text{ years}) - (2.5\% / \text{year})}{14\% / \text{year} + 3\% / \text{year}} = .09.
\]

This disparity suggests that a rationale for the GAAP index remains to be found.
Just as new cars should be able to run on old roads, TRE should be able to interpret and guide accounts as the world knows them. One difference to be bridged is that most accounts, meaning GAAP, report nominal rather than disinflated values. Chapter Eight began by outlining a disinflation algorithm. Next we compared TRE’s “output” to GAAP’s “earnings.” We saw that earnings is generally current-dollar rather than constant-dollar, and cash/accrual basis rather than mark-to-market. The effect is to understate current-dollar growth in times when free growth is positive. The “GAAP index” gives the ratio of GAAP growth to mark-to-market nominal growth, and allows transformations from GAAP’s equations to TRE’s when inflation and free growth rate are known.
IX. TRE AND NIPA

two perspectives

TRE is a chain of logic derived from the prime assumptions. NIPA is a system of accounts. While TRE finds inferences from definitions and from these assumptions, with testability to be considered later, NIPA is constrained toward a more empirical focus from the start. NIPA too claims a rationale from definitions in principle, meaning CIPP, but it seems to me that those definitions are sometimes allowed to fit the data findable, rather than conversely, with less than thorough criticism.

GNP, GDP, NNP and NDP were offered as examples. A nation’s product, meaning product by nationals (national product) or product within the nation (domestic product), is defined in principle as its creation of new goods and services, gross or net of depreciation. In practice it is taken as consumption plus gross or net investment, again meaning gross or net of depreciation, by individuals and by government. The second output rule shows that these definitions in principle and in practice do not correspond well. The argument which interprets creation of value as consumption plus investment stops short. What happens to the consumption? The architects of CIPP and of NIPA certainly realized that a part is invested in “human capital” and that another part is likely to be wasted. But it should also have been obvious, on reflection, that maintenance consumption (maint) is an input into product value which should not be counted again as part of what is produced.

It is fine to move the target into the range of the arrows as a conscious policy. But the policy carries a risk of forgetting where the target really belongs. If we are in a rush to get on to practical applications, we might easily accept the idea that creation of value is consumption plus investment. So says tradition, and so say intuition and common sense when not pushed hard. Since consumption and investment are easy targets to grasp and count, as economic quantities go, we might well feel impatience at a suggestion that the truth lies in the shadows behind. TRE is the gadfly and killjoy that says it does.

pay comparisons

TRE’s “pay” is a more comprehensive counterpart to NIPA’s compensation of employees. Pay, as the whole realized and imputed sales of self-value, also includes an appropriate share of proprietors’ income. Pay is specifically imputed to include the value of all work services performed for the worker himself or for his property except for the unsold thrift work of study and exercise.

consumption comparisons

TRE’s “consumption” compares to the sum of NIPA’s personal consumption expenditures and the consumption part of government consumption expenditures and gross investment. All these flows are meant to include insumption. But NIPA, as a system of accounts, tends to capture only the traded components of insumption. These include food and housing of the young, plus textbooks and tuition and whatever other inputs into self-value are
literally sold or are normally imputed. What escapes measurement most significantly is unsold thrift work, meaning the work of study and exercise which has no market substitute. Thus NIPA consumption, in practice, approximates consumption less unsold thrift work.

net investment comparisons

NIPA’s counterpart to TRE’s “net investment” can be calculated as the sum of gross private domestic investment plus a share of government consumption expenditures and gross investment, less consumption of fixed capital (CFC). NIPA’s version, like TRE’s, includes investment in residences. The chief difference in principle is that TRE’s includes all free growth. NIPA’s is recorded by the accountant’s method (GAAP), and will include only free growth that happens to be realized in sales.

thrift profit comparisons

What TRE calls “thrift profit” maps into a composite of NIPA rubrics. These are the net-of-depreciation version of corporate profits plus rental income of persons and net interest, and also implicit net-of-depreciation profit shares of proprietors’ income and of government consumption expenditures and gross investment. This NIPA composite is broad, including imputed rental value of residences for example. TRE’s thrift profit does the same. The NIPA version may be larger overall, since it will reflect any free growth which happens to be realized in sales. Thrift profit, on the other hand, excludes all free growth.

thrift output comparisons

The closest NIPA counterpart to TRE’s “thrift output,” as we saw, is net product (NNP or NDP). By the second output rule, NIPA could produce a reasonable proxy for the nation’s thrift output by subtracting yield maint from net product. Authorities may decide whether this is worth the effort and controversy entailed in any identification of some kinds of consumption as maint and some not.

I should note however that the gift term in the second output rule, which is not expressed in the CI equation, is more or less recognized in NIPA in personal transfer payments to the rest of the world. Business transfer payments to the rest of the world means taxes paid by domestic firms to foreign governments, which does not seem to mean the same thing as gift.

gross product and total cash flow

The reader may have come to wonder why gross product, whether national or domestic, is reported at all. As the sum of net product and property depreciation (CFC), it overstates thrift output by the amounts of maint and depreciation combined. And yet it is generally the most popular and most quoted index of the nation’s prosperity. Why?

Any businessman knows that realized profit plus depreciation gives cash flow, and that cash flow gives the size of his budget available for reinvestment or yield. Gross product is a little different. Chapter Three showed that it exceeds total cash flow by the sum of unrealized
thrift output plus prop free growth less gift. NIPA measures it by GAAP methods, which generally capture only the portion of prop free growth realized in sales. Also gift becomes small in proportion as scale approaches closure. We also saw that the whole unrealized thrift output of self-value, meaning unsold thrift work, is likely to be unreflected in NIPA. In practice, then, measured gross product and total cash flow should be much the same.

Work cash flow is probably a formulation unique to TRE, and hence unknown to the businessman or government planner. It would follow that the same is true of total cash flow. But work cash flow is simply pure pay, meaning pay less non-yield expense, and total cash flow is this plus property cash flow. This is roughly the overall stream available for budgeting, whether at the household scale or the national one. Then planners who focus on gross product are wise after all.

This is not to claim that work cash flow and property cash flow can be disposed with equal freedom. Property cash flow, both as to its realized profit component and its depreciation recovery component, can be spent however we wish. The same holds only for the realized output (sold work) component of the worker’s cash flow. His work expense must be spent as a precondition for everything. Thus gross product is not budgeted ad libidem. With this qualifier, it probably deserves its billing as the most useful tabulation in NIPA.

Note however that total cash flow is not a maximand. Output is a true maximand, if not so informative a one as risk-adjusted rate of return, and total cash flow is total output less unrealized output plus liquidations of both factors. The wild card here, it seems to me, is property depreciation (CFC in NIPA terms). Nations building less durable plant and equipment, other things equal, should tend to show higher total cash flow (gross product). Thus the Practical Pig, who built his house of brick, might contribute less depreciation to GNP or GDP than his brothers who build of sticks and straw. Gross output is a very useful measure for planning, but a less reliable one to show a nation’s progress in achieving social goals.

Net product is a flawed proxy for output in the sense of creation of value, meaning total return, because it double-counts maint. It equivalently overlooks the depreciation of self-value. Gross product, which adds property depreciation (CFC), is so much the worse one. But gross product should not be interpreted as a measure of output. As a measure of current spending power, it is well suited to determine the overall size of the nation’s public and private budget.
X. APPLICATIONS

TRE in practice

TRE, like other approaches to economics, is meant as a practical tool as well as a way of understanding. It is not an ideology, and cannot tell individuals and executives and policymakers what their goals should be. It can contribute to the debate as to appropriate strategies and tactics toward a menu of possible ones, and can comment on certain beliefs about economic behavior.

TRE and public policy

In broad concept, the growth dichotomy is old news. The prod index has close analogies in traditional economics. As I said earlier, however, the thrift index is something of a departure. Tradition views consumption in general as the flow which can be curtailed to enable thrift growth. TRE sees this flow as gift plus final yield, and as final yield alone at the scale of closure. In any case, tradition would probably agree that waste consumption is the only flow at closure which is actually available for conversion to thrift growth.

The thrift leverage rule, however, may prove a more substantial departure. Its attraction is in that it derives the thrift growth rate from four easily modeled variables. These are the original waste / final yield ratio, the percent reduction, the conversion efficiency and the stasis return rate. I am not sure that tradition offers anything similar.

More interesting still is the appearance that under any likely estimates of these data, thrift growth cannot exceed a small fraction of the growth we actually experience. Table 4.1 showed that even if original waste was half of final yield, meaning that there was as much waste as maint, and if half that waste were eliminated and half the eliminated part were converted to thrift growth, then thrift growth would still account for no more than a sixth of typical growth over the twentieth century. Under more realistic estimates, maximum thrift growth seems to be something between one fifteenth to one hundredth of actual growth. It is actually negative in the likely event that yield rate is lower in stasis. These considerations lead to the “thrift offset principle,” or hypothesis that thrift growth tends to equal zero at the scale of closure.

The inference is that policies designed to support growth through a shift from consumption to thrift investment are likely to pile shovels while lowering return. Thus economists are invited to review the belief, heretofore scarcely controversial, that tax policy ought to encourage plowback of earnings more than yield. That means, for one thing, that it is time to question the advisability of higher tax rates on ordinary income than on capital gains. TRE suggests that the two should be taxed equally. The problem of taxing unrealized capital gains at the same rate, in order to avoid “capital constipation,” might be managed in reasonable comfort through imposition of continuously compounding tax liens.

TRE includes something to please and offend all ideologies. Leftists and levelers might welcome the idea that ordinary income, most of which is pay, ought to be taxed at a rate no higher than that for capital gains. They might be indignant, however, at TRE’s claim that real profit at several percent per year, plus waste and growth rates, is effectively a law of nature. But
there is not much room for doubt. The duplication rule is solid, although surprising, and it is obvious in hindsight that output must first make up losses to the work force through senescence and mortality. Thus the basic rate of return is in some sense the generation turnover rate.

Although TRE questions consumption-restraining policies, including tax policies, it finds no fault with the idea of luxury tax or “sin tax.” The American economist Robert Frank argues that insofar as luxuries are trophies or status symbols, government all but does the sybarite a favor by making such goods more costly and hence more exclusive.

market prediction

Whether consciously or not, markets price assets by discounting expected yield or cash flow. The discount rate is the risk-adjusted rate of return expected for each asset. Then if we find an asset whose risk is judged average and whose yield schedule is known, we can predict its price if we also know the average-risk rate of return. The economy as a whole is an average-risk asset, and the average-risk rate of return is the social rate. Then if we can estimate its yield schedule or cash flow schedule, and also know the social rate, we can gauge its price. The market meanwhile does not know the social rate, but intuits it from recent and long-term experience. If we have a better algorithm, we might anticipate the market.

TRE offers the senescence rate model of average-risk rate of return, before waste and growth rates are added, and the GAAP rule for measuring the efficiency of the accountant’s method in capturing mark-to-market growth. It seems to me that these are helpful tools even now in appraising markets, and may become more helpful as we come to understand their parameters better.

TRE and microeconomics

Microeconomics is the study of bargains and outcomes among rational actors, which are typically individuals but may also be entities such as corporations. Its main paradigm is the interplay of supply, demand and price illustrated in our discussions of equilibrium pay and of conversion efficiency. It is further enriched with a rationale of production functions and utility functions by which assumptions of preferences can be translated into specific decisions. Game theory is another potent tool of the microeconomist, particularly for deriving equilibria for density-dependent inputs.

TRE has little to add to this already well-developed field, except perhaps in proposing risk-dependent rate of return, rather than capital, as the universal maximand of the rational actor. If no utility functions have appeared in this text, that may be because I have preferred not to assign utilities much beyond the statement that tastes are satisfied from total yield. TRE’s finding that total yield includes gift, and excludes insumption, might however be of some interest to microeconomics in clarifying maximands. The adult second output rule shows that the rational actor is not as selfish as he has sometimes been drawn.
Macroeconomics is largely advice to governments in meeting national priorities and in managing taxes and the money supply to steer between the risks of recession and inflation. I have said nothing about the money supply, partly because I think that money is evolving into something whose supply will prove non-critical so long as it is ample. But national priorities tend to include waste conversion, in many countries including this one, and TRE contributes to that debate by questioning the practicality of the goal.
XI. SUMMARY

why bother?

Why should anyone trouble to read this text? The ultimate reason is that TRE offers a trustier guide to public and private policy and to further learning than is offered by other approaches. But why should the reader invest time to see if that is so? What is the immediate inducement?

TRE is likely to be judged by the novelty and success of its predictions. Two of the most conspicuous are the sold work rule, which implies that maint is double-counted in CIPP and NIPA, and the more complex inference that the stasis rate exceeds the Schumpeterian prediction by several percent per year. These are not minor departures from consensus views. Yet the first is obvious in hindsight, since unsubsidized pay cannot fall below the work expense level even as the self-capital that yields it declines toward zero. The second seems in rough agreement with what is known of the corporate subsector, which is the only sector in which rate of return is convenient to measure. Meanwhile it should have seemed equally obvious that a zero stasis rate would mean no replenishment of the population.

TRE is a structure of surprising predictions which seem to fit the data insofar as data can be found by which to test them. It awaits the verdict of further data, for example from the household sector when rate of return there is better understood. Meanwhile the two predictions just mentioned can serve as TRE’s calling cards, or opening cards, or drawing cards, to entice a look at the rest. The greatest surprise is probably the duplication rule. This rule cannot be tested directly, but attracts interest on a theoretical plane because of its irony. It says that maint not only does not add self-value, but in fact subtracts it. It is good that TRE offers such teasers, for the path it charts can be steep.

But the arguments for the rules themselves, as distinct from the models by which they are evaluated, tend to be easy to follow. Even the duplication rule, which baffles intuition, is reached inevitably and in few steps. No economic system rests on safer assumptions than TRE, meaning the prime ones, or gives safer inferences.

retrospect and context

The writer, as a layman, is not well placed to compare TRE with more familiar doctrines. I will try nonetheless.

TRE resembles modern physics in its apparent indifference to common sense. Modern physics however starts from braver premises. Special relativity is no harder to swallow than its two postulates holding the invariance of the speed of light in all inertial frames. TRE pulls its rabbits from less remarkable hats. All of its rules are forced by the prime assumptions, and these are no more than the conditions for stable markets.

The prime assumptions are somewhat unusual in the focus given to owners. What is implicit in other treatments of economics is made explicit. Owners are mortal and reproduce. Their tastes are similar enough to enable market cohesion, and variable enough to make some of
them buyers and some sellers at any time. The premise of bargaining suggests that owners are members of a common species. Overlapping generations theory is probably the best-known precedent for this population-oriented approach.

Capital is treated almost conventionally. One possible departure is its definition as the means of expected taste satisfaction rather than of production. Thus an ice cream cone is capital which will satisfy tastes directly in the very near future, while the ice cream shop is capital which will satisfy them indirectly and later. This definition, coupled with that of yield as outflow from capital value in satisfying current tastes, compels the quantification of capital as present value of all yields it is expected to enable. That logic applies to both factors, and solves the problem of pricing workers.

I regret any annoyance caused by the unusual names given to the factors. But the conventional physical capital and human capital are unapt and unwieldy. “Property” and “self-value” carry more meaning in half the syllables. Physicists have split the atom partly because the words they must use most often are descriptive and short.

The definition of property as capital whose maintenance does not satisfy owner tastes per se may be original. It distinguishes the factors neatly, and highlights the source of accounting differences between them which lead to the duplication rule. It shows at once why slaves count as livestock to owners but as self-value to the slaves themselves.

Consumption, like capital, is defined conventionally in TRE. It is the sum of worker inputs and property extinguishments not recaptured in other property value. Rather it is TRE’s subdivision of consumption into its three components of insumption, maint and waste which parts from tradition.

TRE’s division of total yield into gift plus final yield seems natural on reflection. Value which leaves an owner’s overall inventory of wealth, including property and self-value both, either is given away or ceases to exist. This inference however may be new to economics. Certainly TRE departs from some traditions by insisting that not all yield is consumption and that not all consumption is yield. And the concept of final yield may in itself be novel. Tradition shows great interest in the creation of capital in output, but has neglected the reverse flow. Once created, how does capital quit the overall pool of wealth? Many assets depreciate, but depreciation is cash flow available for replacement. Thus depreciation per se is not exit from the pool. Consumption marks the final extinguishment of property value, but may be reinvested in self-value. Then what exactly is the pathway out? TRE alone, I think, answers that question. Final yield is not all consumption, but rather consumption bought with positive total yield.

We are not surprised that final yield includes waste. But we are surprised to find that it includes maint. It must because worker maintenance, unlike property maintenance, satisfies final tastes and hence is paid from positive total yield. This means that its value cannot be recovered, and is lost in taste satisfaction.

TRE defines output in principle as creation of new value by old. That much is conventional. TRE also reasons that created value must implicitly equal total return, meaning the sum of growth and yield. Though this is one of several traditional definitions of profit, meaning
output produced by property, it may be a new definition of work or output by workers. Yet the definition is inescapable. If work means creation of value, it means growth plus yield. The surprise in this definition is that work and work yield are not the same. Yield to the worker, as to the property owner, is output plus liquidation rather than output alone. The duplication rule shows that he indeed tends to recover his depreciation, meaning senescence, in the taste satisfaction value of his maint. He must, if only because output without recovery of depreciation is a contradiction in terms. We showed that this logic is confirmed by common observation. Workers near retirement are implicitly depreciated to low values. Less capital is expected to imply less output, here meaning work. Yet pay may well hold steady or increase with every year. Work and work yield clearly differ, and the difference must come from liquidation of self-value as the only remaining component of work yield.

There was also a fresh look at growth. A conventional distinction was made between thrift, meaning growth negatively correlated with yield, and growth not correlated with yield. The latter was called free growth, including fortuity as well as technological change. The first task was to find mathematical definitions to capture these ideas usefully. Free growth rate was defined as decline in rate of return over the period since the last recession or another moment of zero growth. Thrift rate was defined as the remainder of growth rate. Those definitions are not the only ones possible within the TRE perspective. In principle they could allow testing of the writer’s view that all growth at the scale of closure is free growth.

**TRE and ideology**

Ricardo was an idea man, brilliantly right and brilliantly wrong by turns. His labor theory of value, developed from ideas of Adam Smith and expanded in turn by Marx, implied that self-value (labor) produced all property in the first place and hence is the sole ultimate factor of production. That is simply not so. Self-value is the larger factor, but not the older one. The two are conascent. Workers in whatever species produce nothing without maintenance consumption, and maintenance is property until exhausted. Food in the mouth and stomach, and the space one occupies at the moment, give the minimum possible property per individual. No creatures, from people to protozoa, can make do with less. There was never a state of nature in which nothing outside oneself was owned and put to use.

TRE, although it carries no ideology or value judgment, can enable criticism of ideologies by revealing strengths or weaknesses in factual perceptions on which they may be based. This contribution is sometimes subtle. Many writers, including some socialists, have offered critiques much like mine above of Marx’ surplus value theory and its precursors. TRE adds detail in such forms as the sold work and convergence rules, which already imply conascence, and in the last day parable which confirms those rules.

TRE enables blunter criticism of Schumpeter’s doctrine that rate of return in the stationary state is zero. The stasis rate appears to be several percent yearly for creatures with vital statistics and adult learning habits like ours. Data from the business sector broadly fit TRE’s estimate, and cannot be reconciled with Schumpeter’s. Other sectors remain cryptic until researchers find ways to understand them.
Schumpeter’s idea survives, I said, in current beliefs such as overlapping generations theory and the golden rule of accumulations. His idea is not ideology in itself, but is an error which could lead to misinformed policy. Although Schumpeter happened to be a champion of market freedom, his belief would imply that profit rates generally higher than growth rates are abnormal. That inference in turn could justify confiscatory taxes on returns beyond the growth threshold. TRE cannot comment on such confiscation as a social goal, but can criticize claims of fact adduced in its support.

TRE does not so much take positions in growth theory as suggest ways in which to frame debate. It does show, I think, that thrift growth at the scale of closure is limited to waste conversion. I personally have questioned the cost-efficiency of waste conversion in a world of presumably saturated niches. More shovels and more diggers from the old cookie stamps cannot make headway. If belt-tightening can somehow jump-start innovation, let us hear how.

Moderation is common sense. Immoderation is a fact of life. I would rather have dinner with Reggie Van Gleason than with Savanarola or the Practical Pig, though I would rather do business with the last. A thrift ethic is a kind of non-partisan ideology which can inspire republicans and democrats alike. Lenin, by all accounts, was more the Puritan than Cromwell. TRE is mute as to ideology and social goals, as always, but can help appraise factual premises. It suggests a review, I think, of the belief that a large and leading economy can grow by any means but innovation.

These examples show that TRE is neither left nor right, neither Cambridge nor Chicago, but rather is a tool useful to a range of social agendas. If it adds a nail to the coffin of surplus value theory, so does it deflate arguments that the tax burden should weigh less on profit and property and more on consumption if we want more growth. Its practical value is not in guiding the choice among social or private goals, but in finding policies likely to further them.

epilogue

Some researchers much of the time, and all of them some of the time, must wall off the cacophony of data and ask themselves what they really want to know. That is the approach taken in total return economics. The risk is that its definitions take scant account of measurability. The reward is logical congruence in the hope that ways of measurement might appear as the argument unfolds. Where measurement proves possible, TRE becomes a framework of surprising and testable inferences which hold up in tests.

The argument might have been simpler if the modern world were simpler. In fact there is free growth and probably waste consumption. Much of this complexity seems to have come with the trek from the ancestral savannah to farms and cities. The writer preferred not to wish any of it away with simplifying assumptions. Although I have taken shortcuts in modeling the stasis rate, the rules themselves depend on the prime assumptions alone. These in turn are no more than the conditions needed for markets to form and persist. TRE offers no bold hypotheses to test. What can be tested is the logic of its inferences from the prime assumptions. If the logic holds, the duplication rule and other TRE findings are general and unconditional.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>accounting profit</td>
<td>same as earnings or GAAP profit</td>
</tr>
<tr>
<td>accretion</td>
<td>gain in asset value due to shortening of discount periods as time passes</td>
</tr>
<tr>
<td>accretion principle</td>
<td>inference that capital value tends to rise as prospective yields from it draw nearer</td>
</tr>
<tr>
<td>adult</td>
<td>earning positive overall yield; refers to replaceable and depreciating assets of either factor</td>
</tr>
<tr>
<td>adult insumption</td>
<td>insumption after maturity</td>
</tr>
<tr>
<td>adult second output rule</td>
<td>second output rule for the set of adults</td>
</tr>
<tr>
<td>adult pay</td>
<td>average pay of all adult cohorts</td>
</tr>
<tr>
<td>adult span</td>
<td>period from maturity to final zero value; estimated at fifty years for modern humans</td>
</tr>
<tr>
<td>age-time independence</td>
<td>simplifying assumption that all cohorts show the same free growth rate at any given time, and that the thrift growth rate of any given cohort is invariate over time</td>
</tr>
<tr>
<td>agent expense</td>
<td>non-yield maint; compensation of workers which neither satisfies tastes nor is creation of value</td>
</tr>
<tr>
<td>bachelor period (B)</td>
<td>period between maturity and the mean age of reproduction</td>
</tr>
<tr>
<td>base insumption</td>
<td>aggregate insumption by non-adult cohorts</td>
</tr>
<tr>
<td>base insumption ratio</td>
<td>ratio of the maturity cohort’s thrift work to average insumption of non-adult cohorts</td>
</tr>
<tr>
<td>biological age</td>
<td>age from birth</td>
</tr>
<tr>
<td>capital (J)</td>
<td>present value of expected yield or cash flow</td>
</tr>
<tr>
<td>cash flow</td>
<td>plowback plus positive yield</td>
</tr>
<tr>
<td>CFC</td>
<td>Consumption of Fixed Capital; roughly property depreciation</td>
</tr>
<tr>
<td>CI</td>
<td>the doctrine that output equals investment plus consumption</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>CI Equation</td>
<td>doctrine that output equals the sum of consumption and investment</td>
</tr>
<tr>
<td>CIPP</td>
<td>doctrine that output equals consumption plus investment and that income equals pay plus profit</td>
</tr>
<tr>
<td>closure</td>
<td>economic closure; absence of external transactions</td>
</tr>
<tr>
<td>closure rule</td>
<td>output rule at the scale of closure; closure output equals closure growth plus closure exit</td>
</tr>
<tr>
<td>consumption (c)</td>
<td>property extinguishments plus services of both factors not recaptured in property value</td>
</tr>
<tr>
<td>consumption lag</td>
<td>tendency for consumption to occur later than internalization</td>
</tr>
<tr>
<td>convergence rules</td>
<td>rules predicting equilibration of outflow and yield</td>
</tr>
<tr>
<td>conversion efficiency</td>
<td>ratio of increase in investment and insumption to decrease in waste as a result of waste conversion</td>
</tr>
<tr>
<td>cutoff age</td>
<td>economic age at which adult insumption ends</td>
</tr>
<tr>
<td>cutoff return rule</td>
<td>rule deriving thrift return to post-cutoff cohorts</td>
</tr>
<tr>
<td>depreciation</td>
<td>loss of value due to time alone; see EXTINGUISHMENTS</td>
</tr>
<tr>
<td>discretionary pay</td>
<td>pay less work expense; equal to sold work</td>
</tr>
<tr>
<td>disinflation rule</td>
<td>rule for disinflating nominal rates of return</td>
</tr>
<tr>
<td>disinvestment</td>
<td>negative component in thrift growth of property; unthrifty in property</td>
</tr>
<tr>
<td>duplication rule</td>
<td>inference that maint equilibrates to senescence</td>
</tr>
<tr>
<td>earnings</td>
<td>book growth plus yield</td>
</tr>
<tr>
<td>economic age</td>
<td>age from first parental investment; estimated at biological age plus one year</td>
</tr>
<tr>
<td>economic lifespan (λ)</td>
<td>period from first investment received to final return to zero value; estimated at 76 years in modern humans</td>
</tr>
<tr>
<td>economic profit</td>
<td>earnings less rent value of property owned</td>
</tr>
<tr>
<td>equilibrium pay</td>
<td>pay sufficient to maintain population size</td>
</tr>
</tbody>
</table>
equivalence  the principle that value-added equals total return
exhaust  final outflow of value from the economy
expense  the share of sales not currently created by the seller
extinguishments  depreciation, depletion and amortization
factor  (1) factor of production; self-value or property; (2) multiplicand
factor index  ratio of self-value to total capital
final gift  gift simultaneously expended by the receiver in final yield
final tastes  tastes for final yield
final yield  exhaust of capital in satisfaction of tastes
final yield rate \((\phi_{\text{final}})\)  ratio of final yield to capital
flow  a quantity measured in dollar value per unit time
free goods  goods gained effortlessly
free growth \((\Pi)\)  growth due to rise in rate of return; growth times average or marginal productivity index; also called productivity growth
free growth rate \((g_{\Pi})\)  change in rate of return since an origin in stasis
free prop growth  free growth in property
free self-growth  free growth in self-value
GAAP  Generally Accepted Accounting Principles; same as accountant’s method
GAAP index \((\kappa)\)  ratio of growth rate reported in GAAP to mark-to-market growth rate
GAAP gap  one minus the GAAP capture index
GAAP profit  same as earnings
GAAP rule  rule deriving the GAAP index
GDP  Gross Domestic Product
generation length
average age difference between parents and offspring

gift closure
set of owners which experiences no net external gift or ingift

GNP
Gross National Product

gross product
GNP or GDP

growth \( (\dot{j}) \)
growth flow of capital

growth rate \( (g_j) \)
s ratio of growth in capital to the capital growing

growth model
simplifying assumptions of age / time independence and thrift offset

human capital \( \text{(H)} \)
traditional term for what TRE calls self-value

income
rights to output; equal to output

ingift
gift received

insumption \( (\iota) \)
thrift in self-value; conversion of consumption to self-value

insumption cutoff age
same as cutoff age

insumption fund model
derivation of thrift return under an assumption of Ricardian pay equilibrium

intactness index \( (C_{\text{im}}) \)
ratio of value of maturity cohort to aggregate insumption of non-adult cohorts

interim profit
in Ricardian analysis, thrift profit on interim property

interim property
in Ricardian analysis, property bought from discretionary pay

interim thrift profit
same as interim profit

investor
see ALTER EGO

last day parable
illustration that pay shrinks to work expense at redependence

leakage
fortuitous components in free growth
lifespan | same as economic lifespan
---|---
liquidations | recovery of extinguishments
maint | same as maintenance consumption
maint equations | inferences showing that maint exceeds sold work under thrift offset
maint index | ratio of maint to pay
maintenance | (1) same as maintenance consumption; (2) preservation; upkeep
maintenance consumption | share of consumption necessary to enable pay
maturity | the age at which parental ingift ends, estimated at 20 years in biological age for Americans today
maturity cohort | cohort currently at the age of maturity
maturity model | equation estimating thrift return as a ratio of the base insumption ratio to the non-adult span
maturity pay | pay of the maturity cohort
maximand | economic quantity maximized by owners
maximand rule | inference that all owners maximize risk-adjusted rate of return
mean age of reproduction (P) | same as generation length
NDP | Net Domestic Product
neg | a contraction for “negative”
neg work yield | same as negative work yield
neg yield | same as negative yield
neg prop yield | investment from ingift or from the owner’s own work yield
neg work yield | insumption from ingift or from the owner’s own property yield
neg yield | investment received from outside
net cash flow  
  cash flow less negative yield; equivalently plowback plus yield

net insumption  
  thrift growth of self-value; insumption less maint

net insumption rule  
  net insumption equals insumption less maint

net thrift investment rule  
  net thrift investment equals thrift investment less disinvestment

net product  
  NNP or NDP

net product equation  
  same as CI equation

net thrift (Θ)  
  same as thrift growth; thrift less unthrift

net thrift investment  
  thrift growth of property; thrift investment less disinvestment

NIPA  
  the National Income and Product Accounts, also called the National Income Accounts

NNP  
  Net National Product

non-adult  
  younger than the age of maturity

non-adult span  
  period from first investment (economic birth) to maturity

non-yield expense  
  work expense which does not satisfy final tastes

numeraire  
  unit or standard of capital value

output (y)  
  current creation of value; total return; growth in capital plus yield

output rate  
  same as rate of return

output rule  
  same as total return rule

owner  
  the individual economic actor; see ALTER-EGO

outflow  
  decapitalization; extinguishments plus yielded output

outgift  
  positive component in gift

parental investment  
  same as parental gift

parental gift  
  gift from adults to non-adults regardless of relatedness
pay (σ)  
see WORK SALES

pay / age profiles  
data showing pay at successive worker ages

physical capital  
traditional term for what TRE calls property

plowback  
the share of sales reinvested in the asset of either factor generating the sales

plowback insumption  
a worker’s insumption bought from his own pay

plowback investment  
investment from an owner’s property sales in his own property

pos  
in equations, a contraction for “positive”

pos prop yield  
same as positive property yield

pos work yield  
same as positive work yield

pos yield  
same as positive yield

positive property yield  
property sales less plowback investment plus property yield in kind

positive work yield  
pay less plowback insumption

positive yield  
decapitalization for reinvestment or taste satisfaction in gift plus final yield

PP  
the doctrine that income equals pay plus profit

PP Equation  
document that income equals the sum of pay and profit

present value rule  
inference that capital value equals the integral of expected yields discounted at the expected rate of return

prime assumptions  
minimum conditions needed for markets to form and persist; assumption of a generationally replenishable population plus near-agreement in tastes plus good foresight over large averages

prod index  
same as productivity index

productivity  
same as rate of return

productivity index  
ratio of gain in rate of return to growth rate since a past moment of zero growth
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td><strong>profit</strong> (p)</td>
<td>output of property; growth in property plus property yield</td>
</tr>
<tr>
<td><strong>profit rate</strong> (r&lt;sub&gt;k&lt;/sub&gt;)</td>
<td>rate of return to property; ratio of profit to property value</td>
</tr>
<tr>
<td><strong>profit rule</strong></td>
<td>total return rule for property; profit equals property growth plus property yield</td>
</tr>
<tr>
<td><strong>prop</strong></td>
<td>in equations, a contraction for property</td>
</tr>
<tr>
<td><strong>property</strong> (K)</td>
<td>capital whose expense does not satisfy owner tastes and hence is deducted from sales in calculating cash flow</td>
</tr>
<tr>
<td><strong>prop cash flow</strong></td>
<td>plowback investment plus positive property yield</td>
</tr>
<tr>
<td><strong>prop expense</strong></td>
<td>share of property sales produced earlier or by others than the seller or both</td>
</tr>
<tr>
<td><strong>prop sales</strong></td>
<td>sales of property or of property yield</td>
</tr>
<tr>
<td><strong>prop yield</strong></td>
<td>yield from property; property sales less plowback investment plus negative property yield plus property yield in kind</td>
</tr>
<tr>
<td><strong>pure pay</strong></td>
<td>same as work cash flow</td>
</tr>
<tr>
<td><strong>pure senescence rate</strong></td>
<td>rate of senescence to self-value</td>
</tr>
<tr>
<td><strong>rate</strong></td>
<td>a pure number divided by time; normally a flow divided by the capital value generating it</td>
</tr>
<tr>
<td><strong>rate of return</strong> (r)</td>
<td>in TRE, ratio of output of either or both factors to capital employed</td>
</tr>
<tr>
<td><strong>realized output</strong></td>
<td>share of output sold; sales less expense</td>
</tr>
<tr>
<td><strong>realized output rule</strong></td>
<td>inference that realized output equals sales less expense</td>
</tr>
<tr>
<td><strong>realized profit</strong></td>
<td>share of profit sold; property sales less property expense</td>
</tr>
<tr>
<td><strong>redependence</strong></td>
<td>age at which self-value, meaning present value of expected future work yield, returning to zero; estimated at biological age 76 for modern Americans</td>
</tr>
<tr>
<td><strong>Ricardian equilibrium</strong></td>
<td>inference that pay should be just sufficient to allow zero thrift growth of self-capital</td>
</tr>
</tbody>
</table>
Ricardian pay: Pay just sufficient to maintain the worker population in the absence of free growth.

Ricardo model: Inference that thrift work tends to be less than senescence at the scale of closure under Ricardian equilibrium.

Ricardo, David (1772-1823): English economist.

Risk: Standard deviation in rate of return.

Risk parity: Simplifying assumption that the two factors are equal on average in risk and return at the scale of closure.

Sales ($\sigma$): Transactions; transfers or services of value to others or to alter-egos whether compensated or not. Sales includes rent and interest received.

Second duplication rule: Inference that self-value equals expected aggregate maintenance less aggregate insumption over remaining lifespan.

Second output rule: Inference that thrift output equals net thrift investment plus consumption plus gift less maint.

Self-insumed work: Same as unsold thrift work.

Self-liquidation: Recovery of senescence in final yield.

Self-value ($H$): Capital whose expense, called maintenance consumption, satisfies owner tastes and hence is paid from yield.

Senescence ($\delta$): Unthriftness in self-value; attrition in self-value due to age before any offsets of insumption or free growth.

Senescence rate: Ratio of senescence to capital at the scale of closure.

Sold work: Share of work sold; pay less maint.

Sold work rule: Realized output rule for self-value; inference that sold work equals pay less maint.

Stasis: Zero economic growth.

Stasis rate ($r_0$): Rate of return in a chosen moment of stasis.

Stasis final yield rate ($\phi_{\text{final},0}$): Final yield rate in a chosen moment of stasis.

Stasis waste rate ($\phi_{\text{waste},0}$): Waste rate in a chosen moment of stasis.
stock

capital; a quantity measured in dollars alone

tautology

logic of broad application

thrift

the positive component in thrift growth

thrift effect

thrift growth rate times conversion efficiency

thrift growth ($\theta$)
growth due to lower yield rate; growth less free growth; growth times average or marginal thrift index; same as net thrift

thrift growth rate ($g_\Theta$)
decline in yield rate since an origin in stasis

thrift index

ratio of reduction in yield rate to growth rate since an origin in stasis

thrift investment

positive component in thrift growth of property

thrift leverage rule

rule deriving thrift effect

thrift offset

imaginary condition where overall thrift growth is zero at the scale of closure

thrift offset principle

simplifying assumption of thrift offset; inference that this assumption is reasonably realistic

thrift output ($y_\Theta$)
output less free growth; thrift growth plus yield

thrift profit

thrift output of property; thrift growth in property plus property yield

thrift work ($w_\Theta$)
thrift output of self-value; thrift growth in self-value plus work yield

total capital ($J$)
same as capital; self-value plus property

total cash flow

work cash flow plus property cash flow

total return ($y$)
growth plus yield; same as output

total return rule

inference that output (creation of value) equals capital growth plus yield

total yield

an owner’s net yield from both factors available for his taste satisfaction in gift plus exit
total yield rule inference that yield to an owner from both his factors combined equals his maintenance consumption plus waste plus gift

transfer yield yield other than final yield; reinvestment plus gift

transfer tastes tastes for making transfer yield

TRE total return economics

uniformity same free growth rate for both factors and all cohorts

unrealized consumption same as unsold thrift work

unrealized output output not sold; free growth plus unrealized thrift output; includes plowback

unrealized profit profit not sold; free growth in property plus unrealized thrift profit

unrealized thrift profit directly self-invested output of a property asset as distinct from plowback investment

unsold thrift work directly self-insumed work such as exercise and learning by the young

unsold work work not sold; free growth in self-value plus unsold thrift work

unthrift (θ_ ) the negative component in thrift growth, but positive in sign

waste share of consumption recovered neither in sales nor in self-value, but in taste satisfaction alone

waste consumption same as waste

waste conversion thrift growth by means of waste reduction

waste conversion rule inference that gain in thrift growth, at the scale of closure, equals the product of waste reduction and conversion efficiency

waste index ratio of waste to stasis output

waste index rule rule deriving the waste index
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tr>
<td>waste rate ($\phi_{\text{waste}}$)</td>
<td>ratio of waste to capital</td>
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<tr>
<td>work ($w$)</td>
<td>output of self-value; growth in self-value plus work yield</td>
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<td>work cash flow</td>
<td>plowback insumption plus positive work yield; pure pay</td>
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<td>work expense</td>
<td>the worker’s cost recovered in pay</td>
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<td>work expense index</td>
<td>ratio of work expense to pay</td>
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<td>work rate ($r_w$)</td>
<td>rate of return to self-value; ratio of work to self-value</td>
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<tr>
<td>work rule</td>
<td>total return rule for workers; work equals worker growth plus work yield</td>
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<tr>
<td>work sales ($\sigma$)</td>
<td>sales by workers, equal to pay</td>
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<tr>
<td>work yield</td>
<td>yield from self-value; pure pay less plowback insumption plus negative work yield</td>
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<tr>
<td>worker</td>
<td>see ALTER-EGO</td>
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<td>worker value ($H$)</td>
<td>same as self-value</td>
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<tr>
<td>yield ($\eta, \phi_J$)</td>
<td>net transfer of value from an asset to its owner or his donees</td>
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<td>maint</td>
<td>share of work expense which satisfies tastes, and must be decapitalized as yield</td>
</tr>
<tr>
<td>yield rate ($\phi$)</td>
<td>ratio of yield to capital</td>
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<tr>
<td>yielded output</td>
<td>share of yield provided by output as distinct from liquidations</td>
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