Coordinating Channels for Durable Goods: The Impact of Competing Secondary Markets

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Abstract
A large literature in economics and marketing studies the problem of manufacturer’s designing contracts that give a retailer appropriate incentives to make decisions that are optimal from the manufacturer’s point of view (see, for example, Spengler 1950, Jeuland and Shugan 1983, McGuire and Staelin 1983, Lal 1990, Rao and Srinivasan 1995, Desai 1997, among others). An important result from this literature is that the manufacturer can coordinate retail price decisions by choosing a two-part tariff in which the wholesale price equals the manufacturer’s marginal cost and the fixed fee extracts all the rents from the retailer. In other words, the manufacturer sells the firm to the retailer for the fixed fee and, thus, eliminates the double-marginalization problem.

Although this result is well established for non-durables, researchers have not analyzed the coordination issue for durable goods manufacturers who have the added complexity of competition from used goods in secondary markets. In this paper, we show how the coordination problem for a durable goods manufacturer is fundamentally different from the traditional coordination problem of a non-durables manufacturer. In particular, the durable goods manufacturer has to solve not only the coordination problem but also the time-consistency problem (see, for example, Coase 1972, Bulow 1982, Purohit 1995). Our objectives in this paper are to investigate whether or not the insights from the channel coordination literature, that has developed principally with non-durable goods in mind, are also applicable to durable goods. In order to do this, we develop a dynamic, two-period model in which a manufacturer sells its products to a retailer who sells the product to consumers. Products sold in the first period become used goods in the second period and compete with sales of new units. Starting from consumer utilities, we derive inverse demand functions for new and used goods and consider a number of different contracts between the manufacturer and the retailer.

We start with a simple contract in which the manufacturer offers a wholesale price for a period at the beginning of that period. As one would expect, this contract does not solve either the channel coordination problem or the time-consistency problem. We then consider a number of two-part tariff contracts. Given the well-established results from the existing channel coordination literature, we begin with a contract in which the manufacturer offers per-period two-part tariffs in which all wholesale prices are set at marginal cost. We find that not only does this contract fail to achieve channel coordination, but the retailer sells a higher quantity than an integrated manufacturer would sell. This is in contrast to the traditional double-marginalization problem in which the retailer sells a lower quantity than an integrated manufacturer would sell.

We then allow the wholesale prices to be different from marginal costs. We show that using this more general two-part tariff contract, the manufacturer can achieve channel coordination. That is, the total channel profit is the same as the profit of an integrated seller. However, the equilibrium wholesale price in the first period is strictly above the marginal cost.

Next, we consider a contract in which the manufacturer uses a single fixed fee, announced at the beginning of the first period. The per-period wholesale prices are still at the marginal cost level in this contract. This contract is identical to “selling the firm to the retailer” at the price of the fixed fee. Here we find that the contract can achieve channel coordination. However, the contract is not an equilibrium solution. In particular, the manufacturer increases wholesale prices to above marginal cost levels.

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Although some of the contracts above solve the double marginalization problem, none of them mitigates the time consistency problem. In order to solve both these problems, the contract must yield total channel profit equal to an integrated renter’s profit. Because the renter does not have a problem with time consistency, an integrated renter earns the highest profits in a durable goods channel. We derive a contract that solves both of these problems. In this contract, at the beginning of period 1, the manufacturer writes a contract with the retailer specifying a fixed fee and two per-period wholesale prices, both of which turn out to be strictly above the marginal cost. Interestingly, with this contract, the manufacturer makes more money by selling through the retailer rather than selling directly to consumers.

We contribute to the coordination literature by examining coordination issues in a dynamic, durable goods context and identifying a new coordination problem—unlike the traditional coordination models, a durable goods manufacturer may have to provide the retailer incentives to sell less rather than to sell more. Clearly, the traditional “selling the firm to the retailer,” approach does not solve this new problem. We also contribute to the durable goods literature by showing how a durable goods manufacturer can sell its product and solve its time consistency problem. Effectively, this allows the manufacturer to earn the same profits as it would get if it could commit to prices or if it could rent its product. When committing to individual consumers or renting can only be achieved through additional costs, our solution is the optimal strategy for a durable goods manufacturer.
Abstract

In this paper, we show how managing a durable goods channel is significantly different from managing a channel for non-durables. Although both channels have to deal with the problem of coordination, the durable goods manufacturer also has to deal with the problem of time-consistency. We show that the standard contracts that coordinate a channel for non-durables do not coordinate the channels for durables.

Focusing on the marketing of durables, we explore whether the idea of “selling the firm” to the retailer ensures channel coordination. When the manufacturer offers per-period two-part tariffs in which all wholesale prices are set at marginal cost, we find that not only does this contract fail to achieve channel coordination, but the retailer sells a higher quantity than an integrated manufacturer would sell. This is in contrast to the traditional double marginalization problem in which the retailer sells a lower quantity than an integrated manufacturer would sell.

Using a more general two-part tariff contract, we show that the manufacturer can achieve channel coordination. However, the equilibrium wholesale price in the first period is strictly above the marginal cost. Next, we derive a contract that solves both the channel coordination and time consistency problems. In this contract, at the beginning of period 1, the manufacturer writes a contract with the retailer specifying a fixed fee and two per-period wholesale prices, both of which turn out to be strictly above the marginal cost. Interestingly, with this contract, the manufacturer makes higher profits by selling through the retailer than selling directly to consumers.
1 Introduction

Most manufacturers do not have their own retail outlets and instead rely on intermediaries to sell their products to end users. For example, auto manufacturers use independent dealers and PC manufacturers sell through large retailers such as Best Buy or CompUSA. In dealing with intermediaries, manufacturers have to solve the problem of coordinating the channel — that is, ensuring that the downstream players take actions that the manufacturer would like them to take. Although this problem is well known and has been studied extensively, the focus has been on coordinating channels for nondurable products. However, it is not clear if the solution to the coordination problem for nondurables should also apply to durables. In particular, unlike nondurables, manufacturers and retailers of durables have to contend with potential competition from a secondary market — consumers always have the option of buying used cars instead of new ones. The basic question we address in this paper is as follows: In the case of durable products, how should a manufacturer structure its contract with the retailer so that it can coordinate the channel and manage the competition from the secondary market? In the process, we show how the coordination problem in distribution channels for durable products is significantly different from the standard coordination problem for non-durables.

In a channel that consists of multiple decision-makers, the coordination problem arises because, each decision-maker owns and operates a part of the channel and makes decisions that optimize its own profit. Such selfish optimization behavior leads to uncoordinated decisions in that they are not optimal either for the other decision-maker or for the whole distribution channel. For example, the retailer may choose a price that maximizes its own profit, but generally that uncoordinated price is not the best price from the manufacturer’s or from the channel’s point of view. A large literature in economics and marketing studies a manufacturer’s problem of designing contracts that give the retailer appropriate incentives to make decisions that align, or coordinate, the retailer’s and manufacturer’s goals (see, for example, Spengler...
1950, Jeuland and Shugan 1983, McGuire and Staelin 1983, Lal 1990, Rao and Srinivasan 1995, Desai 1997, among others). For example, a manufacturer can coordinate the channel by choosing a two-part tariff in which the wholesale price is set at the manufacturer’s marginal cost and the fixed fee extracts all the profits over the minimum amount needed to retain the participation of the retailer. Said differently, the manufacturer can solve the coordination problem by selling the firm to the retailer at a price equal to the fixed fee. This can also be thought of as making the retailer the residual claimant to the channel profits. When the retailer is the residual claimant, then it has the appropriate incentives to choose a price (or other decision variable) that maximizes the channel’s profits.

Although this result is well established for non-durables, researchers have not analyzed the coordination issue for durable goods that have the added complexity of competition from used goods in secondary markets. In particular, the durable goods manufacturer has to solve not only the channel coordination problem but also the time-consistency problem. We develop a dynamic model to analyze the effect of several different types of contracts in solving the coordination and time consistency problems. In particular, we investigate whether the strategy of selling the firm to the retailer can solve either the coordination or the time-consistency problem. Importantly, we derive a relatively simple contract that solves both these problems, and find that an integrated manufacturer can increase its profits if it markets its product through a retailer.

The coordination problem for a durable goods manufacturer is significantly different from the traditional coordination problem of a non-durables manufacturer, principally because of the competition between used and new goods. In particular, suppose a firm sells a certain quantity of a durable to consumers. Having sold this initial quantity, the firm still faces a residual demand for the good, consisting of those consumers who place a value on the good lower than the current market price. As a result, the firm has an incentive to lower its price to attract
this set of consumers. It is easy to see how this process of skimming the market could continue until the firm lowers price to its marginal cost of production. However, from a consumer’s perspective, each time the firm lowers price, it also lowers the “value” of the product that the consumer had purchased at a higher price in an earlier period. Thus, rational consumers would factor in future price reductions into their current willingness to pay. Coase (1972) conjectured that if the firm were to exploit residual demand in future periods, then rational consumers would anticipate this behavior and price would fall to the competitive level in the “twinkling of an eye.” Thus, the net effect of product durability and rationality on the part of consumers is that the firm’s profits are reduced. Further research in this area has formalized Coase’s conjecture (e.g., Bulow 1982; Stokey 1981) and pointed out conditions under which it does not hold, e.g., if there is a constant inflow of new customers (Conlisk, Gerstner and Sobel 1984), if the good depreciates (Bond and Samuelson 1984), or if the firm has increasing marginal production costs (Kahn 1986).

The central problem with durables arises because once a manufacturer sells a unit it is no longer interested in what happens to the value of that unit. Thus, by lowering prices for new products over time, the firm also lowers the value of old units owned by consumers. However, as argued previously, rational consumers would anticipate this behavior and force the firm to lower prices immediately. As a result, the only way for the firm to maintain a higher price level is if it can convince consumers that it will not lower prices over time. Even though ex ante (at the time of initial sales), the firm prefers to make such a promise, ex post (after consumers have bought the product), it would prefer to break the promise. Thus, the firm’s preferences are not time consistent and consumers would not believe any promises made by the firm. A promise would be credible only if it were a legally binding commitment, for example, through a contract with individual consumers.¹ However, such a strategy may be undesirable because

¹Alternatively, the manufacturer could break the molds, thus ensuring that no additional products can be produced.
of the impracticality of writing contracts with possibly a large number of consumers. Another possible solution is for the firm to rent, rather than sell the product to consumers. When the firm rents the product, it retains the ownership of used units, and therefore, has an incentive to keep the price of used units high. However, renting is not entirely free from problems such as the possibility of consumers potentially abusing the product and the additional costs of marketing the product for subsequent rentals. In addition, renting may also have the added cost of writing legal contracts with individual consumers.

When we add a distribution channel to durables, the problem becomes further exacerbated. Now, not only does the manufacturer have to contend with issues related to time-consistency, but it also has to write a contract that helps it solve the problem of channel coordination. And the channel coordination problem itself is more complex because of the inter-temporal linkages we discussed earlier. Specifically, first-period decisions affect second-period outcomes, and this effect may be different for the retailer than for an integrated seller. As a consequence, the channel coordination for durable goods presents challenges not faced by non-durable goods manufacturers. While some of these issues have been explored in Purohit (1995), the general problem of coordinating a durable goods distribution channel has not been addressed. This paper analyzes different contracts and evaluates their ability to solve the coordination and time-consistency problems. We develop a dynamic, two-period model in which a manufacturer sells its products to a retailer who sells the product to consumers. Products sold in the first period become used goods in the second period and compete with sales of new units. Starting from consumer utilities, we build inverse demand functions for new and used goods and consider a number of different contracts between the manufacturer and the retailer. Some of our key findings are as follows.

- Given the well-established results from the existing channel coordination literature, we begin with a contract in which the manufacturer offers per-period two-part tariffs in
which all wholesale prices are set at the marginal cost. We find that not only does this contract fail to achieve channel coordination, but the retailer sells a higher quantity than an integrated manufacturer would sell. This is in contrast to the traditional double marginalization problem in which the retailer sells a lower quantity than an integrated manufacturer would sell.

- In contrast to the findings in the non-durable goods literature, we find that the strategy of selling the firm to the retailer is not an equilibrium outcome.

- We find that a particular two-part tariff can coordinate the supply chain. However, this contract involves the manufacturer charging a wholesale price strictly greater than its marginal cost in the first-period, and charging a wholesale price equal to its marginal cost in the second period. Although, this contract solves the coordination problem, it does not solve the time-consistency problem — the total distribution channel profits with this contract are the same as an integrated manufacturer’s profit when it sells directly to consumers.

- We show that by pre-committing to a two-part contract that covers both periods, the manufacturer can do better by going through a retailer than selling the product directly to consumers. When the manufacturer employs this contract, the total channel profits are identical to the channel profits when the manufacturer rents its products directly to consumers. These are the highest level of profits that a durable goods distribution channel can achieve — either by renting or by selling. Said differently, this contract solves not only the coordination problem, but also the time-consistency problem.

We contribute to the coordination literature by showing that the inter-temporal linkage in the durable good’s market changes the channel coordination problem. In addition to the double marginalization problem, the manufacturer also faces the problem of inducing the retailer to appropriately consider the competition between used and new goods. The traditional “selling
the firm to the retailer,” or “residual claimancy,” approach does not help the durable goods manufacturer. We also contribute to the durable goods literature by showing how a durable goods manufacturer can sell its product and solve its time consistency problem. Effectively, this allows the manufacturer to earn the same profits as it would get if it could commit to prices or if it could rent its product. When committing to individual consumers or renting have additional costs, our solution may be the optimal strategy for a durable goods manufacturer.

The remainder of this paper is organized as follows: In section 2, we lay out the basic model. Section 3 analyzes the problem of coordinating the durable goods distribution channel, and section 4 explores the role of linear and non-linear contracts. We conclude the paper in section 5.

2 Model

In this section, we lay out our assumptions about the product and the players in our analysis — the manufacturer, the retailer and the consumers. We assume a manufacturer produces a durable product at a constant marginal cost of production, \( c > 0 \). We assume that the retailer’s marginal costs are constant, and set them to zero without further loss of generality. In order to market this product to consumers, the manufacturer uses a retailer who purchases units from the manufacturer and sells them to consumers.

The product that is marketed is a durable that provides two periods of service. While the exact length of product life is not crucial, it is important to allow each unit of the product to last for more than one period.\(^2\) In our analysis, a unit sold in period 1 provides service in periods 1 and 2. In addition, we assume that once the product is sold, then all units of the

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\(^2\)Assuming the product lasts for two periods is equivalent to assuming that it becomes obsolete after \( n \) periods. It is only important to assume that the product last for a finite amount of time (Bulow 1982).
product deteriorate at an exogenous rate. In particular, once a consumer purchases a new unit in period 1, the product deteriorates with usage and becomes a used good in period 2. The extent of deterioration depends on the inherent durability, $\delta \ (0 \leq \delta \leq 1)$ of the product. That is, $\delta$ represents how well a unit sold in period 1 holds up in period 2, and $(1 - \delta)$ represents the extent of deterioration due to usage. If $\delta = 1$, the product does not deteriorate and new units are identical to used units. If $\delta = 0$, then the product has no durability and it deteriorates fully after one period of use.

An important assumption of our model is that consumers who purchase the product in period 1 always have the option of selling their used product in a secondary or used market in period 2. For example, there are active secondary markets for a variety of products such as automobiles, airplanes, white goods, etc. We assume that the used market is competitive and neither the manufacturer nor the retailer have any direct control over this market. As a consequence, used goods compete against new goods and the extent of competition between new and used units is directly related to the durability $\delta$. If the product is perfectly durable ($\delta = 1$), then new and used units are identical and new goods face the strongest competition from used goods. On the other hand, if the product is a non-durable (i.e., $\delta = 0$), used goods have no value and there is no secondary market for the product.

Unlike the case of a non-durable that is consumed in an instant, a durable product is “consumed” over time. In our model, because the product is assumed to provide services for two periods, then a consumer who purchases a product in period 1 gets two periods of use out of the product. We derive per-period prices for the product based on consumers’ valuations for the services provided by the product in a given period. Therefore, consumers who purchase the product in period 1 pay a total price that is the discounted sum of two per-period prices.

Define $r_{ij}$ as the one-period price in period $i$ of product $j$, where $i = 1, 2$ and $j$, can either be new ($n$) or used ($u$). Similarly, define $q_{ij}$ as the quantity sold in period $i$ of product $j$. A
product that is sold by the manufacturer (or retailer) to consumers in period one becomes a used product in period two, i.e., $q_{2u} = q_{1n}$. To capture the one period prices of new and used goods, we use the following inverse demand system:

\[
\begin{align*}
r_{1n} &= \alpha - q_{1n}, \\
r_{2n} &= \alpha - \delta q_{2u} - q_{2n}, \\
r_{2u} &= \delta (\alpha - q_{2n} - q_{2u}),
\end{align*}
\]

where $\alpha$ is a positive constant that captures the size of the potential market. We show in the Appendix that the above demand functions are derived from a well-specified consumer utility function. Because there are no used products available in period 1, we do not have an $r_{1u}$. Note that as $\delta$ increases, used goods deteriorate less and are closer substitutes for new goods; thus $r_{2u}$ increases and $r_{2n}$ decreases with $\delta$. Similarly, the quantity of used cars ($q_{2u}$) negatively affects the new car price in period 2, $r_{2n}$.

Equations (1–3) represent the per-period prices for the services delivered in the that period. Because new products deliver a higher quality of service, their price is also higher, $r_{2n} \geq r_{2u}$. Consumers who purchase the product in period 1 are getting two periods of service from the product — first as a new good and then as a used good. In terms of market prices, this means that the selling price of the new product in period 1, $p_{1n}$, should reflect all future service that the product will provide. Therefore,

\[
p_{1n} = r_{1n} + \rho r_{2u}
\]

where $0 < \rho < 1$ is a discount factor common to consumers and the firm.

The manufacturer in our model is assumed to play the role of a Stackelberg leader who announces a contract to the retailer. Based on this contract, the retailer chooses the number of units to order and sell on the market. We require that any contract that the manufacturer offers to the retailer must provide a non-negative profit to the retailer over the two-period horizon.
To rule out uninteresting cases, we also restrict our attention to those parameter values for which the demand in each period is strictly positive.

3 Integrated Channels

In this section we begin with an analysis of two benchmark cases — one in which the manufacturer sells directly to consumers and the other in which the manufacturer rents directly to consumers. Although both of these results are well established (e.g., Bulow 1982), we briefly review them in this section because they provide useful benchmarks for our subsequent model in which we add a retailer in the distribution channel.

3.1 Integrated Seller

We first consider the case of an integrated manufacturer that sells directly to consumers. In this case, the manufacturer chooses a sales quantity at the beginning of each period. To derive the subgame perfect equilibrium, we solve the model recursively. In period two, the manufacturer’s problem is to maximize profits, \( \pi_2 \), by choosing an optimal \( q_{2n} \). That is, the manufacturer maximizes \( \Pi_2 = (r_{2n} - c)q_{2n} \). This maximization problem yields:

\[
q_{2n}^* = \frac{\alpha - c - \delta q_{1n}}{2}.
\]  

(5)

From Equation (5), note that for any positive level of \( \delta \), as the firm sells more units in period 1, it increases the competition from the used market in period 2. Similarly, for any positive level of \( q_{1n} \), any increase in \( \delta \) increases the competition from the used market. This occurs because an increase in \( \delta \) makes a used product a stronger substitute for the new one. This creates an incentive for the firm to reduce its sales quantity in period 2. It is easy to see that for non-durable goods that are purchased repeatedly by consumers in each period, \( \delta = 0 \)
and first-period sales have no effect on second-period decisions.

Given the optimal $q_{2n}^*$ in period 2, the manufacturer maximizes its total discounted profits in the first period by choosing the optimal first period quantity, $q_{1n}^*$. That is, it maximizes,

$$\Pi_1 = (p_{1n} - c)q_{1n} + \rho \Pi_2^*$$

by choosing the optimal $q_{1n}^*$.

This yields

$$q_{1n}^* = \frac{2[(\alpha - c)(1 - \delta\rho)]}{4 + \delta\rho(4 - 3\delta)}$$

and

$$q_{2n}^* = \frac{1}{2}[\alpha - c - \frac{2\delta[\alpha - c(1 - \delta\rho)]}{4 + \delta\rho(4 - 3\delta)}].$$

(6)

### 3.2 Integrated Renter

The integrated renter rents the product to consumers in each period. In period 1, because there are no “used” units, the firm rents only new ones. However, in period 2, the firm rents both new as well as used units carried over from period 1. The integrated renter’s profit in period 2 is given by

$$\Pi_2 = r_{2n}q_{1n} + (r_{2n} - c)q_{2n}.$$  

The firm maximizes its total profits over the two period horizon,

$$\Pi_1 = (r_{1n} - c)q_{1n} + \rho \Pi_2,$$

by choosing optimal quantities for both periods. This yields:

$$q_{1n}^* = \frac{\alpha - c(1 - \delta\rho)}{2 + 2\delta\rho(1 - \delta)}$$

and

$$q_{2n}^* = \frac{1}{2}\left(\alpha - \frac{\alpha\delta + c[1 - \delta(1 - \rho)]}{1 + \delta\rho(1 - \delta)}\right).$$

Because a renter of a durable good does not have a problem with time consistency, it is well known that an integrated renter is more profitable than an integrated seller (Bulow 1982). In particular, the profits of an integrated renter represent the highest possible profits in the distribution channel. On the other hand, if an integrated seller could credibly commit to the quantities it would market, then it could replicate the results of an integrated renter.

$^3$Recall that $p_{1n} = r_{1n} + \rho r_{2n}$
It is important to emphasize that this result hinges on the manufacturer’s ability to credibly commit to these quantities. In other words, if consumers do not believe the manufacturer, then the results in Equations (8) are not feasible, because consumers anticipate the firm will renege on its commitment in period 2. In this event, the firm’s optimal strategy is given by Equation (6).

# 4 Decentralized Channels

Now we consider the case of a manufacturer that has to rely on the services of a retailer. The retailer purchases the product from the manufacturer and sells it to consumers. Below, we examine various contracts that the manufacturer can offer to the retailer, beginning with simple wholesale prices and then moving to two-part tariffs.

## 4.1 Wholesale Price Contract

First, we consider the case where the manufacturer charges a simple wholesale price in each period. Based upon the wholesale price announced at the beginning of each period, the retailer chooses the optimal quantity to order and sell on the market. In order to achieve the subgame perfect equilibrium, we solve the game recursively, beginning with the decisions in period 2.

In period 2, the retailer maximizes profits, \( \pi_{D2} = (r_{2n} - w_2)q_{2n} \), by choosing an optimal \( q_{2n} \), where \( w_2 \) is the wholesale price in period 2. This yields:

\[
q_{2n}^* = \frac{\alpha - w_2 - \delta q_{1n}}{2}. \tag{9}
\]

Given the retailer’s optimal choice, the manufacturer maximizes its period 2 profits, \( \pi_{M2} = \)
\[(w_2 - c)q_{2n}', \text{ by choosing an optimal } w_2. \text{ This yields:} \]

\[
w_2' = \frac{(\alpha + c - \delta q_{1n})}{2}. \tag{10}\]

In period one, the retailer maximizes its discounted profits,

\[
\pi_{D1} = (r_1 + \rho r_{2u} - w_1)q_{1n} + \rho \pi_{D2}, \tag{11}\]

by choosing an optimal \(q_{1n}\). This yields:

\[
q_{1n}' = \frac{\alpha(8 + 5\delta \rho) - 8w_1 + 3c\delta \rho}{16 + \delta \rho(16 - 5\delta)}. \tag{12}\]

Given the retailer’s optimal decisions, the manufacturer maximizes its discounted profits,

\[
\pi_{M1} = (w_1 - c)q_{1n}' + \rho \pi_{M2}, \tag{13}\]

by choosing its optimal \(w_1\). This yields:

\[
w_1' = \frac{\alpha[128 + 240\delta \rho - 45\delta^3 \rho^2 - 56\delta^2 \rho (1 - 2\rho)] + c \left[128 + 144\delta \rho - 11\delta^3 \rho^2 - 8\delta^2 \rho (5 - 2\rho)\right]}{32 (8 + 8\delta \rho - 3\delta^2 \rho)}. \tag{13}\]

It is straightforward to show that, as one would expect, a simple wholesale price contract does not solve the coordination problem for either durables or non-durables. In the following section, we attempt to solve the coordination problem in a durable goods channel by studying a variety of two-part tariffs.

### 4.2 Two-Part Tariffs

Now we consider the case of a distribution channel in which the manufacturer charges a two-part tariff consisting of a fixed and a variable fee. The fixed fee is a lump-sum payment and the variable fee is a constant wholesale price per unit ordered.
non-durables literature is that a two-part tariff coordinates the channel. In this coordinating contract, the manufacturer sets the wholesale price at marginal cost and extracts all the retailer profit through the fixed fee. This ensures that the retailer faces the same variable cost structure as an integrated manufacturer. In this strategy, the manufacturer essentially “sells” the firm to the retailer at a fixed fee and the retailer acts as the integrated manufacturer. In this section, we explore whether such an approach can solve the problem of managing a durable goods channel. In particular, we look at a series of two-part tariffs that explore whether coordination can occur through selling the firm to the retailer. We begin with a contract in which the manufacturer offers per-period two-part tariffs in which all wholesale prices are set at the manufacturer’s marginal cost. Subsequently, we allow wholesale prices to be different from marginal cost. Finally, we explore contracts in which the manufacturer commits to prices ahead of time.

4.2.1 Two-Part Tariff with \( w_1 = w_2 = c \)

It is well known that a two-part tariff in which the manufacturer sets the wholesale price at its marginal cost and extracts all the rents from the retailer through a fixed fee coordinates the channel for a nondurable. In this section we test whether or not such a contract can achieve the same goal in a distribution channel for a durable product.

We begin by assuming that the wholesale prices are given by \( w_1 = c \) and \( w_2 = c \), and the retailer maximizes its period 2 profits,

\[
\pi_D = (r_2 - c)q_2 - F_2.
\]

This leads to the optimal production quantity, \( q_{2n}^{\circ} \):

\[
q_{2n}^{\circ} = \frac{\alpha - c - \delta q_1 n}{2}.
\]

The manufacturer does not gain from leaving any positive profit for the retailer in period

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2. Therefore, it chooses $F_2$ to extract all the profit from the retailer.⁴

The problem in period 1 is solved in a similar manner and is not detailed here. When wholesale prices are constrained to equal marginal cost, the optimal fixed fees are given by:

$$
F_1^o = \frac{|2(\alpha - c) + (\alpha + c)\delta \rho|^2}{16 + 8\delta \rho(2 - \delta)}
$$

$$
F_2^o = \frac{1}{4}\left[\alpha - c - \frac{\delta(\alpha - c + \frac{1}{2}(\alpha + c)\delta \rho)}{2 + \delta \rho(2 - \delta)}\right]^2.
$$

(14)

This leads to the following proposition:⁵

**Proposition 1** A per-period two-part tariff with wholesale prices set at marginal cost does not coordinate the durable goods channel.

At first blush, it may seem that charging a two-part tariff in each period should coordinate the channel. However, in the case of durables, this turns out not to be the case. Note that the manufacturer always has an incentive to choose a second-period fixed fee that extracts all the second-period rents from the retailer. However, the retailer in period 1 anticipates this move by the manufacturer in period 2. Therefore, in making the first-period quantity decision, the retailer does not consider any effect of its decision on the outcomes in period 2. As a result, the retailer ends up choosing a first-period quantity that is too high from the manufacturer’s and channel’s perspectives. In other words, the optimal two-part tariff leaves the retailer with no stakes in the period 2 outcome, and recognizing this, the retailer acts as a single-period optimizer in period 1. This is the reason for the sub-optimality of the two-part tariff with marginal cost pricing.

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⁴We only require that the retailer earns a non-negative profit over the model horizon. Therefore, it is possible that the manufacturer can leave a negative profit for the retailer in period 2 and a positive profit for the retailer in period 1 such that the net present value of the two profits is non-negative. It turns out that such a solution leads to the same outcome as the solution described in here. More details are available from the authors.

⁵All proofs are in the Appendix.
An interesting aspect of this solution is that even while restricting the wholesale prices to marginal cost, “selling the firm” does not emerge as a solution. Under the current form of contract, selling the firm emerges as a solution only if the optimal second-period fixed fee were zero. This, however, is not an equilibrium because we find that the second-period fixed fee is always positive.

An interesting corollary to the above proposition is that the retailer’s period 1 selling quantity is in fact higher than that of an integrated manufacturer. In traditional channel coordination settings, because of the double marginalization problem, the retailer sells too low a quantity; hence the optimal two-part tariff coordinates the system by increasing the quantities to the optimal amount. What we discover in our setting is that a two-part tariff with marginal cost pricing leads the retailer to sell too much! Proposition 1 highlights why the channel coordination problem is different in the durable goods markets. In addition, this suggests that there are two possible approaches for the manufacturer to reach an equilibrium solution. The first solution is to charge a wholesale price above the marginal cost — a higher wholesale price can potentially bring down quantities to the appropriate level. The other possible solution is not to extract all the profits from the retailer in period 2, so that the retailer has some stakes in period 2 outcome. We consider both these possibilities next.

### 4.2.2 General Two-Part Tariff Contract

Now we consider the optimal two-part tariff without imposing any restrictions on the variable fee. As in the previous cases considered, we solve this model recursively, beginning with the retailer’s problem in period 2. The retailer maximizes period 2 profits,

\[ \pi_{D2} = (r_{2n} - w_2)q_{2n} - F_2, \]
by choosing the optimal \( q_{2n} \), subject to \( \pi_{D2} \geq 0 \). This yields:

\[
q_{2n}^* = \frac{\alpha - w_2 - \delta q_{1n}}{2}.
\]

Based upon this choice, the manufacturer maximizes its profits in period 2,

\[
\pi_{M2} = (w_2 - c)q_{2n} + F_2,
\]

by choosing the optimal \( w_2 \) and \( F_2 \). This yields: \( w_2^* = c \) and, as before, the manufacturer chooses \( F_2 \) such that the retailer earns no rents in period 2, \( \pi_{D2}^* = 0 \). The problem in period 1 is solved in a similar manner and is not detailed here. The period 1 wholesale price and fixed fee are given by:

\[
w_1^* = \frac{\alpha \rho \left[ 4 - 3\delta^2 \rho - 2\delta (1 - 2\rho) \right] + c \left[ 8 + 4\delta \rho + \delta^3 \rho^2 - 4\delta^2 \rho (1 + \rho) \right]}{8 + 2\delta \rho (4 - 3\delta)}, \quad (15)
\]

\[
F_1^* = \frac{2 \left[ \alpha - c (1 - \delta \rho) \right]^2 (2 + 2\delta \rho - \delta^2 \rho)}{(4 + 4\delta \rho - 3\delta^2 \rho)^2}. \quad (16)
\]

Note that for any \( \delta > 0 \), the optimal wholesale price in period 1 is greater than marginal cost, \( w_1^* > c \). When \( \delta = 0 \), then the product we are considering is a non-durable, and we get the standard result of marginal cost pricing, \( w_1^* = c \).

The retailer’s optimal quantities are given by:

\[
q_{1n}^* = \frac{2[\alpha - c (1 - \delta \rho)]}{4 + 4\delta \rho - 3\delta^2 \rho}, \quad (17)
\]

\[
q_{2n}^* = \frac{\alpha \left[ 4 - 3\delta^2 \rho - 2\delta (1 - 2\rho) \right] - c[4 - 2\delta (1 - 2\rho) - \delta^2 \rho]}{2(4 + 4\delta \rho - 3\delta^2 \rho)}. \quad (18)
\]

Comparing the quantities in Equations (17-18) with the quantities in Equation (6), it is clear that the above contract induces the retailer to choose coordinated quantities.

**Proposition 2** There exists a two-part tariff that coordinates the distribution channel. In this contract:
The manufacturer charges \( w_1^{**} \geq c \) in the first period and \( w_2^{**} = c \) in the second period.

The manufacturer’s profits are equal to those of an integrated manufacturer selling directly to consumers.

Thus, similar to the case of non-durables, there exists a two-part tariff that can coordinate the distribution channel for a durable product. However, there is an important difference — the manufacturer sells at marginal cost only in period 2; in period 1, it sells strictly above marginal cost. The rationale for setting wholesale price above marginal cost in period 1 can be understood from our discussion in the previous section. In particular, note that when the manufacturer sets the first-period wholesale price at marginal cost, the retailer’s optimal sales quantity in period 1 is higher than the coordinated quantity. Therefore, the manufacturer raises its wholesale price above its marginal cost, effectively increasing the retailer’s per-unit costs and, in turn, inducing it to choose a lower sales quantity. Although this two-part tariff effectively coordinates the distribution channel to the level of an integrated seller, the distribution channel as a whole still suffers from the same time-consistency problem that an integrated seller of a durable good would face.

### 4.2.3 Fixed Fee Commitment

Proposition 1 shows that when the manufacturer extracts all the retailer’s profits in period 2, the retailer has no stake in the outcome of period 2. As a result, it ignores the inter-temporal linkages, and sells more than what the manufacturer would want it to. Earlier we showed how selling above marginal cost is a solution to this problem. Below we consider another possible solution — by not extracting all the profits from the retailer in period 2, the manufacturer gives the retailer a stake in the outcome in period 2. If the manufacturer has to choose the period 2 fixed fee, \( F_2 \), at the end of period 1, then its incentives are to choose \( F_2 \) to extract all the period 2 profit from the retailer. Therefore, in period 1 the retailer will anticipate this
behavior of the manufacturer, and we will revert back to the result in Proposition 1. The only way the manufacturer can convince the retailer that it will not extract all the retailer profits in period 2 is to commit to a specific value of $F_2$ at the beginning of period 1, before the retailer chooses the first-period quantity. In other words, the manufacturer needs to offer a contract that specifies fixed fees for both periods at the beginning of period 1. It is easy to see that this approach is equivalent to charging a single fixed fee. Note that we see similar arrangements in practice in which retailers pay a one-time fixed (lump sum) fee to obtain the rights to sell a manufacturer’s products and then pay per-unit wholesale fees for the products that they buy.

Our specific interest in analyzing this type of contract is to see if the equilibrium wholesale prices are equal to the marginal cost. The reason is that if the equilibrium solution involves the manufacturer setting the wholesale prices at marginal cost, then this solution is equivalent to one where the manufacturer simply sells the firm to the retailer at a price equal to the fixed fee. This leads to the following:

**Proposition 3** *When the manufacturer commits to a fixed fee, it charges wholesale prices above marginal cost in both periods.*

Proposition 3 shows that even when the manufacturer can commit to fixed prices, selling the firm is not an equilibrium outcome. The manufacturer who sells through a retailer charges wholesale prices above marginal cost in both periods. The reason is that once the manufacturer sets the fixed fee at the beginning of period, it gains from charging a wholesale price above the marginal cost in period 2. As a result, the period 1 wholesale price is also above marginal cost. However, this suggests that if the manufacturer can commit to both — the wholesale prices and fixed fees — for both periods at the beginning of period 1, then it can better alleviate the channel coordination problem. We discuss such a contract next.
4.2.4 A Commitment Contract

In this case, we consider the case of a manufacturer who can write a contract in period 1 that covers both periods. In particular, the manufacturer announces a fixed fee, $F$, and wholesale prices, $w_1$ and $w_2$, at the beginning of period 1. Note that charging one fixed fee for both periods versus charging two different fixed fees — one for each period — does not make any difference in the current contract. It is straightforward to see that the single fixed fee in the current contract can be broken up into two fixed fees without changing the outcome.

The game is solved recursively by first solving the retailer’s problem in period 2, followed by its problem in period 1. The retailer’s problem is solved recursively because the retailer is not able to credibly commit to future prices to the market as a whole. On the other hand, the manufacturer can make a credible price commitment to a single retailer. Given the retailer’s optimal choices, the manufacturer maximizes its total profits by choosing the optimal contract — the fixed fee and both wholesale prices.

The retailer’s optimal quantities are:

\[ q_{1n}^{***} = \frac{\alpha - c (1 - \delta \rho)}{2 + 2\delta \rho - 2\delta^2 \rho}, \tag{19} \]

\[ q_{2n}^{***} = \frac{\alpha (1 - \delta)(1 + \delta \rho) - c[1 - \delta (1 - \rho)]}{2 + 2\delta \rho - 2\delta^2 \rho}. \tag{20} \]

Comparing Equations (19-20) with Equation (8), it is clear that the current contract results in the same quantities that an integrated renter would choose and results in the highest profits for the distribution channel.
The optimal wholesale prices and fixed fee are as follows

\[ w_1^{****} = \frac{\alpha \delta^2 \rho + c (4 + 4 \delta \rho + \delta^3 \rho^2 - 5 \delta^2 \rho)}{4 + 4 \delta \rho - 4 \delta^2 \rho}, \]
\[ w_2^{****} = \frac{\alpha \delta + c (2 - \delta) (1 + \delta \rho)}{2 + 2 \delta \rho - 2 \delta^2 \rho}, \]
\[ F^{****} = \frac{2A c \alpha + B c^2 + D \alpha^2}{8[1 + \delta \rho (1 - \delta)]^2}, \]

where \( A, B, D \) are as given in the Appendix.

It is easy to see that \( w_1^{****} \geq c \) and \( w_2^{****} \geq c \) with the equalities holding only for \( \delta = 0 \) (the product is non-durable). Thus, when \( \delta = 0 \), our results reduce to the traditional marginal cost pricing results for non-durable manufacturers.

**Proposition 4** In a distribution channel that consists of a durable product manufacturer who sells to a retailer who sells to consumers, a two-part tariff with commitment coordinates the distribution channel and achieves the highest level of profits. The optimal wholesale price in this contract is above marginal cost in each period.

By committing not only to a specific fixed fee but also to specific wholesale prices, the manufacturer induces the selling retailer to choose the same quantities as an integrated renter. In the absence of the retailer, an integrated seller would not be able to choose these quantities without making credible commitments to individual consumers. It is often the case that a commitment to a retailer is less costly, more credible and ultimately more enforceable than commitments to numerous individual consumers. In such cases, having a retailer helps the manufacturer solve the time consistency problem and allows it to earn greater profits than it would if it were an integrated seller.

Interestingly, in non-durable goods settings, the best that a decentralized manufacturer can do is obtain the profits of an integrated manufacturer. On the other hand, a durable goods seller can do even better by selling through a retailer — it can earn the same profits as an
integrated renter. If renting the same unit multiple times involves additional marketing costs, then it may be the case that selling through a retailer may even dominate renting directly to consumers.

Finally, we note that in this type of contract also, selling the firm to the retailer is a feasible solution. However, it doesn’t turn out to be the equilibrium outcome.

5 Conclusions

This paper introduces the problem of managing a channel for a durable good. This is a complex problem because not only does the manufacturer have to deal with the problem of coordination but also the problem of time-consistency. The problem of coordination arises because of the need to align the manufacturer’s and retailer’s goals. On the other hand, the time-consistency problem arises because of consumers beliefs that future market prices of the durable will fall. For a manufacturer that can only sell its product to consumers, the only way out of this problem is to make a commitment not to lower prices in the future. While a commitment such as this is certainly possible — and it may even be honored by a firm that cares about its reputation — the fact remains that, *ex post* it is not time-consistent and the manufacturer has an incentive to deviate from its *ex ante* commitment. In addition, it is difficult, although not impossible, for a firm to make numerous commitments to individual consumers in the market. Similarly, although renting the product eliminates the problem of time-consistency, it suffers from other problems such as consumer moral hazard and the need to market the product multiple times in its lifetime. In this paper, we offer a simpler way out of this problem: Make a price commitment with a two-part tariff to a retailer. The beauty of this contract is that it solves not only the coordination problem but also the retailer’s time-consistency problem. More importantly, the price commitment is also credible, because it is made through a contract between a manufacturer and a retailer. Business-to-business contracts such as this are readily
verifiable and enforceable. Thus, our analysis shows that selling can be as profitable as renting. And if we consider additional costs of renting or leasing, selling can be more profitable. This also offers another explanation for why leasing is not as widespread as theoretical predictions would suggest.

We also derive interesting insights about the channel coordination problem for a durable goods manufacturer. We show that under certain conditions, a retailer may sell more than an integrated manufacturer. To induce the retailer to sell fewer quantities of the durable, the wholesale price exceeds the manufacturer’s marginal cost. We also show that a manufacturer can gain by committing to future terms of trade with the retailer. The reason is that the channel coordination problem is worsened by the manufacturer’s incentives to act opportunistically in the future. Once the retailer makes his first-period decisions, the manufacturer has incentives to extract all the profits from the retailer in period two. By making a contractual commitment not to act in such a way, the manufacturer makes the channel coordination task easier.

The results in this paper are important in establishing how firms can coordinate the channels for durable products. We find that the specifics of the contract vary with the inherent durability of the product and the extent of competition posed by the secondary market. Our analysis has implications for any durable product category in which manufacturers sell through intermediaries. By incorporating the strategic impact of consumer strategies in a multi-period durable-goods setting, we are able to add insights to the channels literature. In particular, we argue that product durability plays a crucial role in manufacturer relationships with retailers. We acknowledge that we have treated durability as an exogenous variable and assumed that there is no uncertainty about demand. Clearly manufacturers have some control over the durability that they build into a product and they often face uncertain demand. We intend to explore both these avenues in future research.
References


Appendix

Derivation of Demand Functions

We model consumers who are heterogeneous in their valuations of the durable good. In this vertical differentiation model, we use the parameter $\phi \in (0, \alpha)$ to represent a consumer’s valuation of the per-period service provided by a new product. Note that a consumer with a higher $\phi$ values the product more than a consumer with a lower $\phi$. Finally, we assume that $\phi$ is distributed uniformly in the interval $[0, 1]$ and, in any period, each consumer uses at most one product. Recall from our discussion earlier that the product deteriorates as it ages. Thus, a consumer’s valuation of the per-period services from a used product are $\delta \phi$.

The net utility from using a product for a single period is:

$$U = \delta^m \phi - r \quad (A1)$$

where $m$ is an indicator variable such that $m = 0$ if the product is new and $m = 1$ if the product is used, and $r$ is the one-period price.

In equilibrium, consumers choose one of the following four strategies: (i) buy a new product in period 1 and, in period 2, sell their used product and buy a new product (BB); (ii) buy a new product in the first period and hold onto it in the second period (BH); (iii) remain inactive in period 1 and buy a used product in period 2 (IU); and (iv) be inactive in both periods (II). In terms of consumer utility, it can be shown that if all four strategies are observed in equilibrium, then consumers who follow a BB strategy value the product more (i.e., have a higher $\phi$) than consumers who follow a BH strategy, who value it more than consumers who follow an IU strategy, who value it more than consumers who follow an II strategy.

First consider the consumers in period 2. Following the notation in the text, let $q_{1n}(= q_{2u})$
and \( q_{2n} \) be the number of products sold in period 1 and 2, respectively. Consider the lowest valuation consumer who adopts an IU strategy. This consumer is located at a point \( \phi_3 = \alpha - q_{1n} - q_{2n} \) on the \([0, \alpha]\) line and has to be indifferent between following an IU and an II strategy. From Equation (A1), this consumer’s net utility from an IU strategy is \( \delta(\alpha - q_{1n} - q_{2n}) - r_{2u} \), and the utility from following an II strategy is zero. Equating these two utilities, we get the following demand for used products:

\[
r_{2u} = \delta(\alpha - q_{2u} - q_{2n}). \tag{A2}
\]

Now consider the lowest valuation consumer who adopts a BB strategy. This consumer is located at a point \( \phi_1 = \alpha - q_{2n} \) and has to be indifferent between BB and BH strategies. In period 2, the net utility from a BB strategy is \( \alpha - q_{2n} - r_{2n} + r_{2u} \).\(^6\) Similarly, the net utility from holding onto the product in period 2 is \( \delta(\alpha - q_{2n}) \). Equating these two utilities yields the one-period price for new products in period 2:

\[
r_{2n} = \alpha - q_{2n} - \delta q_{2n}. \tag{A3}
\]

Now consider the consumers in period 1 and assume that they are making a decision about using the services of the product for a single period. The last consumer who uses a product in period 1 is located at a point \( \alpha - q_{1n} \). This consumer has to be indifferent between using this product for one period and staying out of the market in this period. Equating the utilities from these two strategies yields the one-period price in period 1:

\[
r_{1n} = \alpha - q_{1n}. \tag{A4}
\]

Now consider the two-period horizon when consumers are buying the product in period 1. At this stage, the marginal consumer (located at \( \alpha - q_{1n} \)) should be indifferent between following following

\(^6\)Recall that a BB strategy means that the consumer buys a new product in period 1, sells it at a price \( r_{2u} \) in period 2 and buys another new product at a price of \( r_{2n} \).
a BH strategy and an IU strategy. That is, this consumer can either buy a product now and continue using the same product in period 2, or buy nothing now and wait until period 2 to buy a used product. Equating the net utilities of the BH and IU strategies of this consumer implies that

$$\alpha - q_1n - p_1n + \rho[\delta(\alpha - q_1n)] = \rho[\delta(\alpha - q_1n) - r_{2u}].$$

This yields the selling price of the product in period 1:

$$p_{1n} = r_{1n} + \rho r_{2u}.$$  \hfill (A5)

## Proofs of Propositions

### Proposition 1

An integrated seller’s profit, $\Pi^s$, is given by:

$$\Pi^s = \frac{4(\alpha - c)^2 + 4\rho(\alpha - c)(\alpha + c(-1 + 2\delta)) + \delta\rho^2(4(\alpha - c)^2 + \delta(c^2 + 6c\alpha - 3\alpha^2))}{16 + 4\delta\rho(4 - 3\delta)}. \hfill (A6)$$

A decentralized seller’s profit when charging a two-part tariff with $w_1 = w_2 = c$, $\Pi^\circ$, is given by

$$\Pi^\circ = \frac{(2\alpha - 2c - \rho\delta(c + \alpha))^2}{16 + 8\delta\rho(2 - \delta)} + \frac{\rho}{4} \left(\alpha - c - \frac{\delta(\alpha - c + \frac{1}{2}(\alpha + c)\delta\rho)}{-2 + \delta\rho(2 - \delta)}\right)^2. \hfill (A7)$$

In addition, the difference

$$\Pi^s - \Pi^\circ = \frac{\delta^2\rho^2[c(-4 + \delta(2 - 4\rho) + \delta^2\rho) + \alpha(4 - 3\delta^2\rho + \delta(-2 + 4\rho))]^2}{16(2 + 2\delta\rho - \delta^2\rho)^2(4 + 4\delta\rho - 3\delta^2\rho)} > 0. \hfill (A8)$$
**Proposition 2**

From Equations (6), (17) and (18), it is clear that the \( \{w_1^{\ast\ast}, w_2^{\ast\ast}, F_1^{\ast\ast}, F_2^{\ast\ast}\} \) contract results in coordinated selling quantities. The manufacturer’s profit, \( \Pi^{\ast\ast} \), is given by,

\[
\Pi^{\ast\ast} = \frac{4(\alpha - c)^2 + 4\rho(\alpha - c)(\alpha + c(-1 + 2\delta)) + \delta\rho^2(4(\alpha - c)^2 + (c^2 + 6\alpha - 3\alpha^2)\delta)}{16 + 4\delta\rho(4 - 3\delta)} = \Pi^* \quad (A9)
\]

and the manufacturer’s margin is given by

\[
w_1^{\ast\ast} - c = \frac{\delta\rho[\alpha (4 - 2\delta + 4\delta\rho - 3\delta^2\rho] - c (4 - 2\delta + 4\delta\rho - \delta^2\rho)]}{8 + 2\delta (4 - 3\delta)\rho}. \quad (A10)
\]

For the second-period sales,  \( q_2^{\ast\ast} \), to be positive, \( \alpha > c \frac{4 - 2\delta + (4 - \delta)\delta\rho}{4 - 2\delta + (4 - 3\delta)\rho}\). Therefore, for any value of \( \delta > 0 \), \( w_1^{\ast\ast} - c > 0 \).

**Proposition 3**

The fixed fee and the wholesale prices that the manufacturer charges in equilibrium are given by:

\[
F^{***} = \frac{256(\alpha - c)^2 + 16\rho(\alpha - c)G + 4\delta\rho^2H + \delta^2\rho^3J}{4 (-16 + \delta (-16 + 7\delta)\rho)} \quad (A11)
\]

\[
w_1^{***} = \frac{\alpha\delta\rho[8 - \delta(4 - 8\rho) - 5\delta^2\rho] + c[32 + 24\delta\rho + \delta^2\rho^2 - 2\delta^2\rho(5 + 4\rho)]}{32 + 2\delta\rho(16 - 7\delta)},
\]

\[
w_2^{***} = \frac{1}{2}\left[\alpha + c - \frac{\delta[\alpha(8 + 3\delta\rho) - c(8 - 5\delta\rho)]}{16 + 16\delta\rho - 7\delta^2\rho}\right],
\]

where

\[
G = \alpha [4 + \delta(28 - 5\delta)] - c [4 - \delta(4 + 5\delta)],
\]

\[
H = \alpha^2 [32 + \delta(43 - 15\delta)] - 2c \alpha [32 - 5\delta (9 - \delta)] + c^2 [32 - \delta (69 - 25\delta)],
\]

\[
J = \alpha^2 (16 - \delta) (4 - \delta) + c^2 [64 - \delta (44 - 19\delta)] - 2c \alpha [64 - \delta (116 - 31\delta)].
\]
Because the margins

\[ w_1^{**} - c = \delta \rho \frac{4 (\alpha - c) (2 - \delta) + \delta \rho (\alpha (8 - 5 \delta) - c (8 - \delta))}{32 + 2 \delta \rho (16 - 7 \delta)} > 0, \]

\[ w_2^{**} - c = \frac{1}{2} \alpha - 3 c - \delta \frac{\alpha (8 + 3 \delta \rho) - c (8 - 5 \delta \rho)}{16 + \delta \rho (16 - 7 \delta)} > 0, \]

it is clear that the manufacturer does not sell the firm to the retailer.

**Proposition 4**

The optimal wholesale prices and franchise fee with commitment are:

\[ w_1^{***} = \frac{\alpha \delta^2 \rho + c (4 + 4 \delta \rho + \delta^2 \rho^2 - 5 \delta^2 \rho)}{4 + 4 \delta \rho - 4 \delta^2 \rho}, \]  
(A12)

\[ w_2^{***} = \frac{\alpha \delta + c (2 - \delta) (1 + \delta \rho)}{2 + 2 \delta \rho - 2 \delta^2 \rho}, \]  
(A13)

\[ F^{***} = \frac{2 A \alpha + B c^2 + D \alpha^2}{8 [1 + \delta \rho (1 - \delta)]^2}, \]  
(A14)

where

\[ A = (2 + (2 - \delta^2) \rho + 2 \delta (2 - 3 \delta + \delta^2) \rho^2 + 2 (1 - \delta)^2 \delta^2 \rho^3, \]

\[ B = (2 + (2 - 4 \delta - \delta^2) \rho + 2 \delta (2 - 3 \delta + 2 \delta^2) \rho^2 + (2 - \delta^2) \delta^2 \rho^3, \]

\[ C = (2 + (2 - 2 \delta - \delta^2) \rho + \delta (4 - 7 \delta + 3 \delta^2) \rho^2 + (2 - 3 \delta + \delta^2) \delta^2 \rho^3. \]

The optimal sales quantities are:

\[ q_1^{***} = \frac{\alpha - c (1 - \delta \rho)}{2 + 2 \delta \rho - 2 \delta^2 \rho}, \]  
(A15)

\[ q_2^{***} = \frac{\alpha (1 - \delta) (1 + \delta \rho) - c [1 - \delta (1 - \delta)]}{2 + 2 \delta \rho - 2 \delta^2 \rho}. \]  
(A16)

It is easy to see that the selling quantities are identical to the quantities of a monopolist retailer in Equation (8). It can also be shown that the manufacturer’s profit under this contract, \( \Pi^{***} \),

\[ \Pi^{***} = \frac{\alpha^2 [1 + (1 - \delta) \rho] (1 + \delta \rho) + c^2 [1 + (1 + \delta (-2 + \rho)] \rho] - 2 c a [1 + (1 - \delta) \rho (1 + \delta \rho)]}{4 + 4 (1 - \delta) \delta \rho} \]  
(A17)
is identical to the profit of the manufacturer when it rents the good directly to consumers. In addition, the margins in this case are given by:

\[
\begin{align*}
    w_{1}^{****} - c &= \frac{\delta^{2} \rho [\alpha - c (1 - \delta \rho)]}{4 + 4 (1 - \delta) \delta \rho}, \quad (A18) \\
    w_{2}^{****} - c &= \frac{\delta [\alpha - c (1 - \delta \rho)]}{2 + 2 \delta \rho - 2 \delta^{2} \rho}. \quad (A19)
\end{align*}
\]

For the first-period quantity to be positive, \( \alpha > c (1 - \delta \rho) \). With this restriction, it is easy to see that \( w_{1}^{****} > c \) and \( w_{2}^{****} > c \).