12-6-2002

Personalized Pricing and Quality Differentiation on the Internet

Anindya Ghose  
*Carnegie Mellon University, aghose@andrew.cmu.edu*

Vidyanand Choudhary  
*University of California, Irvine, veecee@uci.edu*

Tridas Mukhopadhyay  
*Carnegie Mellon University, tridas@andrew.cmu.edu*

Uday Rajan  
*Carnegie Mellon University, urajan@umich.edu*

Follow this and additional works at: [http://services.bepress.com/roms](http://services.bepress.com/roms)

Recommended Citation  
Available at: [http://services.bepress.com/roms/vol2/iss1/paper3](http://services.bepress.com/roms/vol2/iss1/paper3)
Personalized Pricing and Quality Differentiation on the Internet

Abstract
No changes were made in the Abstract. Please use the previous Abstract that was submitted

Keywords
Vertical Differentiation, Personalization, Price Discrimination, Electronic Commerce
Personalized Pricing and Quality Differentiation on the Internet

Vidyanand Choudhary† Anindya Ghose‡ Tridas Mukhopadhyay§ Uday Rajan¶

GSIA
Carnegie Mellon University
Pittsburgh, PA 15213
20th May 2002

---

*We are grateful to seminar participants at WISE 2001 and Carnegie Mellon University, and to Kannan Srinivasan for helpful comments. All errors remain our own responsibility.

†Tel: (412) 268-4224, E-mail: veecee@uci.edu
‡Tel: (412) 268-5798, E-mail: aghose@andrew.cmu.edu
§Tel: (412) 268-2307, E-mail: tridas@andrew.cmu.edu
¶Tel: (412) 268-5744, E-mail: urajan@andrew.cmu.edu
Vidyand Choudhary is an Assistant Professor of MIS at the Graduate School of Management, University of California at Irvine. He received his Ph.D. in Management from Purdue University. His research interests are in the Economics of Information Systems, Impact of emerging technologies on firm’s business strategies, analytical modelling of electronic marketplaces, product differentiation and price discrimination, Inter-temporal pricing of software products and upgrades.

Anindya Ghose is a Doctoral candidate in Information systems GSIA, Carnegie Mellon University. His primary area of interests is in the application of Game Theory to Industrial Organization in areas such as Personalization in Online Business-to-Consumer commerce and modelling the impact of Internet Intermediaries on marketing channel structures.

Tridas Mukhopadhyay is Deloitte Consulting Professor of e-Business at Carnegie Mellon University. He is also the Director of the Institute for eCommerce and MSEC (Master of Science in Electronic Commerce) program at CMU. He received his Ph.D. in Computer and Information systems from the University of Michigan in 1987. His research interests include business-to-business commerce, Internet use at home, business value of information technology, and software development productivity. In addition, Professor Mukhopadhyay has worked with several organizations, such as General Motors, Ford, IBM, Diamond Technology Partners, Chrysler and Texas Instruments. He has been on the committee of several doctoral candidates in various disciplines, including marketing and information systems. He has published in several journals, including Management Science and Information Systems Research.

Uday Rajan is an Associate Professor of Economics and Finance at GSIA, Carnegie Mellon University. He received his Ph.D. in Economics from Stanford University. He has been on the committee of several doctoral candidates in various disciplines, including economics, finance, marketing, and information systems. His main research areas include Game Theory and applications of Game Theory to Finance and Industrial Organization. He has published in several journals, including the American Economic Review, the Journal of Economic Theory, and the Journal of Financial Economics.
Corresponding Author: Anindya Ghose, Phone: (412) 268-5798.
Abstract

In uncertain environments, e-tailers learn about the most profitable prices through price experimentation. Indeed, in this electronic environment online retailers can easily experiment with different prices to offer to different consumers. Retailers using the Internet as a medium for commerce can gather a remarkable wealth of information about their existing and potential customers, and hence better estimate a consumer’s reservation price. As Bakos (2001) reports, technology allows firms to identify and track individual consumers, both within an online store and across different websites. This leads to the creation and sharing of consumer profiles, matching of consumer identities with relevant demographic information and comparison with the preferences of similar consumers through various collaborative and content filtering techniques. Based on such information, the computing power of the Internet retailers’ web server can be used to deploy complex pricebots and algorithms to determine prices to approach first-degree price discrimination. Spurred partly by the low menu cost of changing prices on the Internet and partly as a response to consumer use of price-comparison shopbots, firms are exploring the idea of personalized prices for goods and services that are currently sold at posted prices.

Personalized pricing requires some knowledge of each consumer’s preferences, and an ability to charge different prices to different consumers. The price offered to a consumer whose valuation for a product is known may be higher or lower than the posted uniform price charged by firms who lack the sophistication to target individual consumers. In this paper, we use the term personalized pricing, or PP, to refer to technologies that facilitate such first-degree price discrimination. A retailer that invests in PP can identify individual consumers, infer consumer valuations, and determine a consumer’s willingness to pay for its product. This retailer can therefore offer a personalized price that may provide the consumer greater surplus relative to the potential surplus from competitors’ products.

We develop an analytical game-theoretic framework to investigate the competitive implications of such personalized pricing technologies (PP). Shaked & Sutton (1982) and Gabszewicz and Thisse (1986), building on research by Mussa and Rosen (1978), develop duopoly models of vertical differentiation. Moorthy (1988) extends the basic model by incorporating variable production costs and allowing consumers the opportunity to not buy a product. Our analysis extends Moorthy’s model by allowing that one, or both, firms can observe consumer valuations, and thereby engage in first-degree price discrimination.

We first show that, even though a monopolist makes a higher profit with PP, its optimal quality is the same with or without PP. However, the monopolist makes a higher profit with PP. Second, in a duopoly, unless a firm engages in product differentiation, information about
consumer preferences and valuations by itself does not provide any strategic advantage. Whether or not any of the firms have PP, the outcome is still Bertrand competition, and firms do not earn any profit. Hence, firms should not invest in this technology unless they can differentiate themselves from competitors.

We then consider a model of vertical product differentiation, and show how personalized pricing on the Internet affects firms’ choices of quality differentiation in a competitive scenario. First, if the firm with PP has a low quality, its optimal price is non-monotonic in consumers’ willingness to pay. That is, some high valuation consumers are offered lower prices than some low valuation ones. Second, when one firm adopts PP, the other firm responds by lowering its price. This is a competitive response: a firm with PP knows the valuation of each consumer, and can therefore charge prices as low as its own marginal cost to a specific consumer. It therefore encroaches into the market share of the other firm, which responds to the increased competition by reducing its price.

Third, when only one of the firms adopts PP, it is optimal for it to increase product differentiation. While it is optimal for the firm adopting PP to increase product differentiation, the non-PP firm seeks to reduce differentiation by moving in closer in the quality space. Fourth, we find that, when the PP firm has a high quality both firms raise their qualities, relative to the uniform pricing case. Conversely, when the PP firm has low quality, both firms lower their qualities. PP provides the low quality firm with an opportunity to penetrate an untapped market segment further to the left than where it presently is. Hence it lowers its quality to extend its reach in the direction of decreasing consumer type. As a competitive response to differentiate itself, the high quality firm initially moves to the right as long as moving away is relatively inexpensive due to low convexity of its costs. But when the costs start increasing at a much faster rate, the potential loss per unit of market share on the right is outweighed by the gains from moving to the left.

Fifth, depending on the convexity of the marginal cost function, we outline the incentives of firms to deploy such technologies. Our model shows it is an optimal strategy for the low quality firm to adopt PP, if the other firm does not. Regardless of whether the low quality firm has PP, the high quality firm should adopt PP only if the cost function is not too convex. Sixth, if both firms acquire PP, then both firms earn lower profits than in the case where neither firm has PP. Essentially, they are trapped in a prisoner’s dilemma.

Finally, consumer surplus falls (compared to the no PP case) if the PP firm has low quality, but rises if the PP firm has high quality. In fact, consumer surplus is highest when both firms have PP. This is due to the fact that since both firms can now price at marginal costs.
Thus this leads to the case of maximum market coverage and most intensified price competition. Thus, despite the threat of first-degree price discrimination, personalized pricing with competing firms can lead to an overall increase in consumer welfare. Thus our analysis offers interesting strategic insights for managers about how to address the competitive problems associated with personalized pricing and quality choices.

**Keywords:** Vertical Differentiation, Personalization, Price Discrimination, Electronic Commerce
1 Introduction

In uncertain environments, e-tailers learn about the most profitable prices through price experimentation. By its very nature, the Internet is well adapted to such a learning process. Indeed, in this electronic environment the menu costs of changing prices are negligible, and sellers can easily experiment with different prices to offer to different consumers. In the near future, firms will have the wherewithal to use the waves of personal profile and consumer and supplier activity data to set personalized prices. The state of the e-tailing industry now has put the consumer in control. He is able to surf from site to site to compare prices, or even use automated agents that ferret out the lowest prices, playing merchants against each other and forcing prices into the cellar. Such a situation may render the prevailing “one price fits all” model obsolete.

Interestingly, the Internet is a double-edged sword. Retailers using the Internet as a medium for commerce can also gather a remarkable wealth of information about their existing and potential customers, and hence better estimate a consumer’s reservation price. As Bakos (2001) reports, technology allows firms to identify and track individual consumers, both within an online store and across different websites. This leads to the creation and sharing of consumer profiles, matching of consumer identities with relevant demographic information and comparison with the preferences of similar consumers through various collaborative and content filtering techniques. Based on such information, the computing power of the Internet retailers’ web server can be used to deploy complex pricebots and algorithms to determine prices to approach first-degree price discrimination (Bailey, 1998). Spurred partly by the low menu cost of changing prices on the Internet and partly as a response to consumer use of price-comparison bots, firms are exploring the idea of personalized prices for goods and services that are currently sold at posted prices.

Personalized pricing requires some knowledge of each consumer’s preferences, and an ability to charge different prices to different consumers. The price offered to a consumer whose valuation for a product is known may be higher or lower than the posted uniform price charged by firms who lack the sophistication to target individual consumers. In this paper, we use the term personalized pricing, or PP, to refer to such first-degree price discrimination. A retailer that invests in PP can identify individual consumers, infer consumer valuations, and determine a consumer’s willingness to pay for its product. This retailer can therefore offer a personalized price that provides the consumer greater surplus relative to the potential surplus from competitors’ products. If the retailer’s
information indicates a particular consumer has a high reservation price, the price discrimination algorithm will increase the price for that consumer and vice versa.

There are many recent examples of personalized pricing among online retailers. In 1998-99, Books.com adopted a price discrimination strategy where different buyers were paying different prices for the same item based on their shopping behavior. A well-known example, of course, is Amazon.com, which varied prices to different consumers on its popular Diamond Rio MP3 player by up to $50 from the original $233 retail tag (Morneau, 2000). Later on, over a five-day period, Amazon offered discounts of twenty to forty percent off the list price on 68 of its 100 most popular DVD titles, which again differed by consumers. This promotion resulted in the same title being sold at a price ranging from $24 to $39.

One way for a retailer to engage in price discrimination is through intelligent agents dynamically inserting personalized discounts on pop-up windows on a consumer’s screen. Software for this is provided by, among others, iChoose, Dash, and zBubbles (Johnson, 2000). Chen & Iyer (2001) report that, in the North American long-distance telephone market, the major competitors (AT&T, MCI and Sprint) have been able to improve the sophistication of consumer databases that helps them to provide specialized discounts to a majority of the population. Further, Ford plans to move towards pricing its automobile financing products dynamically, based on consumer profiles and choices, and expects to cut its $10 billion spending on non-targeted promotions significantly (Aron, et al., 2001).

E-tailers are now using data-mining software and click-stream analysis products to track consumer behavior. Of late quite a few software companies have begun to offer personalized pricing within e-commerce products. Calico’s Dynamic Custom Price application enables sellers to offer personalized prices (www.calico.com). Zilliant software provides online businesses with real-time feedback on consumer behavior and competitive pricing to support personalized pricing decisions. Many firms believe that the concept of making the right offer to the right consumer will be the way of the future. In this paper we intend to examine the following questions. How does competition between online retailers, in the presence of intelligent agents and price bots that can extract buyer preferences and implement personalized pricing, affect equilibrium outcomes in a

---

1NetGenesis’ NetAnalysis, for example, gleans behavioral data from Web server log files, a network sniffer (software that sits on the network and logs traffic) and from Web server plug-ins.
competitive scenario? What are the variables of strategic interest? When do firms competing on the quality of value added services benefit from personalized pricing and what are the incentives for investing in such technologies for competing firms? Does the improvement in firms’ knowledge of individual consumers alleviate the pricing pressure on retailers or does it intensify price competition in the industry such that all competing firms become worse off?

We consider these questions in a duopoly framework in which one or both firms can perfectly identify valuations of heterogenous consumers.\textsuperscript{2} Recent work on price discrimination and customization includes Ulph and Vulkan (2001), who find that a firm that first-degree price discriminates is also better off if it mass-customizes. In a monopoly setting, Aron, et al., (2001) analyze the pricing, profitability and welfare implications of agent-based technologies that engage in pricing based upon product preference information revealed by consumers. In the context of consumer addressability, where firms can reach individual customers, Chen and Iyer (2001) find that when product differentiation and the cost of incremental addressability become small, firms strategically differentiate in their choice of addressability to mitigate destructive competition. Chen, et al., (2001), have shown that mistargeting can soften price competition in the market, and qualitatively change the incentives for competing firms engaged in individual marketing.

We derive a number of analytical results on firm pricing and quality differentiation, and on consumer welfare, when one or both firms have PP. First, the optimal quality for a monopolist is the same with or without PP. However, the monopolist makes a higher profit with PP. Second, in a duopoly, unless a firm engages in product differentiation, information about consumer preferences and valuations by itself does not provide any strategic advantage. Whether or not any of the firms have PP, the outcome is still Bertrand competition, and firms do not earn any profit. Hence, firms should not invest in this technology unless they can differentiate themselves from competitors. Third, if the firm with PP has a low quality, its optimal price is non-monotonic in consumers’ willingness to pay. That is, some high valuation consumers are offered lower prices than some low valuation ones. Fourth, when one firm adopts PP, the other firm responds by lowering its price. This is a competitive response: a firm with PP knows the valuation of each consumer, and can therefore charge prices as low as its own marginal cost to a specific consumer. It therefore

\begin{footnote}{2This assumption is made in order to provide benchmark results. Future potential notwithstanding, current technologies can only approximately determine consumer valuations.}

3

Published by Berkeley Electronic Press Services, 2002
encroaches into the market share of the other firm, which responds to the increased competition by reducing its price. Fifth, when only one of the firms adopts PP, it is optimal for it to increase product differentiation. This can be interpreted as a move to reduce competition with the other firm.

In addition to the above results, for a wide range of cost parameters, we demonstrate some properties of firm profit and consumer surplus with PP. First, within this range, it is a dominant strategy for the low quality firm to acquire PP. That is, regardless of whether the high quality firm acquires PP or not, the low quality firm makes a higher profit with PP. Conversely, the high quality firm should acquire PP only if the costs of quality are not too steep. Next, if both firms acquire PP, then both firms increase profits (if costs are not too convex). However, if marginal costs sharply increase in quality, then both firms earn lower profits compared to the case where neither has PP. Essentially, they are trapped in a prisoner’s dilemma. Finally, consumer surplus falls (compared to the no PP case) if the PP firm has low quality, but rises if the PP firm has high quality. In fact, consumer surplus is highest when both firms have PP.

The rest of the paper is organized as follows. Section 2 describes the model in detail and analyzes a monopolist’s choices. Section 3 provides the intuition behind why firms would sell heterogeneous products and thus lays the groundwork for further analysis of quality based competition. Section 4 presents a preliminary result that acts as a benchmark for comparative statics. Section 5 considers the cases of the PP firm choosing a low and a high quality respectively. That is, we consider two equilibria, with the PP firm being the lower quality firm in one equilibrium, and the higher quality firm in the other. We then proceed to Section 6 to analyze the equilibrium when both firms have PP. In Section 7 we provide some interesting observations using numerical analysis. We then discuss some implications of our findings, with some concluding remarks in Section 8. All proofs are relegated to the Appendix in Section 9.

---

3Shafer and Zhang (1995) had a similar result in their model of coupon targeting.
4We do not consider the question of which equilibrium will emerge. In our model, neither firm has the option of forcing the other into a particular equilibrium.
2 Model

We consider vertical differentiation in a personalized pricing context. Shaked & Sutton (1982) and Gabszewicz and Thisse (1986), building on research by Mussa and Rosen (1978), develop duopoly models of vertical differentiation. These papers have shown that the strategic effect of the desire to reduce price competition results in a product equilibrium where firms seek maximal product differentiation. Moorthy (1988) extends the basic model by incorporating variable production costs and allowing consumers the opportunity to not buy a product. This results in less than maximal product differentiation.

Firms compete in both the quality and price of the products they offer. Formally, we model their competition as a three-stage game. At the first stage, firms simultaneously choose the quality levels of their products. At stage 2, the two firms simultaneously choose their prices. Finally, at the last stage, consumers decide which, if any, product to buy.

Our analysis extends Moorthy’s model by allowing that one, or both, firms can perfectly observe consumer valuations, and thereby engage in first-degree price discrimination. Consistent with his model, whether firms have PP or not, the pure strategy equilibria are characterized by one firm choosing a high quality (call this firm \( h \)), and the other one a low quality (call this firm \( l \)).

Consumers are modelled as utility maximizers. If a consumer purchases a product of quality \( q \) at price \( p \), his utility is \( U(\theta) = \theta q - p \), where \( \theta \in [0, 1] \). A consumer has positive utility for one unit only. The type parameter \( \theta \) indicates a consumer’s marginal valuation for quality. For any given quality, a consumer with a higher \( \theta \) is willing to pay more for the product than one with a lower \( \theta \). If either of the two products offers a positive net utility, a consumer buys the one that maximizes his surplus. Otherwise, he chooses not to buy either product. As a benchmark, we consider the case of firms setting a single posted price (which we call “uniform pricing”) at stage 2. We then examine outcomes under personalized pricing (PP), or first-degree price discrimination.

Consistent with prior literature, we assume that firms have a marginal cost for production which is invariant with the quantity, but depends on the quality of the product. That is, both firms have the same cost function, but, depending on the quality levels they choose, their marginal costs may

---

5 Armstrong & Vickers (2001) provide an elegant framework that incorporates much of the earlier work on price competition in an environment with multiple firms. In a model of horizontal differentiation, Bhaskar & To (2002) find that, with perfect price discrimination and free entry, there is excessive entry.
differ in equilibrium.

**Assumption 1**  
(i) Each firm has a constant marginal cost for producing the good, denoted by $c$.  
(ii) $c(\cdot)$ is twice differentiable, strictly increasing and strictly convex in $q$. That is, $c' > 0$ and $c'' > 0$.

Quality in this model is a broad notion that encompasses any features that may affect a consumer’s willingness to pay for a good. These could include features intrinsic to the product itself (such as durability and functionality) and those related to the quality of the online shopping experience or the service level provided by the firm (such as warranties, delivery times, and consumer service). Quality is observed perfectly by all consumers at no extra cost.

Although cross-merchant product comparisons are a threat to merchant profitability, they are characteristic of the retail marketplace and are here to stay. Knowing this, retailers add value to manufacturers’ products to distinguish themselves from their competitors. These value-added services include extended warranties, forgiving return policies on defective items, special gift services, superior customer service and support contracts, fast delivery times with low costs, cross-manufacturer product configurations, and so on. Depending on the product, these value-added services can be critical to a consumer’s buying decision regardless of the manner of shopping.

Given the quality levels and prices offered by the two firms, consumers make their choices. Suppose, in the benchmark case of uniform pricing, firm 1 offers $(q_1, p_1)$, and firm 2 offers $(q_2, p_2)$. There will be a subset of consumers (possibly null) who buy from each of firms 1 and 2. The profit of firm $j$ is its market coverage times $(p_j - c(q_j))$. In the case of PP, we allow one or both firms to be equipped with a technology that perfectly reveals the consumer’s type before the price is disclosed to the consumer. While the firm offers the same quality product to all consumers, it can choose a customized price, and hence engage in first-degree price discrimination. In this case, a firm’s profit from consumer $\theta$ is $(p(\theta) - c(q_j))$.

In practice, a personalized pricing technology of this nature is likely to incur some fixed costs. Most such technologies involve developing software to employ intelligent agents to infer consumer valuations, as well as algorithms to provide personalized prices based on the estimated valuations. However, such costs are independent of the quality of the product being offered by the firm, and hence are fixed costs when considering the firm’s choice of quality. For simplicity, in this model,
we treat these costs as zero. Adding a fixed cost does not change the qualitative nature of our results. However, depending on the nature of the variable costs, we provide guidelines as to when firms should or should not invest in PP.

We consider pure strategy subgame-perfect equilibria of this three-stage game. That is, for any strategies the firms may choose at stages 1 and 2, consumers behave optimally at stage 3. Firms, in turn, not only anticipate this behavior, but also choose optimal prices, given quality levels, at stage 2. Before considering the duopoly case, we first consider the effect of PP on a monopolist’s choice of quality.

### 2.1 Monopoly Case

Consider first, a monopoly with uniform pricing. Let \( q^m_n \) and \( p^m_n \) be the quality and price, respectively, offered by the firm in this case where the superscript \( n \) refers to the fact that the firm does not have PP and the subscript \( m \) denotes the fact that it is a monopolist. Define \( \theta^m_n = \frac{p^m_n}{q^m_n} \). Then, consumers with types \( \theta \geq \theta^m_n \) will buy the product (because this leads to higher utility than not consuming) and those with types \( \theta < \theta^m_n \) will not. The monopolist’s profit function, therefore, is

\[
\pi^m_n(p^m_n, q^m_n) = (1 - \theta^m_n)(p^m_n - c(q^m_n)).
\]

Next, define \( q^d_m \) and \( p^d_m \) as the quality level and price, respectively, offered by a monopolist with PP where the superscript \( d \) denotes the fact that the monopolist has PP technology. Since this firm observes consumer types before choosing its price, \( p^d_m \) will be a function of consumer type, \( \theta \). Since marginal cost is constant in sales volume, a product sold to one consumer will have no effect on the price to any other consumer. Hence the firm charges each consumer his entire surplus from consuming the good. That is, \( p^d_m(\theta) = \theta q^d_m \). This price function is, trivially, increasing in \( \theta \): higher consumer types pay higher prices.

Further, the firm is willing to price as low as marginal cost, \( c(q^d_m) \) to persuade a consumer to buy the product. At this price, the lowest consumer type willing to buy is \( \theta^d_m = \frac{c(q^d_m)}{q^d_m} \). All types higher than \( \theta^d_m \) buy the product, and pay the price \( p^d_m(\theta) = \theta q^d_m \). Hence, the profit function of the PP monopolist is

\[
\pi^d_m(q^d_m) = \int_{\theta^d_m}^{1} (p^d_m(\theta) - c(q^d_m))d\theta = \int_{\theta^d_m}^{1} (\theta q^d_m - c(q^d_m))d\theta
\]
We first show that, regardless of the availability of PP, a monopolist firm chooses the same quality. This result immediately implies that a monopolist with PP earns a higher profit than a monopolist without PP. In fact, at the same quality, a monopolist with PP will achieve a higher market share than one without PP. The ability to customize prices according to consumers’ willingness to pay ensures that the firm is now able to reach many more consumers than it could before.

Proofs for Proposition 1 and all other Propositions, are provided in the Appendix.

**Proposition 1** In equilibrium, regardless of PP, a monopolist sets the same quality level: \( q_m^{d*} = q_m^n \). Further, \( \pi_m^d = 2\pi_m^n \).

Increasing (decreasing) quality implies a trade off between increasing (decreasing) costs and decreased (increased) market penetration for the firm. Personalized Pricing gives it the ability to reach a previously untapped portion of the market, without changing its product quality. By pricing at marginal cost for the threshold consumer, the firm ensures that it is able to penetrate an additional market segment without incurring additional costs. This case provides a benchmark to the one in which one of two duopolists obtains a PP technology. As we show in the next section, in the latter case, qualities of both firms typically change in response to the availability of PP.

3 Duopoly with personalized pricing: No Quality Differentiation

We next turn to the duopoly case, with two firms in the market. We first show that the ability to price discriminate, by itself, is of no value unless firms also differentiate in quality.

Suppose that one or both firms have access to PP. Suppose the two firms have products of identical quality, so that \( q_1 = q_2 \). Then, regardless of access to personalized pricing, Bertrand competition is inevitable, and each firm earns zero profit.

**Proposition 2** Suppose both firms offer the same quality, so that \( q_1 = q_2 \). Then, in equilibrium, regardless of the availability of PP, \( p_1 = p_2 = c(q_1) \), so that each firm earns zero profit.

Thus, even when one of the firms possesses a technology of extracting consumer valuations and pricing accordingly (PP), it does not have any competitive advantage in the absence of some
form of product differentiation. This result implies that online retailers will choose not to indulge in personalized pricing, unless they can also provide value-added services to differentiate their product. Without product differentiation, retailers are reduced to competing at marginal cost, and are unable to earn a profit. We therefore turn to the case in which firms first choose the quality of their product, and then the price.

4 Differentiated Duopoly: Neither Firm has PP

As a benchmark case, we first assume that neither firm has access to PP. We call this case the no-PP case. As Moorthy shows, in any pure strategy equilibrium, the firms choose different quality levels. This feature continues to prevail when one or both firms have PP. As we have shown, if firms choose the same quality, both firms earn zero profit. Hence, each firms prefer quality differentiation, regardless of whether it has higher or lower quality than the other.

When there is no access to PP, firm $i$, $(i = h, \ell)$, chooses a quality $q^n_i$ and price $p^n_i$, where the superscript $n$ indicates that neither firm has PP. In equilibrium $q^*_h > q^*_\ell$ and $p^*_h > p^*_\ell$. Henceforth the superscripts $^l, ^h$ and $^b$ will indicate the scenarios when only one firm has PP and the PP firm chooses low quality or high quality or when both firms have PP.

The subgame-perfect equilibrium in this case is determined by backward induction, starting with stage 3. As shown by Moorthy (1988, Proposition 1, part 3), in equilibrium at stage 3, the firms share the market in the following manner.\(^6\) Consumers with valuations greater than a cutoff level $\theta^n_h$ and less than 1 purchase product $h$, and those with valuations between a second cutoff level $\theta^n_\ell$ and $\theta^n_h$ purchase product $\ell$. $\theta^n_h$ is defined by the consumer exactly indifferent between products $h$ and $\ell$, and $\theta^n_\ell$ by the consumer indifferent between product $\ell$ and not consuming at all. That is,

$$\theta^n_h q^n_h - p^n_h = \theta^n_\ell q^n_\ell - p^n_\ell,$$

or,

$$\theta^n_h = \frac{p^n_h - p^n_\ell}{q^n_h - q^n_\ell}.$$

$$\theta^n_\ell q^n_\ell - p^n_\ell = 0,$$

or,

$$\theta^n_\ell = \frac{p^n_h - p^n_\ell}{q^n_\ell}.$$

This situation is depicted in Figure 1 below.

\(^6\)Though Moorthy assumes quadratic costs, this result depends only on consumer preferences, and not on costs.
4.1 Price Competition at Stage 2

Therefore, we can write the profit function of firm $\ell$ as $\pi^\ell_n = (\theta^h_n - \theta^\ell_n)(p^\ell_n - c^\ell_n)$, and that of firm $h$ as $\pi^n_h = (1 - \theta^h_n)(p^n_h - c^n_h)$. Now, consider stage 2 of the game, after firms have chosen their respective qualities, $q^n_h$ and $q^n_\ell$. Let $c^n_h = c(q^n_h)$, and $c^n_\ell = c(q^n_\ell)$. We solve for the prices chosen by the firms at stage 2.

**Lemma 1** In equilibrium, the prices of the two firms are 
\[ p^*_h = \frac{q^n_h(2q^n_h - q^n_\ell) + c^n_h + 2c^n_\ell}{4q^n_h - q^n_\ell}, \]
\[ p^*_\ell = \frac{q^n_\ell(q^n_\ell + 2c^n_\ell) + q^n_h(c^n_h - q^n_\ell)}{4q^n_h - q^n_\ell}. \]

Given these prices, we now consider firms’ choices of quality levels at stage 1.

4.2 Quality Competition at Stage 1

At this stage, firms anticipate the prices they will choose at stage 2 (as a function of the qualities chosen at stage 1), and their resulting profits at stage 3. Profits for each firm are hence written as a function of quality levels $q^n_h$ and $q^n_\ell$ alone. Denote these profit functions as $\pi^n_h$ and $\pi^n_\ell$. In particular, suppose firm $h$ chooses a quality $q^n_h^*$, and firm $\ell$ chooses $q^n_\ell^*$. Then,

\[ \pi^n_h = \frac{c^n_h(-2q^n_h + q^n_\ell) + q^n_h(2q^n_h - 2q^n_\ell + c^n_\ell)^2}{(q^n_h - q^n_\ell)(4q^n_h - q^n_\ell)^2}, \]
\[ \pi^n_\ell = \frac{q^n_\ell(q^n_\ell(q^n_\ell - q^n_h + c^n_\ell) + (c^n_\ell(-2q^n_h + q^n_\ell)^2))}{q^n_\ell(q^n_h - q^n_\ell)^2(4q^n_h - q^n_\ell)^2}. \]

The first-order condition for firm $h$, $\frac{\partial \pi^n_h}{\partial q^n_h} = 0$, defines a reaction function for firm $h$; i.e., it determines the optimal quality level of firm $h$ as a function of $q^n_\ell$. Similarly, the first-order condition
for firm $\ell$, $\frac{\partial \pi^n_\ell}{\partial q^n_\ell} = 0$, denotes the reaction function of firm $\ell$. The equilibrium quality levels, $q^n_{h*}$ and $q^n_{\ell*}$, are determined by simultaneously solving these two equations.

In the next section, we consider the case in which one firm has PP, and compare the resulting quality levels to this benchmark case.

5 Duopoly with personalized pricing: Only one Firm has PP

We now consider the situation in which one firm has access to PP i.e., it can infer consumer valuations and form a perfect estimate of each consumer’s willingness to pay for its product. As Proposition 2 suggests, there is no pure strategy subgame perfect equilibrium in which both firms choose the same quality level, since this results in zero profits for both. Instead, each firm prefers to have a different quality (either higher or lower). Hence, there are two equilibria in this case; one in which the PP firm chooses a lower quality than the other firm, and a second one in which the PP firm chooses a higher quality. We consider each of these equilibria in turn.

5.1 PP Firm Offers Low Quality

We denote the equilibrium qualities in this case as $q^\ell_{h*}$ and $q^\ell_{h*}$, with $p^\ell_{h*}$ and $p^\ell_{h*}(\theta)$ denoting the equilibrium prices. In this case, firm $\ell$ knows the type of each consumer, and hence can offer prices that depend on $\theta$. In equilibrium, it must be willing to offer a price as low as its marginal cost, $c^\ell = c(q^\ell)$, to each consumer, if necessary. Further, consistent with price discrimination, it will charge as high a price as it can from each consumer it sells to. As before, we solve this game by backward induction. At stage 3, firm $h$ (which does not have PP in this case) will operate in a market segment $[\theta^h_{h}, 1]$, and firm $\ell$ in a market segment $[\theta^\ell_{h}, \theta^\ell_{h}]$.

Consider first the location of the marginal consumer $\theta^\ell_{h}$, who is indifferent between buying from either firm. This consumer must obtain the same utility from either product. If $p^\ell(\theta^\ell_{h}) > c^\ell$, then firm $\ell$ would lower its price for this consumer, to ensure that he strictly prefers to buy product $\ell$. Hence, it must be that $p^\ell(\theta^\ell_{h}) = c^\ell$. Therefore, this consumer is defined by

$$\theta^\ell_{h} q^\ell_{h} - p^\ell_{h} = \theta^\ell_{h} q^\ell_{\ell} - c^\ell, \quad \text{or} \quad \theta^\ell_{h} = p^\ell_{h} \frac{q^\ell_{\ell} - c^\ell}{q^\ell_{h} - q^\ell_{\ell}}.$$

Similarly, $\theta^h_{h}$ is defined by the consumer who is indifferent between buying product $\ell$ and not consuming at all. Again, it must be that $p^h(\theta^h_{h}) = c^\ell$, else firm $\ell$ could increase its profit by reducing
its price for this consumer. Hence,
\[ \theta^\ell q^\ell - c^\ell = 0, \quad \text{or} \quad \theta^\ell = \frac{c^\ell}{q^\ell}. \]

We show that the equilibrium price function of firm \( \ell \) is non-monotonic in consumer type; that is, it charges some high valuation consumers less than it charges some low valuation consumers. Define \( \hat{\theta} = \frac{p^\ell}{q_h} \).

**Proposition 3** In equilibrium, at stage 2, firm \( h \) charges \( p^h = \frac{1}{2} (q^h - q^\ell + c^h + c^\ell) \). For consumers in the range \([\theta^\ell, \hat{\theta}]\), firm \( \ell \) sets \( p^\ell(\theta) = \theta q^\ell \), and for those in the range \((\hat{\theta}, \theta^h]\), it sets \( p^\ell(\theta) = p^h - \theta(q^h - q^\ell) \).

This situation is depicted in Figure 2.

![Figure 2: Prices of \( \ell \) and \( h \) when \( \ell \) alone has PP](image)

The intuition for the non-monotonicity of \( p^\ell(\theta) \), is that in the market segment \([0, \hat{\theta}]\), firm \( \ell \) faces no competition from firm \( h \). These consumers are not willing to buy product \( h \) at the offered quality and price. Hence, firm \( \ell \) is able to extract their entire consumer surplus, and consumers in this range are left with no surplus. However, consumers in the range \([ \hat{\theta}, 1]\) obtain a positive utility from consuming product \( h \) as well. Hence, firm \( \ell \) faces competition in this range, and must offer consumers at least as high a surplus as firm \( h \), to induce them to buy product \( \ell \). Thus, these consumers have a positive surplus that is monotonically increasing in consumer type.
Substituting in the optimal price of firm \( h \), the equilibrium price schedule for firm \( \ell \) is

\[
p^\ell_r(\theta) = \begin{cases} \theta q^\ell_r & \text{if } \theta \in [\theta^\ell_r, \hat{\theta}] \\ \frac{1}{2}(q^h_{r,\ell} - q^\ell_r + c^\ell_r + c^h_r) - \theta(q^h_{r,\ell} - q^\ell_r) & \text{if } \theta \in (\hat{\theta}, \theta^h_r] \end{cases}
\]  

(1)

Now, consider the choice of qualities at stage 1. Suppose firm \( \ell \) chooses \( q_r \), and firm \( h \) chooses \( q_h \). Further, suppose firm \( h \) chooses \( p^h_r \) optimally (as given by Proposition 1), given the two qualities. Then, the profit function of firm \( \ell \) is

\[
\pi^\ell_r(q^h_r, q^\ell_r) = \int_{\theta^\ell_r}^{\hat{\theta}} (\theta q^\ell_r - c^\ell_r)d\theta + \int_{\theta^\ell_r}^{\theta^h_r} \left( \frac{1}{2}(q^h_{r,\ell} - q^\ell_r + c^\ell_r + c^h_r) - \theta(q^h_{r,\ell} - q^\ell_r) - c^\ell_r \right)d\theta = \frac{(p^h_r q^h_{r,\ell} - q^h_r q^\ell_r)^2}{2(q^h_{r,\ell} - q^\ell_r)^2 q^h_r q^\ell_r}.
\]  

(2)

The first-order condition for firm \( \ell \), therefore, is \( \frac{\partial \pi^\ell_r}{\partial q^\ell_r} = 0 \). Recalling that \( p^h_r \) is also a function of \( q^\ell_r \), this yields

\[
\frac{(p^h_r q^h_{r,\ell} - q^h_r q^\ell_r)(c^h_r q^\ell_r(q^h_{r,\ell} - 2q^\ell_r) + q^h_r(p^h_r q^h_{r,\ell} - (q^h_{r,\ell} - q^\ell_r)(2(c^\ell_r)'q^h_{r,\ell} + q^\ell_r - (c^\ell_r)'q^\ell_r)))}{2(q^h_{r,\ell} - q^\ell_r)^2 (q^\ell_r)^2 q^h_r} = 0
\]  

(3)

Let \( \psi^\ell_r(q^h_r, q^\ell_r) \) denote the left-hand side of the above equation.

Since \( p^h_r > c^h_r \), and \( q^h_r > q^\ell_r \mapsto \frac{c^h_r}{q^h_r} > \frac{c^\ell_r}{q^\ell_r} \) (since \( c(\cdot) \) is convex), the optimal quality \( q^\ell_r \) is given by the solution to

\[
c^\ell_r q^\ell_r(q^h_{r,\ell} - 2q^\ell_r) + q^\ell_r(p^h_r q^h_{r,\ell} - (q^h_{r,\ell} - q^\ell_r)(2(c^\ell_r)'q^h_{r,\ell} + q^\ell_r - (c^\ell_r)'q^\ell_r)) = 0
\]  

(4)

The solution to this equation yields the reaction function of firm \( \ell \). Denote this by \( r^\ell_r(q^h_r) \).

Similarly, the profit function of firm \( h \) is

\[
\pi^h_r(q^h_r, q^\ell_r) = (p^h_r - c^h_r)(1 - \theta^h_r(p^h_r, q^h_r, c^\ell_r, q^\ell_r)) = \frac{(q^h_{r,\ell} - q^\ell_r - c^\ell_r + c^h_r)^2}{2(q^h_{r,\ell} - q^\ell_r)^2}
\]  

The corresponding first-order condition is \( \frac{d \pi^h_r}{d q^h_r} = 0 \), or

\[
\frac{(q^h_{r,\ell} - q^\ell_r - c^\ell_r + c^h_r)(q^h_{r,\ell} - q^\ell_r + c^\ell_r - c^h_r - 2(q^h_{r,\ell} - q^\ell_r)(c^\ell_r)'\)q^h_{r,\ell} + q^\ell_r - (c^\ell_r)'q^\ell_r}{4(q^h_{r,\ell} - q^\ell_r)^2 (q^\ell_r)^2} = 0
\]  

(5)

Let \( \psi_h(q^h_r, q^\ell_r) \) denote the left-hand side of this equation.

Since \( q^h_r > q^\ell_r \), it cannot be that \( (q^h_{r,\ell} - q^\ell_r - c^\ell_r + c^h_r) = 0 \). Hence, the optimal quality of firm \( h \), \( q^h_r \) is given by the solution to

\[
q^h_{r,\ell} - q^\ell_r + c^\ell_r - c^h_r - 2(q^h_{r,\ell} - q^\ell_r)(c^\ell_r)' = 0
\]  

(6)
Let \( r_h^\ell(q^\ell_h) \), the solution to this equation, denote the reaction function for firm \( h \).

Denote \( (c^\ell_h)' = c'(q^\ell_h) \) and \( (c^\ell_{\ell})' = c'(q^\ell_{\ell}) \). First, we show that, each of these derivatives must be that less than 1 in equilibrium. This will be useful in showing properties of the reaction and profit functions in the next two results.

**Lemma 2** *At the equilibrium qualities, \((c^\ell_h)' < 1\), and \((c^\ell_{\ell})' < 1\).*

We can now demonstrate that the reaction functions of both firms are upward sloping. That is, if firm \( \ell \) increases its quality, firm \( h \) should raise its quality, and vice versa.

**Proposition 4** *The reaction functions of both firms are upward-sloping; that is, \( \frac{dr^\ell}{dq^\ell_h} > 0 \) and \( \frac{dr^\ell_{\ell}}{dq^\ell_{\ell}} > 0 \).*

We show that, when firm \( \ell \) acquires PP, the competitive response of firm \( h \) is to reduce its price. PP allows firm \( \ell \) to price as low as marginal cost to a particular consumer, to induce him to buy product \( \ell \). This leads to an immediate increase in the market share of firm \( \ell \), both amongst low valuation consumers, and those who were previously buying product \( h \). In response to this heightened competition from firm \( \ell \), firm \( h \) reduces its price. This response of firm \( h \), in turn, induces firm \( \ell \) to lower its own quality, to reduce the competition with firm \( h \).

**Proposition 5** *Suppose both firms offer the no-PP qualities, \( q^n_h, q^n_{\ell} \). If firm \( \ell \) now acquires PP, compared to the no-PP case, (i) firm \( h \) charges a lower price: \( p^\ell_h < p^n_h \), and (ii) if firm \( h \) remains at its original quality, \( q^n_h \), then firm \( \ell \) lowers its quality.*

Of course, in equilibrium, both firms change their qualities from the no-PP case. We expect the price of firm \( h \) to be lower, and the quality of firm \( \ell \) to be lower. What quality does firm \( h \) choose?

**Observation 1** *Given a cost function of the nature \( c(q) = q^\alpha \), if the cost function is not too convex (in particular, \( \alpha \leq 1.2 \)), firm \( h \) chooses a higher quality in equilibrium. Conversely, if the cost function is highly convex (\( \alpha > 1.2 \)), it chooses a lower quality.*

This is demonstrated in Figure 3 below.
PP provides firm \( \ell \) with an opportunity to penetrate an untapped market segment further to the left than where it presently is. Hence it lowers its quality to extend its reach in the direction of decreasing consumer type. As a competitive response to differentiate itself, firm \( h \) initially moves to the right as long as moving away is relatively inexpensive due to low convexity of its costs. But when the costs start increasing at a much faster rate, the potential loss per unit of market share on the right is outweighed by the gains from moving to the left. By moving towards the low quality firm, \( h \) increases the uncontested portion of its market share on the right where it faces no competition from \( \ell \). Thus, for a wide range of \( \alpha \), both firms reduce their qualities when the PP firm chooses a low quality. Further, to give consumers a positive surplus and remain competitive, firm \( h \) also reduces its price, after lowering its quality. We show in Section 7 that the non-monotonicity of \( \ell \)'s price also implies that consumers are worse off than in the no-PP case.

5.2 PP Firm Offers High Quality

We denote the equilibrium qualities in this case as \( q^h_* \) and \( q^\ell_* \), with \( p^h_\theta(\theta)^* \) and \( p^\ell_* \) denoting the equilibrium prices. In this case, firm \( h \) knows the type of each consumer, and is hence willing to
price as low as \( p_h(\theta) = c_h^h \) if need be.

At stage 3, firm \( \ell \) (which does not have PP in this case) will operate in a market segment \([\theta_h^h, \theta_h^h]\), and firm \( h \) in a market segment \([\theta_h^h, 1]\). Consider first the location of the marginal consumer \( \theta_h^h \), who is indifferent between buying from either firm. This consumer must obtain the same utility from either product. If \( p_h^\ell(\theta_h^h) > c_h^h \), then firm \( \ell \) would lower its price for this consumer, to ensure that he strictly prefers to buy product \( \ell \). Hence, it must be that \( p_h^\ell(\theta_h^h) = c_h^h \). Therefore, this consumer is defined by

\[
\theta_h^h q_h^h - c_h^h = \theta_h^\ell q_h^\ell - p_h^\ell, \quad \text{or} \quad \theta_h^h = \frac{c_h^h - p_h^\ell}{q_h^\ell - q_h^h}.
\]

Similarly, \( \theta_h^\ell \) is defined by the consumer who is indifferent between buying product \( \ell \) and not consuming at all. Again, it must be that \( p_h^\ell(\theta_h^\ell) = c_h^\ell \), else firm \( \ell \) could increase its profit by reducing its price for this consumer. Hence,

\[
\theta_h^\ell q_h^\ell - c_h^\ell = 0, \quad \text{or} \quad \theta_h^\ell = \frac{c_h^\ell}{q_h^\ell}.
\]

Consider firm \( \ell \)'s profit function at stage 2: \( \pi_h^\ell = (p_h^\ell - c_h^\ell)(\theta_h^h - \theta_h^\ell) = (p_h^\ell - c_h^\ell)\left(\frac{c_h^\ell - c_h^h}{q_h^\ell - q_h^h} - \frac{c_h^h}{q_h^h}\right) \).

The first-order condition for profit-maximization, \( \frac{\partial \pi_h^\ell}{\partial p_h^\ell} = 0 \), directly yields

\[
p_h^\ell = \frac{(c_h^\ell q_h^h + c_h^h q_h^\ell)}{2q_h^\ell} \quad (7)
\]

Substituting in the optimal price of firm \( \ell \), the equilibrium price schedule for firm \( h \) is

\[
p_h^\ell(\theta) = \frac{(c_h^\ell q_h^h + c_h^h q_h^\ell)}{2q_h^\ell} + \theta(q_h^h - q_h^\ell)
\]

Now, consider the choice of qualities at stage 1. Suppose firm \( \ell \) chooses \( q_\ell \), and firm \( h \) \( q_h \). Further, suppose firm \( \ell \) chooses \( p_h^\ell \) optimally, given the two qualities. Then, the profit function of firm \( h \) is \( \pi_h(q_h^h, q_h^\ell) = \int_{q_h^h}^{q_h^\ell} (p_h^\ell(\theta) - c_h^h) d\theta \).

Replacing the values of \( p_h^\ell(\theta) \), \( \pi_h(q_h^h, q_h^\ell) = \frac{(p_h^\ell + q_h^h - q_h^\ell - c_h^h)^2}{2(q_h^h - q_h^\ell)^2} \). The corresponding first-order condition for firm \( h \), therefore, is \( \frac{\partial \pi_h}{\partial q_h} = 0 \), which gives us

\[
\frac{(p_h^\ell + q_h^h - q_h^\ell - c_h^h)}{q_h^h - q_h^\ell} \left\{ 1 - (c_h^h) + \frac{q_h^h (c_h^\ell)^2 - c_h^h}{2(q_h^\ell)^2} + \frac{p_h^\ell + q_h^h - q_h^\ell - c_h^h}{2(q_h^h - q_h^\ell)^2} \right\} = 0 \quad (8)
\]

Let \( \pi_h(q_h^\ell) \), the solution to this equation, denote the reaction function for firm \( h \).
Similarly, the profit equation for firm $\ell$ is given by \( \frac{(c_h^\ell q_h^h - c_h^\ell q_h^H)^2}{2q_h^\ell q_h^\ell (q_h^h - q_h^\ell)} \). The corresponding first-order condition for firm $\ell$, therefore, is \( \frac{\partial \pi_h^\ell}{\partial q_h^\ell} = 0 \), which gives us

\[
\frac{(c_h^\ell q_h^h - c_h^\ell q_h^H)(c_h^\ell(q_h^h - 2q_h^h) + q_h^\ell(c_h^\ell - 2(c_h^\ell)'(q_h^h - q_h^\ell)))}{(2q_h^\ell(q_h^h - q_h^H))^2} = 0 \tag{9}
\]

Since \( \frac{c_h^\ell}{q_h^\ell} > \frac{c_h^\ell}{q_h^H} \), it cannot be that \( (c_h^\ell q_h^h - c_h^\ell q_h^H) = 0 \). Hence, the optimal quality of firm $\ell$, $q_h^{\ell*}$ is given by the solution to

\[
c_h^\ell(q_h^h - 2q_h^\ell) + q_h^\ell(c_h^\ell - 2(c_h^\ell)'(q_h^h - q_h^\ell)) = 0. \tag{10}
\]

Let \( r_h^\ell(q_h^H) \), the solution to this equation, denote the reaction function for firm $\ell$.

**Proposition 6** Suppose both firms are at the no-PP qualities, $q_h^n, q_h^n$. If firm $h$ now acquires PP, compared to the no-PP case, (i) firm $\ell$ charges a lower price: $p_h^\ell < p_h^n$, and (ii) if firm $\ell$ remains at its original quality, $q_h^n$, then firm $h$ chooses a higher quality.

Of course, in equilibrium, both firms change their qualities from the no-PP case. We expect the price of firm $\ell$ to be lower, and the quality of firm $h$ to be lower. What quality does firm $\ell$ choose? Similar to the previous case, if the cost function is not too convex (in particular, $\alpha \leq 1.55$), firm $\ell$ chooses a lower quality in equilibrium. Conversely, if the cost function is highly convex ($\alpha > 1.55$), it chooses a higher quality.

Thus, for a wide range of $\alpha$, both firms increase their qualities when the PP firm chooses a higher quality. Further, the price of firm $\ell$ falls. We show in Section 7 that this implies that consumers are better off than in the no-PP case.

The firm with PP has a strategic advantage, since it knows consumer valuations and can price at marginal cost for the threshold consumer. To maximize this strategic advantage, it seeks to differentiate itself further from the other firm and avoid head-to-head competition. As long as costs are increasing at a moderate rate, the non-PP firm seeks to increase product differentiation by moving away. However when cost function becomes steep, the non-PP firm seeks to reduce quality differentiation and come closer to the PP firm in the quality space. That is, if the PP firm has a low quality, in equilibrium, both firms end up with lower qualities than previously. The converse outcome occurs if the PP firm chooses high quality; that is, in equilibrium both firms end up with higher qualities.
We further note that, when only one firm has PP, regardless of whether it chooses a high or low quality, the other firm offers a lower price than the corresponding price in the case in which neither firm has PP. Since it knows that PP immediately equips the other firm with an ability to lower prices wherever necessary, the initial strategic response by the non-PP firm, is to lower its own price in order to remain competitive.

6 Both Firms have PP

Suppose, as before, that firm $h$ chooses $q^b_h$ and firm $\ell$ chooses $q^b_\ell$. Then, $\theta^h = \frac{c^b_h - c^b_\ell}{q^b_h - q^b_\ell}$, $\theta^b = \frac{c^b_h}{c^b_\ell}$, and $\hat{\theta} = \frac{c^b_h}{q^b_h}$. Recall that firm $h$ sells to consumers in the region $[\theta^b_h, 1]$ and firm $2$ in the region $[\theta^b_\ell, \theta^b_h]$. As before, $\hat{\theta}$ represents the point beyond which firms compete for consumers, so that the price offered by the low firm is declining in consumer type.

Consider stage 2 of this game, where the firms choose their price schedule, given qualities $q^b_h, q^b_\ell$. The profit function of firm $h$, is given by $\pi^h = \int_{\theta^b_h}^{1} (p^b_h(\theta) - c^b_h)d\theta$. The maximal price firm $h$ can charge any consumer $\theta$ is the price at which he is exactly indifferent between buying the low quality
product at \( c^b_h \) (the lowest price firm \( \ell \) is willing to charge) and the high quality product \( h \) at \( p^b_h(\theta) \). Therefore,
\[
\theta q^b_h - p^b_h(\theta) = \theta q^b_{\ell} - c^b_{\ell},
\]
or \( p^b_h(\theta) = c^b_{\ell} + \theta(q^b_{\ell} - q^b_h) \). Thus,
\[
\pi^b_h = \int_{\theta^b_h}^{\theta^b_{\ell}} (c^b_{\ell} + \theta(q^b_{\ell} - q^b_h) - c^b_h) d\theta = \frac{(q^b_{\ell} - q^b_h - c^b_h + c^b_{\ell})^2}{2(q^b_h - q^b_{\ell})}.
\]
Notice that this is exactly the same as the profit function of firm \( h \) when only firm \( \ell \) has PP, equation (5). Hence, it follows that the reaction function of firm \( h \) is identical to that expressed by equation (5).

Next, consider the profit function of firm \( \ell \). We have \( \pi^b_{\ell} = \int_{\theta^b_{\ell}}^{\theta^b_h} (p^b_h(\theta) - c^b_{\ell}) d\theta \). Now, as before, in the region \([\theta^b_{\ell}, \theta^b_h]\), firm \( \ell \) faces no effective competition from firm \( h \) (since these consumers will not buy good \( h \) even at a price \( c^b_h \)). Hence, in this region, it charges \( p^b_h(\theta) = \theta q^b_c \). In the region \([\theta, \theta^b_h]\), firm \( h \) is willing to price as low as \( c^b_{\ell} \). Hence, firm \( \ell \) must price so that \( \theta q^b_{\ell} - p^b_{\ell}(\theta) \geq \theta q^b_h - c^b_{\ell} \), or \( p^b_{\ell}(\theta) \leq c^b_{\ell} - \theta(q^b_h - q^b_{\ell}) \). The optimal price in this region is, therefore, \( p^b_{\ell}(\theta) = c^b_{\ell} - \theta(q^b_h - q^b_{\ell}) \).

Hence, its profit function is
\[
\pi^b_{\ell} = \int_{\theta^b_{\ell}}^{\theta} (\theta q^b_{\ell} - c^b_{\ell}) d\theta + \int_{\theta}^{\theta^b_h} (c^b_{\ell} - \theta(q^b_h - q^b_{\ell}) - c^b_{\ell}) d\theta = \frac{(c^b_{\ell} q^b_{\ell} - q^b_h c^b_{\ell})^2}{2q^b_{\ell} q^b_{\ell} (q^b_h - q^b_{\ell})}.
\]
Notice that this profit function is exactly the same as the profit function of firm \( \ell \) when only firm \( h \) has PP. Hence, it follows that its reaction function is identical to equation (9).

To compare the qualities in this case with the cases in which only one firm has PP, we parameterize the cost function as \( c(q) = Aq^\alpha \). For \( \alpha > 1 \), this function is convex. We assume that \( A > \frac{1}{\alpha} \). This function satisfies assumption 1.

We show that both firms offer a higher quality than in the case in which only firm \( \ell \) has PP, but offer a lower quality than in the case in which only firm \( h \) has PP. Interestingly, compared to the case when neither firm had PP, we observe that the high quality firm lowers its quality and the low quality firm raises its quality. This implies that both firms actually come closer to each other.

The intensified competition leaves both firms worse off.

**Proposition 7** Suppose \( c(q) = Aq^\alpha \), where \( \alpha > 1 \) and \( A > \frac{1}{\alpha} \). Then, the equilibrium qualities, \( q^h_{\ell}^* \) and \( q^h_{\ell}^* \), satisfy \( q^h_{\ell}^* > q^h_{\ell}^* > q^h_{\ell}^* \) and \( q^h_{\ell}^* > q^h_{\ell}^* > q^h_{\ell}^* \).
7 Observations

In this section, we examine which firms are likely to adopt PP, and the resultant consumer welfare. Suppose neither firm has PP. We assume that after one or both firms adopt PP, the quality rankings of the firms do not change. That is, the low quality firm when neither firm has PP remains the low quality firm when one or both firms have PP. Quality levels are tantamount to brand equity, and significant changes to quality are likely to be costly. This is especially true when quality rankings are reversed. By contrast, local or marginal changes to quality can be made in continuous fashion. Hence, we now consider firm \( \ell \) acquiring PP, or firm \( h \) acquiring PP, or both.

We demonstrate these observations numerically in the model, assuming a cost function \( c(q) = q^\alpha \) (that is, \( A = 1 \)).

Observation 2 For \( \alpha \in [1, 4] \), it is a dominant strategy for firm \( \ell \) to acquire PP. That is, regardless of whether firm \( h \) has PP, firm \( \ell \) should adopt PP.

Figure 5 demonstrates the increase in profit to firm \( \ell \) when it acquires PP. The figure on the left considers the case of neither firm having PP, and the figure on the right the case of firm \( h \) having PP.

We emphasize that the cost of building or acquiring a personalized pricing technology is not factored into this calculation. Such a cost can be incorporated as follows. The vertical line between the dashed and shaded line indicates the gain to firm \( \ell \) from PP. It will adopt PP if and only if this
gap exceeds the fixed cost of adopting PP.

**Observation 3** Regardless of whether firm $\ell$ has PP, firm $h$ should adopt PP only if the cost function is not too convex. In particular, there exists an $\hat{\alpha} \in [2, 3]$ such that, if $\alpha > \hat{\alpha}$, and firm $h$ acquires PP, its profits decrease.

Figure 6 demonstrates this.

How can the profit of firm $h$ decrease when it acquires PP? Recall that, when firm $h$ acquires PP and firm $\ell$ does not have PP, firm $\ell$ responds by reducing its price. This induces firm $h$ to increase its quality. Increasing quality is especially costly when the cost function is steep; indeed, it is costly enough in this case to outweigh the benefits of charging consumers their willingness to pay. A similar intuition holds when firm $\ell$ has PP. If firm $h$ acquires PP in this situation, the new equilibrium sees both firms at a higher quality, which is again correspondingly costly for firm $h$. Again, note that this result does not factor in a cost for acquiring PP. With such a cost, firm $h$ has even less incentive to acquire PP.

**Observation 4** If both firms acquire PP, then, for low $\alpha$, both firms have higher profits compared to the case when neither firm has PP. However, for $\alpha \in [2, 3]$, both firms have lower profits, compared to when neither firm has PP. However the market shares increase for both firms, for all $\alpha$.

The result on profits can be seen by comparing the profits of the two firms in Figures 5 and 6, between the cases “Neither firm has PP” and “Both firms have PP.” Personalized pricing intensifies...
the competition between the firms. If the cost function is steep, both firms are worse off as a result.

**Observation 5** Consumer surplus is higher when both firms have PP, as compared to any of the other cases.

This points out the benefits of competition when there is first-degree price discrimination, in contrast to the monopoly case, in which consumer surplus is zero.\(^7\)

The total consumer surplus can be written as

\[
C = \int_{\theta_{t}}^{\theta_{h}} (\theta q_{t} - p_{t}(\theta))d\theta + \int_{\theta_{h}}^{1} (\theta q_{h} - p_{h}(\theta))d\theta
\]

When both firms have PP, firm \( h \) charges a price \( p_{h}(\theta) = c_{h} + \theta q_{h}(\theta) - q_{h}(\theta) \) to its consumers. Compare this to the price it charges when firm \( \ell \) does not have PP: \( p_{\ell}(\theta) = p_{\ell} + \theta (q_{\ell}(\theta) - q_{\ell}(\theta)) \).

If firm \( \ell \) now acquires PP, the greater competition leads to a lower price for consumers of firm \( h \), and a corresponding increase in welfare. Consumer surplus falls (compared to the no personalized pricing case) if the PP firm has low quality, but rises if the PP firm has high quality. When firm \( \ell \) has PP, it extends its market reach to a segment previously untapped, since it can price as low as marginal cost. However, a segment of its consumers receive no surplus, since they pay a price exactly equal to their willingness to pay. Conversely, if firm \( h \) has PP, it faces competition from firm \( \ell \) throughout its market segment, so it is forced to concede some surplus to consumers. In fact consumer surplus is highest when both firms have PP. This scenario represents the case of most intense competition between the two firms. Figure 7 demonstrates the consumer surplus in all the cases.

Finally, in Table 1, we consider the quadratic cost case \( (c(q) = q^{2}) \) case in greater detail, to provide some benchmarks on prices when firms have PP. Notice that the results in the case when neither firm has PP correspond exactly to those of Moorthy (1988). The average price displayed in the table is the average of the prices paid by different consumers for the good. For example, in the case when neither firm has PP, of course, all consumers pay the same price. When firm \( \ell \) has PP, \( p_{\ell}(\theta) = 0.201, p_{\ell}(\theta) = 0.164\theta \) for \( \theta \in [0.164, 0.517] \), and \( p_{\ell}(\theta) = 0.201 - 0.224\theta \) for \( \theta \in [0.517, 0.776] \).

This leads to a maximum price of 0.0848 and a minimum one of 0.0269. When both firms have PP, because of the intensified competition the average price of both firms is the lowest of all cases.\(^7\)

---

\(^7\)Bhaskar and To (2002) obtain a similar result in their framework.
Further, their overall market coverage is at its highest. Hence, consumer surplus is maximized in this case.

<table>
<thead>
<tr>
<th></th>
<th>Neither firm has PP</th>
<th>Firm $\ell$ has PP</th>
<th>Firm $h$ has PP</th>
<th>Both firms have PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualities</td>
<td>0.409, 0.199</td>
<td>0.388, 0.164</td>
<td>0.444, 0.222</td>
<td>0.4, 0.2</td>
</tr>
<tr>
<td>Market Shares</td>
<td>0.28, 0.35</td>
<td>0.224, 0.612</td>
<td>0.444, 0.222</td>
<td>0.4, 0.4</td>
</tr>
<tr>
<td>Average Price</td>
<td>0.227, 0.075</td>
<td>0.201, 0.056</td>
<td>0.247, 0.074</td>
<td>0.2, 0.02</td>
</tr>
<tr>
<td>Profits</td>
<td>0.0164, 0.012</td>
<td>0.0112, 0.0177</td>
<td>0.022, 0.0055</td>
<td>0.016, 0.008</td>
</tr>
</tbody>
</table>

Table 1: Summary of equilibrium results when $c(q) = q^2$

8 Discussion

In this section we derive managerial implications of our results. Electronic retailers are beginning to explore the potential benefits of “smarter pricing” on the Internet and as such are pursuing
strategies which encourage the adoption of personalized pricing. Firms able to gather information about consumer needs and willingness to pay can customize their offerings and prices to give their consumers exactly what they want, at exactly the price they are willing to bear.

In a recent survey of online retailers (Johnson, 2000), 57% of retailers surveyed planned to offer multiple prices for the same item, and 71% expected to have preferred pricing for regular consumers. The credit rating agency Experian has launched a software package that enables e-commerce sellers to recognize customers instantly. It can send their profile to retailers, including details of their wealth and the products they are most likely to buy. A number of financial services companies are using similar technology to price products and deliver services to their banking and credit card consumers. Capital One uses profiles based on hundreds of variables to tailor products and prices for specific clients (see McDonnell 2001). How this might impact other major players such as First Union Bank and Bank of America and their incentives to invest in personalized pricing technologies is worth pondering about.

Our model points out certain interesting pricing strategies for firms. If the low quality firm deploys PP then it is optimal for it to use a non monotonic price schedule. This result implies that certain high valuation consumers are charged lower prices than some lower valuation consumers. This counter-intuitive result holds because in a segment of high valuation consumers, the firm with PP finds itself competing with a high quality firm (that does not have PP). To induce these consumers to buy its product, the firm needs a declining price schedule. Conversely, in a segment of the market with low valuations, the PP firm is a local monopolist, and can afford to charge consumers exactly their willingness to pay.

It is important to note that given these technologies, it is easy to get into a spiraling price war. In order to avoid this, firms need to increase product differentiation either by providing some value added services or by adding features to the product to enhance its durability or functionality. Retailers can charge quality-sensitive shoppers premium prices if the overall value proposition is clearly exposed and appealing enough. Compared to Buy.com, Amazon provides added features such as customized book recommendations, editorial reviews, and easy site navigability.

While ours is a static model, the intuition extends to a dynamic setting, in which firms can react to each other’s actions. Suppose first that there is no lag in acquiring PP, so that firms can immediately respond to each other’s actions. Our results in the static model suggest that a low
quality firm would benefit from adopting PP irrespective of what its high quality competitor does. However, in a scenario in which the high quality firm can observe its action and respond, a low quality firm may prefer to not adopt PP. Suppose, for example, that $\alpha \in [1.5, 3]$, so that costs are only moderately convex. In this scenario, if either firm adopts PP, it is a best response for the other firm to follow suit. However, both firms are better off in the scenario where neither has PP, as compared to both having PP. Hence, it is optimal for neither firm to adopt PP. Conversely, if $\alpha > 3$, only the low quality firm will adopt PP, since the high quality firm reduces its own profit by adopting PP.

On the other hand, if adopting PP requires considerable lead time, an early adopter can exploit its first mover advantage to increase profits in the interim period before any competitor can follow. This offers an extra incentive to adopt PP. Compared to the previous paragraph, firms should now adopt PP for a wider range of $\alpha$.

We also identify the diverse scenarios under which firms make different product quality choices, given that one firm has decided to acquire PP. When a low quality firm acquires PP, its best response is to lower its quality level. This can be done through removal of additional product features or value-added services. In such a scenario the high quality firm is better off by also reducing its quality level. Conversely, if the high quality firm acquires PP, both firms should provide additional product features or services to increase their quality levels. However when both firms decide to acquire PP, the manner in which they would change their qualities also merits attention. If the high quality firm has PP and the low quality firm decides to deploy PP as well, then both firms should reduce their quality levels. Conversely if the low quality firm has PP and the high quality firm decides to acquire it, then both firms should raise their quality levels.

Finally, our model also demonstrates that consumers would benefit if higher quality firms adopt PP. In the event that all firms adopt PP, consumers would benefit the most. Thus we conclude that strategies approaching first degree price discrimination on the Internet should eventually lead to an overall increase in consumer welfare, which is quite in contrast to popular perceptions.

Our primary objective in this paper has been to provide an analytical framework to investigate the competitive implications of technologies which allow for precise inferring of consumers’ valuations for firms’ products and their ability to charge different prices from different consumers plus the firms’ resultant choice of product differentiation.
Our model of vertical differentiation in the online retail B2C market, shows how first degree price discrimination on the Internet will affect firms’ choice of quality or service differentiation in a competitive scenario. There are quite a few extensions which we have not considered but can be worked upon. In particular, we have not incorporated product customization in our model. Firms may be able to combine PP with customization and then firms can choose locations with respect to the degree of customization as well.  

9 Appendix

Proof of Proposition 1

Consider the monopolist without PP. Since \( \theta_m^n(p, q) = \frac{p_m^n}{q_m^n} \), the firm’s profit function is

\[
\pi_m^n(p, q) = (1 - \frac{p_m^n}{q_m^n})(p_m^n - c(q)).
\]

Consider first the optimal choice of \( p_m^n \), given \( q \). The first-order condition \( \frac{\partial \pi_m^n}{\partial p_m^n} = 0 \) leads to \( (p_m^n)^* = \frac{q_m^n + c(q)}{2} \). The function is obviously concave in \( p \), so this yields a maximum. Substituting this into the profit function, we have \( \pi_m^n(q) = \frac{(q_m^n - c(q))^2}{4q_m^n} \).

Next, consider the PP monopolist. From the text, its profit function is

\[
\pi_m^d(q) = \int_{q_m^d}^{q_m^d} (\theta q_m^d - c(q))d\theta = \frac{q_m^d}{2} + \frac{c(q)^2}{2q_m^d} - c(q) = 2\pi_m^n(q).
\]

Since \( \pi_m^d(q) = 2\pi_m^n(q) \) for all \( q \), it must be that \( \pi_m^d \) and \( \pi_m^n \) have the same maximizer; that is, \( q_d = q_m \).

Proof of Proposition 2

Suppose that \( q_1 = q_2 \). If neither firm has access to PP, the Proposition is immediate. Consider the case that firm 1 has PP, and firm 2 does not. Suppose \( p_2 > 0 \). Firm 1 will never charge \( p_1(\theta) > p_2 \) to any consumer \( \theta \), since the consumer will buy product 2 instead. Further, by the usual Bertrand argument, firm 1 will not charge \( p_1(\theta) = p_2 \) either. By charging a price \( \epsilon \) below \( p_2 \), firm 1 ensures that consumer \( \theta \) buys its product.

---

8We are grateful to seminar participants at WISE 2001 and Carnegie Mellon University, and to Kannan Srinivasan for helpful comments. All errors remain our own responsibility.
Suppose $p_1(\theta) > 0$ for some consumer $\theta$. Then, firm 2 can capture a positive market share by charging $p_2 \in (0, p_1(\theta))$. Hence, the only equilibrium is the Bertrand one, $p_1(\theta) = p_2 = c(q_1)$ for all consumers $\theta$.

The same argument holds if firm 2 has PP instead of firm 1. Consider next the case in which both firms have PP. The argument above then applies to each consumer type $\theta$: it must be that, for each $\theta \in [0, 1]$, $p_1(\theta) = p_2(\theta) = c(q_1)$.

**Proof of Lemma 1**

From the definitions of $\theta^n_h$ and $\theta^n_\ell$, the profit of firm $h$ is $\pi^n_h = (1 - \frac{p^n_h - p^n_\ell}{q^n_h - q^n_\ell})(p^n_h - c^n_h)$. Differentiating with respect to $p^n_h$ and setting equal to zero, we have

$$2p^n_h - p^n_\ell = q^n_h - q^n_\ell + c^n_h.$$  

(11)

The second derivative is $\frac{-2}{q^n_h - q^n_\ell} < 0$, so we have a maximum.

Similarly, the profit of firm $\ell$ is $\pi^n_\ell = (\frac{p^n_h - p^n_\ell}{q^n_h - q^n_\ell} - \frac{p^n_\ell}{q^n_\ell})(p^n_\ell - c^n_\ell)$. Differentiating with respect to $p^n_\ell$ and setting equal to zero, we have

$$-p^n_\ell q^n_\ell + 2p^n_h q^n_\ell = c^n_\ell q^n_\ell.$$  

(12)

Again, it is immediate to show that the second order condition for maximization is satisfied as $\frac{-2q^n_\ell}{q^n_h - q^n_\ell} < 0$.

The equilibrium prices, $(p^n_{h*}, p^n_{\ell*})$, are found by solving simultaneously equations (11) and (12), which yields $p^n_{h*} = \frac{q^n_h(2q^n_h - q^n_\ell + c^n_h)}{4q^n_h - q^n_\ell}$, and $p^n_{\ell*} = \frac{q^n_\ell(q^n_\ell + 2c^n_\ell) + q^n_h(c^n_h - q^n_\ell)}{4q^n_h - q^n_\ell}$.

**Proof of Proposition 3**

Consider firm $h$ first. Its profit at stage 2, if it charges price $p_h$, is $\pi^\ell_h = (1 - \theta^\ell_h) (p^\ell_h - c^\ell_h) = (1 - \frac{p^\ell_h - c^\ell_h}{q^\ell_h - q^\ell_\ell}) (p^\ell_h - c^\ell_h)$. The first-order condition for profit-maximization, $\frac{\partial \pi^\ell_h}{\partial p^\ell_h} = 0$, directly yields

$$p^\ell_{h*} = \frac{1}{2} (q^\ell_h - q^\ell_\ell + c^\ell_h + c^\ell_\ell).$$

Next, consider firm $\ell$. Firm $\ell$ will set its price for each consumer, $p^\ell_\ell(\theta)$, as high as possible to satisfy two restrictions: (i) the consumer buys product $\ell$ instead of product $h$, so that

$$\theta q^\ell_\ell - p^\ell_\ell(\theta) \geq \theta q^\ell_h - p^\ell_h,$$

$$p^\ell_\ell(\theta) \leq p^\ell_h - \theta(q^\ell_h - q^\ell_\ell),$$

27
and (ii) the consumer buys product \( \ell \), rather than not consume at all. That is,

\[
\theta q^\ell - p^\ell(\theta) \geq 0, \quad \text{or} \quad p^\ell(\theta) \leq \theta q^\ell.
\]

Further, firm \( \ell \) must set \( p^\ell(\theta) \geq c^\ell = c^\ell(q^\ell) \) for each consumer, else it makes a loss on that consumer, and would prefer to not sell to him. Hence, we have \( p^\ell(\theta) \geq c^\ell \), and \( p^\ell(\theta) \leq \min\{\theta q^\ell, \ p^\ell - \theta(q^\ell - q^\ell)\} \).

The first term in the latter inequality is defined by the consumer’s reservation utility (i.e., zero), and the second term can be interpreted as his incentive compatibility constraint: if this is violated, then he buys product \( h \) instead.

Given that \( \hat{\theta} = \frac{p^\ell}{q^\ell} \) as defined, it is immediate that \( p^\ell - \theta(q^\ell - q^\ell) > \theta q^\ell \) for \( \theta > \hat{\theta} \), and \( p^\ell - \theta(q^\ell - q^\ell) < \theta q^\ell \) for \( \theta < \hat{\theta} \). The pricing function for firm \( \ell \) now follows.

**Proof of Lemma 2**

Consider equation (6), which defines \( q^\ell_h \). Dividing throughout by \( (q^\ell_h - q^\ell) \), we have

\[
1 + \frac{c^\ell_h - c^\ell}{q^\ell_h - q^\ell} - 2(c^\ell)^{'} = 0,
\]

or \( (c^\ell)^{'} = \frac{1}{2}(1 + \frac{c^\ell_h - c^\ell}{q^\ell_h - q^\ell}) \). Now, \( \theta^\ell < 1 \) implies \( \frac{p^\ell - c^\ell}{q^\ell_h - q^\ell} < 1 \). Since \( c^\ell_h < p^\ell \), it follows that \( \frac{c^\ell_h - c^\ell}{q^\ell_h - q^\ell} < 1 \). Hence, \( (c^\ell)^{'} < 1 \). Since \( q^\ell_h < q^\ell \) it follows that \( (c^\ell)^{'} = c^\ell(q^\ell_h) < c^\ell(q^\ell) < 1 \).

**Proof of Proposition 4**

Recall that \( \psi_\ell(c^\ell_h, q^\ell) \) is the left-hand side of equation (3), the first-order condition for firm \( \ell \). Here, \( p^\ell_h \) is a function of \( q^\ell_h, q^\ell \) (that is, we assume \( p^\ell_h^{*} \) is optimally chosen by firm \( h \)). Then,

\[
\frac{\partial \psi_\ell}{\partial q^\ell_h} = \frac{(c^\ell)^{'}(1)(c^\ell_h + c^\ell - (c^\ell)^{'}q^\ell)}{4(q^\ell_h - q^\ell)^2} + \frac{(c^\ell_h - c^\ell)(c^\ell)^{'}(c^\ell_h + c^\ell - (c^\ell)^{'}q^\ell)}{4(q^\ell_h - q^\ell)^2} - \frac{(c^\ell_h - c^\ell)^2}{4(q^\ell_h - q^\ell)^2}
\]

Now, the first and the fifth terms sum to \( \frac{(c^\ell)^{'}(1)(c^\ell_h + c^\ell - (c^\ell)^{'}q^\ell)}{4(q^\ell_h - q^\ell)^2} > 0 \), since \( (c^\ell)^{'} < 1 \) by Lemma 2, \( (c^\ell)^{'} > \frac{c^\ell}{q^\ell_h} \), and \( c^\ell < q^\ell \) (else firm \( \ell \) has zero sales).

Adding the second, third and fourth terms, and simplifying, we have

\[
\frac{(c^\ell_h - c^\ell)(c^\ell)^{'}(q^\ell_h - q^\ell)(c^\ell)^{'} - (c^\ell_h - c^\ell)(c^\ell)^{'}(q^\ell_h - q^\ell))^3}{4(q^\ell_h - q^\ell)^3}.
\]
Then, original optimal quality of firm for firm expression is also positive. Therefore, the first term by Lemma 2 is 

\[
(i) \text{ Suppose both firms choose the same quality levels as in the no-PP case; that is, } q^c_h = q^n_h \text{ and } q^c_\ell = q^n_\ell. \text{ Given } p^n_h \text{ from Lemma 1 and } p^c_h \text{ from Proposition 3, we have } p^c_h < p^n_h \text{ if and only if }
\]

\[
\frac{1}{2}(q^n_h - q^n_\ell + c^n_h + c^n_\ell) < \frac{2(q^n_h c^n_h + q^n_\ell c^n_\ell - 2q^n_h q^n_\ell + 2q^n_\ell^2)}{(4q^n_h - q^n_\ell^2)}
\]

\[
(4q^n_h - q^n_\ell^2)(q^n_h - q^n_\ell + c^n_h + c^n_\ell) < (4q^n_h c^n_h + 2q^n_\ell c^n_\ell - 4q^n_h q^n_\ell + 4q^n_\ell^2)
\]

\[
q^n_h c^n_\ell - q^n_\ell c^n_h - q^n_h q^n_\ell + q^n_\ell^2 + q^n_h c^n_\ell - q^n_\ell c^n_h < 0
\]

\[
(q^n_h c^n_\ell - q^n_\ell c^n_h) + (c^n_h - c^n_\ell)(q^n_h - q^n_\ell) < 0.
\]

Consider the last inequality. \(q^n_h > q^n_\ell\), and \(c^n_h \leq c^n_\ell\) (else firm \(\ell\) sells zero units). Further, since \(c(\cdot)\) is convex, \(c^n_\ell > c^n_h\), so \(q^n_h c^n_\ell - q^n_\ell c^n_h < 0\). Hence, the last inequality holds, so that \(p^c_h < p^n_h\).

(ii) Recall the definition of \(\psi_\ell\) as the left-hand side of equation (3), the reaction function of firm \(\ell\). Then,

\[
\frac{\partial \psi_\ell}{\partial p^c_h} = \frac{1 - (c^\ell_\ell)'}{2q^c_\ell} + \frac{c^\ell_h - c^\ell_\ell - (c^\ell_\ell)'(q^c_\ell - q^c_h)}{2(q^c_\ell - q^c_h)^2}
\]

The first term by Lemma 2 is also positive. Since \(c(\cdot)\) is convex, \(c^\ell_h > c^\ell_\ell + (c^\ell_\ell)'(q^c_\ell - q^c_h)\), so the second term is also positive. Thus \(\frac{\partial \psi_\ell}{\partial p^c_h} > 0\).
Now, suppose the firms choose their no-PP quality levels, \(q^n_h, q^n_\ell\). If firm \(h\) keeps its quality level at \(q^n_h\) and reduces its price from \(p^n_h\) to \(p^\ell_h\), we will have \(\psi_\ell > 0\). Hence, firm \(\ell\) must reduce its quality to ensure that \(\psi_\ell = 0\).

**Proof of Proposition 6**

We proceed with a series of steps.

**Step 1:** Suppose both firms choose the same quality levels as in the no-PP case; that is, \(q^h_h = q^n_h\) and \(q^h_\ell = q^n_\ell\). Then, \(p^h_h < p^n_\ell\).

**Proof of Step 1:** Given \(p^n_h\) from Lemma 1 and \(p^h_h\) from equation 7, we have \(p^\ell_h < p^n_\ell\) if and only if

\[
\frac{c^n_h c^n_\ell + c^\ell_h q^n_h}{2q^n_h} < \frac{2q^h_h c^n_\ell + q^n_h c^n_\ell + q^\ell_h q^n_\ell - q^\ell_h 2}{4q^n_h - q^\ell_h}
\]

\[
q^n_h q^n_\ell (c^n_h - c^n_\ell) + (q^n_h c^n_\ell - 2q^n_h q^n_\ell)(q^n_h - q^\ell_h) < 0
\]

\[
q_\ell((q^n_h - q^n_\ell)(c^n_h - 2q^n_h) + q^n_\ell(c^n_\ell - c^n_\ell)) < 0
\]

\[
q_\ell(q^n_h - q^n_\ell)((c^n_h - q^n_h) + q^n_\ell\left(\frac{c^n_\ell - c^n_h}{q^n_h - q^n_\ell} - 1\right)) < 0
\]

Consider the last inequality. \(q^n_\ell > q^n_h\), and \(c^n_h \leq q^n_h\) (else firm \(h\) sells zero units). Further, \(\frac{c^n_\ell - c^n_h}{q^n_h - q^n_\ell} < 1\), so \((c^n_h - q^n_h) + q^n_\ell\left(\frac{c^n_\ell - c^n_h}{q^n_h - q^n_\ell} - 1\right) < 0\). Hence, the last inequality holds, so that \(p^\ell_h < p^n_\ell\).

**Step 2:** Suppose firm \(\ell\) chooses a quality \(q_\ell = q^n_\ell\), but reduces its price from \(p^n_\ell\) to \(p^h_\ell\). Then, firm \(h\) increases its quality in response to the lowering of the price of the low quality firm.

**Proof of Step 2:** The profit function of the high quality firm is given by the equation \(\pi^h_h(q^h_h, q^n_\ell) = \frac{(p^h_\ell + q^h_h - q^n_\ell)^2}{2(q^n_h - q^n_\ell)}\). Consider \(r^h_h(q^n_\ell)\) the reaction function for firm \(h\).

Taking the partial of this expression with respect to \(p^h_\ell\) and replacing the optimal price gives the following

\[
\frac{\partial}{\partial p^h_\ell} \frac{d\pi^h_h}{dq^n_h} = \frac{1}{2} \left( \frac{c^n_h - q^n_\ell}{q^n_h} \frac{c^n_\ell - (q^n_\ell - q^n_h)(c^n_h)'}{(q^n_h - q^n_\ell)^2} \right)
\]

The first term is clearly negative. From the convexity of the cost function, the second term < 0, from which the inequality holds.

**Step 3:** The reaction functions \(r^h_h\) is upward sloping. Hence, when the high firm increases its quality \(q^n_h\), the low firm will increase \(q^n_\ell\).
Proof of Step 3:
Recall that $\psi_\ell(q^h_\ell, q^h_\ell)$ is the left-hand side of equation (9), the first-order condition for firm $\ell$. Here, $p^h_\ell$ is a function of $q^h_\ell, q^h_\ell$ (that is, we assume $p^h_\ell$ is optimally chosen by firm $h$). Then,

$$\frac{\partial \psi_\ell}{\partial q^h_\ell} = \frac{(c^h_\ell - c^h_\ell - (c^h_\ell)(q^h_\ell - q^h_\ell))(c^h_\ell - c^h_\ell + (c^h_\ell)(q^h_\ell - q^h_\ell))}{2(q^h_\ell - q^h_\ell)^3}$$

Convexity of $c(\cdot)$ directly implies $c^h_\ell > c^h_\ell + (c^h_\ell)'(q^h_\ell - q^h_\ell)$ and $(c^h_\ell)' > \frac{c^h_\ell - c^h_\ell}{q^h_\ell - q^h_\ell}$. Hence, $\frac{\partial \psi_\ell}{\partial q^h_\ell} > 0$, so that to reach $\psi_\ell = 0$, firm $\ell$ must increase its quality.

Step 4: The reaction function $r^h_\ell$ is upward sloping. That is, when the low firm increases its quality $q^h_\ell$, the high firm will increase $q^h_\ell$.

Proof of Step 4: Recall equation (8), the first order condition of firm $h$. Here, again $p^h_\ell$ is a function of $q^h_\ell, q^h_\ell$ (that is, we assume $p^h_\ell$ is optimally chosen by firm $h$). Then

$$\frac{\partial \psi_\ell}{\partial q^h_\ell} = \frac{1}{4} \left( \frac{(c^h_\ell - q^h_\ell)(c^h_\ell)'(c^h_\ell - (2 - (c^h_\ell)'(q^h_\ell))}{q^h_\ell^2} + \frac{(c^h_\ell + c^h_\ell)(c^h_\ell - c^h_\ell)}{(q^h_\ell - q^h_\ell)^2} - \frac{(c^h_\ell - c^h_\ell)^2}{(q^h_\ell - q^h_\ell)^3} - \frac{(c^h_\ell)(c^h_\ell)'}{(q^h_\ell - q^h_\ell)^3} \right)$$

Consider the first term. $c^h_\ell < q^h_\ell(c^h_\ell)'$ and from lemma 2 $c^h_\ell < 1$. So the first term is $> 0$.

Adding terms 2, 3 and 4 we have

$$\frac{\partial \psi_\ell}{\partial q^h_\ell} = \frac{(c^h_\ell - c^h_\ell - (c^h_\ell)'(q^h_\ell - q^h_\ell))(c^h_\ell - c^h_\ell + (c^h_\ell)'(q^h_\ell - q^h_\ell))}{4(q^h_\ell - q^h_\ell)^3}$$

Convexity of $c(\cdot)$ directly implies $c^h_\ell > c^h_\ell + (c^h_\ell)'(q^h_\ell - q^h_\ell)$ and $(c^h_\ell)' > \frac{c^h_\ell - c^h_\ell}{q^h_\ell - q^h_\ell}$. Hence, $\frac{\partial \psi_\ell}{\partial q^h_\ell} > 0$. Hence, to reach $\psi_h = 0$, firm $h$ must increase its quality.

Proof of Proposition 7
Step 1: Suppose the firms charge the optimal qualities in the case in which the PP firm is firm $\ell$, $q^*_h$ and $q^*_\ell$. Then, firm $\ell$ will increase its quality if firm $h$ also has PP.

Proof of Step 1: Note that $q^*_h, q^*_\ell$, are the solutions to the two equations (6) and (4), respectively. From equation (6), we have $2(c^h_\ell)' - 1 = \frac{c^h_\ell - c^h_\ell}{q^h_\ell - q^h_\ell}$. From equation (4), we have

$$\frac{2(c^h_\ell)' - 1}{q^h_\ell} - 1 = \frac{2(c^h_\ell - 2(c^h_\ell)'q^h_\ell)}{q^h_\ell} = \frac{c^h_\ell - c^h_\ell}{q^h_\ell - q^h_\ell}$$

Hence, $\frac{c^h_\ell - 2(c^h_\ell)'q^h_\ell}{q^h_\ell} = \frac{q^h_\ell(1 - (c^h_\ell)')}{q^h_\ell} - (c^h_\ell)'$.
Now, consider the left-hand side of the reaction function of firm $\ell$ when both firms have PP, as given by equation 10. Denote this as $\psi_\ell(q_h^b, q_\ell^b)$. We have

$$\psi_\ell(q_h^b, q_\ell^b) = c_\ell^b(q_h^\ell - 2q_\ell^\ell) + q_\ell^\ell(c_h^\ell - 2(c_\ell^\ell)(q_h^\ell - q_\ell^\ell)) = q_\ell^\ell(q_h^\ell - q_\ell^\ell)(\frac{c_h^\ell - 2(c_\ell^\ell)q_\ell^\ell}{q_h^\ell - q_\ell^\ell} + \frac{c_h^\ell - c_\ell^\ell}{q_h^\ell - q_\ell^\ell})$$

Now, $\frac{c_h^\ell - c_\ell^\ell}{q_h^\ell - q_\ell^\ell} = 2(c_h^\ell)' - 1$, and $\frac{c_h^\ell - 2(c_\ell^\ell)q_\ell^\ell}{q_h^\ell} = q_\ell^\ell(1 - (c_\ell^\ell)') - (c_\ell^\ell)'$. Substituting these, we have

$$\psi_\ell = q_\ell^\ell(q_h^\ell - q_\ell^\ell)\left\{\frac{q_h^\ell(1 - (c_\ell^\ell)')}{q_h^\ell} - (c_\ell^\ell)' + 2(c_\ell^\ell)' - 1\right\} = q_\ell^\ell(q_h^\ell - q_\ell^\ell)\left\{(q_h^\ell(c_\ell^\ell)' - q_\ell^\ell(c_\ell^\ell)'') - (q_h^\ell - q_\ell^\ell)\right\}.$$

Now, the first term on the right-hand side is clearly positive. Consider the second term. Using $c(q) = Aq^\alpha$, denote the second term as $\phi(\alpha) = A\alpha((q_h^\ell)^\alpha - (q_\ell^\ell)^\alpha) - (q_h^\ell - q_\ell^\ell)$.

At $\alpha = 1$, we have $\phi(\alpha) = 0$. Further, $\phi'(\alpha) = A\alpha(q_h^\ell)^\alpha \ln q_h^\ell - q_\ell^\ell \ln q_\ell^\ell) + A(q_h^\ell - q_\ell^\ell)^\alpha$. Hence,

$$\frac{1}{A} \phi'(\alpha) = q_h^\ell(1 + \alpha \ln q_h^\ell) - q_\ell^\ell(1 + \alpha \ln q_\ell^\ell).$$

Now, the function $q^\alpha(1 + \alpha \ln q)$ is increasing in $q$, since both terms are increasing in $q$. Hence, $\phi'(\alpha) > 0$. Therefore, $\phi(\alpha) > 0$ for all $\alpha > 1$.

Hence, $\psi_\ell(q_h^\ell, q_\ell^\ell) > 0$. Therefore, starting with the case in which firm $\ell$ has PP, if firm $h$ acquires PP, firm $\ell$ will increase its quality, to reach $\psi_\ell = 0$.

**Step 2:** From the proofs of Propositions 4 and 6, it follows that both reaction functions are upward-sloping, so that, in response to firm $\ell$ increasing its quality, firm $h$ will increase its own quality, and so on. Hence, the equilibrium qualities satisfy $q_h^{\ell *} < q_h^b$ and $q_\ell^{\ell *} < q_\ell^b$.

**Step 3:** Next, suppose both firms offer the quality levels $q_h^h, q_\ell^h$, the equilibrium qualities when only firm $h$ has PP. We show that firm $h$ will decrease its quality in this instance if firm $\ell$ also has PP. From this, it will follow (as in Step 2 above) that $q_h^{\ell *} < q_h^b$ and $q_\ell^{\ell *} < q_\ell^b$.

**Proof of Step 3:** From equation (6), we know that the optimal quality of firm $h$ is the solution to the equation

$$\psi_h(q_h^b, q_\ell^b) = 1 - 2(c_\ell^b)' + \frac{q_h^b - q_\ell^b}{c_h^b - c_\ell^b} = 0.$$

We evaluate $\psi_h$ at the qualities $q_h^{b *}, q_\ell^{b *}$, which are the solutions to the two equations (8) and (10), respectively.
From equation (8),

\[
1 - 2(c_h') + \frac{c_h - c_h}{q_h - q_h} = -\frac{q_h'((c_h') - (c_h^b / q_h^b))}{q_h^b} - 2 + \frac{c_h^b - p_h^b}{q_h^b - q_h^b} + \frac{c_h^b - c_h}{q_h - q_h^b}.
\]

Now, \((c_h') > \frac{c_h^b}{q_h^b}\) since \(c(\cdot)\) is convex. Further, \(q_h^b - c_h^b > q_h^b - c_h\), else firm \(h\) has zero sales. Since \(p_h^b > c_h^b\), this yields \(\frac{c_h^b - p_h^b}{q_h^b - q_h^b} < \frac{c_h^b - c_h^b}{q_h^b - q_h^b} < 1\).

Therefore, at the qualities \((q_h^b, q_h^b), \psi_h < 0\). Hence, to reach a quality at which \(\psi_h(q_h^b, q_h^b) = 0\), firm \(h\) must decrease its quality. Since both reaction functions are upward sloping, firm \(\ell\) will also decrease its quality.

References


