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Do international roaming alliances harm consumers?

Benno Bühler †

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Abstract

We develop a model of international roaming in which mobile network operators (MNOs) compete both on the wholesale market to sell roaming services to foreign operators and on the retail market for subscribers. The operators own a network infrastructure only in their home country. To allow their subscribers to place or receive calls abroad, they have to buy roaming services provided by foreign MNOs. We show that in absence of international alliances and capacity restrictions, competition between foreign operators would drive wholesale unit prices down to marginal costs. However, operators prefer to form international alliances in which members mutually provide roaming services at inefficiently high wholesale prices. Alliances serve as a commitment device to soften competition on the retail market and harm consumers through excessively high per call prices. Although operators compete in two-part tariffs for subscribers, wholesale roaming prices do not exhibit profit-neutrality as do access prices in related models of network interconnection. We also show that international alliances are endogenously formed if not prevented by regulation.

Keywords: International roaming, vertical relations, regulation

JEL classification: D43; L13; L42; L96

1 Introduction

The European market for international roaming accounts for approximately €8.5 billion or 5.7% of the estimated total mobile industry revenues in 2005 (European Commission, 2006). According to the European Commission the market for international roaming is highly profitable and expected to further grow during the next years. By January 2006, roaming contributed about 12% to the European mobile industry profits (European Commission, 2006, p.78). This paper considers the formation of international alliances...
as an explanation for high and persistent roaming profits even though operators compete on the retail and wholesale level.

International roaming provides subscribers with the possibility to use their mobile phone outside their own country, where their home network operator has no coverage. More precisely, international roaming allows subscribers to use the infrastructure of a visited network in order to make and receive calls abroad. In order to provide their subscribers with this possibility, Mobile Network Operators (MNOs) need to conclude international roaming agreements with other MNOs in the foreign countries. When a MNO allows subscribers of a foreign operator to access its network it acts as host operator. For roaming services, the foreign operators charge wholesale prices to the roaming subscribers’ home operator which in turn charge retail prices to their subscribers. As wholesale roaming charges appear as costs for home operators they have a direct impact on the retail prices. Summing up, operators are typically active on two markets: They offer roaming services to foreign operators and buy roaming services abroad on the wholesale market. In addition, they compete in their home country on the retail market for subscribers.

In 2006, the European Commission assessed that both the average roaming retail and wholesale prices were unjustifiably high (European Commission, 2006). For example, it estimated that the per-minute costs (including a margin for fixed costs) of an outgoing roaming call are approximately 20 cents, while the wholesale prices are on average about 75 cents and retail prices are roughly €1.10 (European Commission, 2006, p. 20). Hence the wholesale prices are estimated to amount roughly 4 times the costs for originating, transmitting and termination of outgoing roaming calls. This raises the question why competition has not been effective in the case of roaming.

In this paper we argue that international alliances of MNOs may result in collusively high wholesale prices which would not be sustainable otherwise. Recently, such alliances have been formed claiming to facilitate the provision of roaming services. Affiliate operators typically agree on special roaming wholesale conditions based on the promise to direct roaming traffic preferably to other alliance members. We show that because of strategic considerations MNOs prefer to form alliances in order to commit to procure and to sell roaming services at high prices. Setting high wholesale prices within an alliance allows to soften competition on the retail market and thereby increases total profits.

In our model, in each of two equally sized countries two MNOs compete on the retail market à la Hotelling for subscribers. We ignore nationwide calls and focus instead on subscribers’ demand for roaming calls abroad. To provide this service,
each operator needs to access to foreign operators’ infrastructure. Operators may form international alliances and promise to procure roaming services exclusively from their partner network. In this case they negotiate jointly on wholesale prices. Operators may also post wholesale prices and buy roaming services without being affiliated in an alliance. They first set the wholesale roaming prices and decide from which foreign operator to buy roaming services. Then they offer two-part retail tariffs to potential subscribers in their home country.

One important peculiarity of roaming compared to models of network-interconnection is that there is no “competitive bottleneck” in the sense that no particular foreign operator has to provide the roaming services. Operators may thus freely choose with which foreign MNO they conclude a roaming agreement. Competition among foreign operators to act as host operator for a MNO should therefore drive down wholesale prices for roaming services. However, we show that a preferred alternative is to form international alliances and to mutually agree on high wholesale roaming prices. Within an alliance each member acts as home operator for its own subscribers and as host operator for the partner’s foreign subscribers. High wholesale prices are perceived as high marginal costs and hence render it optimal to set higher retail prices, thereby softening price competition in the retail market. However, expenses on the wholesale market are retrieved by providing roaming services to the subscribers of the partner network as long as the international traffic is balanced. In short, as retail prices are strategic complements when competing for subscribers on the home market, committing to high retail prices via high wholesale prices allows to raise the equilibrium profits.

Our findings are interesting in the light of recent technological developments that have increased the strategic importance of roaming alliances. Until recently, operators had limited technical instruments to determine which foreign network their subscribers would use. Subscribers that did not manually register in a particular foreign network were assigned almost randomly among foreign operators. Having little control over the foreign network which traveling subscribers would use, operators could hardly commit to keep the roaming traffic within partner networks. In addition, not being able to direct subscribers to networks that offer cheaper roaming services induced MNOs to charge high wholesale prices. By now, technologies have been developed that allow to direct roaming traffic. In 2006, the European Commission estimated that roughly 80% of the roaming traffic was already actively directed by the use of these technologies (European Commission, 2006, p. 24). If operators have the ability to select the host network they may commit by help of alliances to use the networks of other affiliated operators even if these charge higher wholesale prices.

Relative to the existing literature, we find novel and surprising results. Our model

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7In models of interconnected networks subscribers usually become member at one particular network such that this operator becomes monopolist for the access to this subscriber. The fact that there is ex ante competition for subscribers but an de-facto monopoly of access ex post is denoted as “competitive bottleneck”. See e.g. Armstrong (2002); Armstrong and Wright (2007).

8For an detailed technical description, see e.g. Stumpf (2001), Salsas and Koboldt (2004) or European Commission (2005).

9We show in section 7, that in the absence of control regarding the host network the wholesale prices may even exceed the monopoly level.
exhibits what Carter and Wright (1994) call *symbiotic production*: Each operator offers roaming services as intermediate products to foreign operators, and resells roaming services from foreign operators to own subscribers. Similar to Carter and Wright (1994) but unlike the models of national telecommunication, operators of different countries do not compete for the same subscribers. Carter and Wright (1994) assume that there is only one operator in each country and find that double marginalization leads to inefficiently high retail prices. They conclude that both operators and consumers would be better off if operators cooperated and bilaterally reduced their wholesale prices. In contrast, we show that this argument does not apply when there is competition both on the retail and intermediate product markets. In absence of alliances, competition between foreign operators drives down wholesale prices and competition in two-part tariffs on the home market induces operators to avoid dead-weight losses by offering calls at perceived marginal costs and assure profits via a fixed payment. Hence in our model, subscribers could place roaming calls at an efficient price level if international alliances were forbidden.

The role of the wholesale roaming prices in our setup is similar to that of the access prices in the two-way network interconnection model of Laffont, Rey, and Tirole (1998a). They find that higher access prices may soften competition and produce higher equilibrium prices if network operators compete on the retail market in linear prices. However, they also show that if operators compete in two-part tariffs, the collusive power of access prices vanishes. In contrast, we find that higher wholesale prices allow to raise profits even though firms compete in two-part tariffs on their home market. In the roaming market, directly competing operators of one country cannot soften competition simultaneously as they need access to foreign infrastructure. Therefore, if one operator enters into an international alliance and agrees on high wholesale roaming prices, the competitor's perceived costs for roaming services remain unchanged. In contrast to Laffont, Rey, and Tirole (1998a), our line of reasoning relies on strategic complementarity of retail tariffs: If one operator commits to higher wholesale prices, it optimally offers less favorable contracts to subscribers. The domestic competitor reacts by offering also less attractive tariffs and due to softer competition, both operators' profits increase. This might explain why domestic competitors did rarely complain when international alliances were formed.

There are also conceptual similarities to the literature of vertical relationships.\footnote{See e.g. Armstrong (2002) for an overview.} In particular, Shaffer (1991) shows that downstream firms might prefer to pay higher unit prices for intermediate goods and receive a fixed compensation instead of low unit prices if this serves as a commitment device to soften downstream competition. Similarly, in our model, operators prefer to commit to high wholesale price to soften competition. However, our reasoning does not rely on fixed payments to compensate higher unit prices since operators mutually provide roaming services in an alliance. In addition, the existing literature has analyzed competition in linear prices on the downstream market so far. To our knowledge, we are the first who show that operators may also exploit strategic complementarity even though competing in nonlinear prices in the downstream market.

Recently, a small literature that also analyzes the international roaming market

\footnote{See e.g. Bonanno and Vickars (1988), Shaffer (1991) and Rey and Stiglitz (1995).}
emerged. Salsas and Koboldt (2004) as well as Lupi and Manenti (2006, 2008) also consider a setup of two operators in each of two countries.\footnote{Tsyganok (2008) also analyzes international roaming.} However, Salsas and Koboldt (2004) do not explicitly take into account that each operator is active both on the wholesale market and on the retail market and therefore do not consider the possibility of international alliances. Another difference of their base setup to our model is their assumption that roaming traffic cannot be directed to a particular foreign network. They find that if roaming traffic is allocated randomly across foreign networks, the resulting wholesale price even exceeds that of a monopolist.\footnote{In an extension, Salsas and Koboldt (2004) assume that traffic can be (partially) directed to the cheapest foreign operator and they find that this assumption drives wholesale prices down.} Compared to Salsas and Koboldt (2004), our contribution is to show that it may be advantageous to reciprocally commit to direct roaming traffic to a partner network. Lupi and Manenti (2006, 2008) assume that operators act as local monopolists on the retail market. Therefore, they do not analyze operators’ incentives to set high wholesale prices in order to soften retail competition. In their setup, alliances optimally set wholesale prices at marginal costs, which is not in line with the current evidence.\footnote{Lupi and Manenti (2008) also analyze wholesale prices that arise in case operators may grant loyalty discounts if all addressable traffic is directed to the same visited network. They find that loyalty discounts may lead to high wholesale prices.} In contrast to Lupi and Manenti (2008), in our model alliances arise endogenously.

The remainder of the paper is organized as follows: In the next section, we formally introduce our basic model where alliances are exogenously given. Section 3 characterizes the equilibrium retail tariffs for given wholesale prices. In section 4 we show that wholesale prices would be set equal to marginal costs in the absence of international alliances. Section 5 considers the case where all operators have formed competing international alliances and shows that operators set inefficiently high wholesale prices. Section 6 extends our basic model by adding a first stage in which alliances can be formed. As a result two competing alliances may emerge endogenously in absence of regulatory constraints. In section 7 we formally derive that recent improvements in the technology of network selection have augmented the role of international alliances. Section 8 offers various extensions that mainly serve as robustness checks before we conclude in section 9.

2 The Model

There are two countries $A$ and $B$ as well as two MNOs with index 0 and 1 in each country. Operator $x_i$ is active in home country $x \in \{A, B\}$ and has position $i \in \{0, 1\}$. We assume that each operator’s network covers only its home country. Every operator participates in two related markets: Firstly, each operator competes with its domestic competitor on the retail market for subscribers which live in the operator’s home country. Secondly, in the wholesale market each operator offers roaming services to foreign operators and buys these services in order to resell them to own subscribers. These wholesale agreements are thus established between two operators of different nationality.

Cost structure: Each of the four operators incurs the same marginal cost $c \geq 0$ when a subscriber places a roaming call. This marginal cost includes origination, trans-
fer and termination. For simplicity, we assume that all roaming calls are terminated at some third party fixed network so that we can abstract from traffic generated by the termination of roaming calls. In addition, operators have to incur monthly fixed costs $C_F$ per subscriber, e.g. for billing.

**Retail pricing structure:** We focus on outgoing roaming calls that subscribers may place while traveling abroad and assume that it is the only service which MNOs offer to their subscribers. In particular, we abstract from nationwide calls. Operators offer a two-part tariff: Operator $x_i$ charges a usage price $p_{xi} \in \mathbb{R}$ per roaming call from abroad and a (monthly) fixed fee $F_{xi} \in \mathbb{R}$. When a consumer places $q$ roaming calls, she has to pay in total $p_{xi}q + F_{xi}$.

**Retail demand structure:** As in Laffont, Rey, and Tirole (1998a), networks are differentiated à la Hotelling. In each country, consumers are uniformly located on the segment $[0, 1]$. The operators are located at the two extremities and the index $i \in \{0, 1\}$ also indicates their position. Each consumer may join at most one network. Being connected to any network generates a fixed surplus $v_0$. Placing roaming calls generates a *gross surplus* $u(q)$. Consumers have quasilinear preferences in wealth such that the (incremental) utility of a consumer with taste $l$ who joins operator $x_i$ and places $q$ roaming calls is:

$$-\frac{1}{2\sigma}|l - l| + u(q) - p_{xi}q - F_{xi} + v_0$$

The term $-\frac{1}{2\sigma}|l - l|$ expresses the loss of utility in case the joined network does not correspond exactly to the consumers taste where $\sigma > 0$ parameterizes the degree of taste differentiation. A consumer that does not join either network receives utility that is normalized to 0. For technical convenience, we assume that joining a network is sufficiently attractive (i.e. $v_0$ is high enough) so that all subscribers join a network on the relevant range of prices. Preferences are the same in both countries. Note that consumers care only about their domestic operator, not about which foreign operator handles their roaming calls.

The optimal individual demand and the resulting consumers’ value from roaming calls are defined as follows:

$$q(p) \equiv \arg \max_q \{u(q) - pq\}$$

$$v(p) \equiv u(q(p)) - pq(p)$$

---

15Further services such as nationwide calls could be included in the model at the cost of tractability. Due to competition in two part tariffs, usage prices would be set equal to perceived marginal costs. See also footnote 16 below.

16While we use $v_0$ to assure that the market is covered, this term may represent the net surplus generated by services other than roaming which we do not model explicitly.

17This assumption is commonly made the literature of network interconnection. See e.g. Laffont, Rey, and Tirole (1998a, p. 7) for further discussion.

18The assumption that consumers do not care which foreign network provides the roaming services can be justified in several ways. One plausible reasoning relies on a heterogenous coverage. While a subscriber usually lives and works at a priori known places, she prefers to join a network that offers good coverage at these focal points. However, when signing a mobile phone contract, a subscriber is usually less aware of the foreign places where she will use roaming services.
Since subscribers have quasilinear preferences concerning wealth, the value function \( v : \mathbb{R} \rightarrow V \)\(^{19}\) satisfies the envelope condition \( v'(p) = -q(p) \). We maintain the following mild assumption throughout the paper:\(^{20}\)

**Assumption 1** Per customer demand \( q(p) \) is non-negative, continuously differentiable and non-increasing on \( \mathbb{R} \): \( q(\cdot) \in \mathbb{R}_+, \; q'(\cdot) \leq 0 \). Subscribers have a strictly positive demand for roaming services at the true marginal cost: \( q(c) > 0 \).

For future reference we define the net surplus of a tariff as

\[
w(p, F) \equiv v(p) - F
\]

Economically, the net surplus indicates how much of the value \( v(p) \) created by placing roaming calls retains with the subscriber.

If the difference between the net surpluses offered by competing retail contracts in country \( x \) is not too large \( (|w_{xi} - w_{xj}| < \frac{1}{2\sigma}) \), both operators achieve a strictly positive market share.\(^{21}\) The market share \( n_{xi} \) of operator \( i \) in country \( x \) is then:

\[
n_{xi} = n(w_{xi}, w_{xj}) \equiv \frac{1}{2} + \sigma (w_{xi} - w_{xj})
\]

If instead operator \( i \) offers a contract that is far more attractive than its competitor’s tariff \( (w_{xi} \geq w_{xj} + \frac{1}{2\sigma}) \), then it corners the whole market.

**Wholesale prices:** As each operator’s infrastructure only covers its home country, its subscribers have to be hosted by another operator while traveling abroad. We assume that the home operator is able to determine on which network its subscribers register once they are abroad, since roughly 80% of the roaming traffic was indeed directed to the desired foreign network by 2006. For each roaming call, the host operator bills the wholesale price \( a_{yj} \) to the subscriber’s home operator.\(^{22}\) So the host operator’s profit on one roaming call is \( a_{yj} - c \).

**International alliances:** Mobile operators may also form international alliances. Within an alliance, the operators negotiate on a wholesale price at which they mutually provide roaming services. Alliance members commit to direct their subscribers to the partner network abroad. It will become clear that the appeal of alliances lies precisely in the commitment that the subscribers are possibly not hosted by the cheapest operator

\(^{19}\)Define \( V \equiv \{ \hat{v} \mid \exists p \in \mathbb{R} \text{ s.t. } \hat{v} = v(p) \} \) as the set of values that can be possibly achieved.

\(^{20}\)This assumption essentially imposes restrictions on \( u(\cdot) \). We state this assumption directly on \( q(\cdot) \) for notational convenience.

\(^{21}\)See e.g. Lafont, Rey, and Tirole (1998a).

\(^{22}\)Under the rules of the GSM Association, when a roaming subscriber uses the services of a visited network, the roaming subscriber’s home network is responsible for payment of all charges incurred for services used in accordance with the published *Inter-Operator-Tariffs* (IOT) of the visited network. The introduction of the IOT in 1998 dissociated wholesale roaming prices from the standard retail tariffs applied by the visited network. Thus, the competitive conditions prevailing on the retail market were no longer reflected on the wholesale market for international roaming. Prior to 1998, wholesale roaming charges were calculated on the basis of the so-called *Normal Network Tariff* (NNT) of a visited MNO. The NNT was based on the standard user tariff charged by MNOs at the retail level. In 1995 visited MNOs started charging foreign MNOs a multiplier up to a maximum of 1.15 to the NNT. This cap was supposed to reflect subscription charges that would otherwise have not been reflected in the wholesale roaming charges for outgoing calls. See also [http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/05/44&format=HTML&aged=0&language=EN&guiLanguage=en](http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/05/44&format=HTML&aged=0&language=EN&guiLanguage=en).
abroad. After a wholesale price has been negotiated, it becomes public knowledge. This assumption reflects that the wholesale prices, which are also called Inter-Operator-Tariffs, are published by the GSM Association. International alliances can be formed only between operators from different home countries.  

Figure 1 summarizes the structure of the model. It shows the equilibrium configuration of two competing alliances. In this figure, MNOs with the same index form alliances. The dashed line illustrates the possibility to offer roaming services to foreign operators that are outside of an alliance.

Timing: The base model consists of the following stages:

1. Members of an alliance negotiate wholesale prices for roaming calls within the alliance.
2. MNOs simultaneously set wholesale roaming prices for operators that are not affiliated with an alliance.
3. Operators set retail tariffs.
4. Consumers subscribe to their preferred network and place their calls.

The sequential structure allows MNOs to set their wholesale prices strategically. It reflects that due to legal and practical reasons, wholesale prices can be changed less easily than retail tariffs. The model is solved by backward induction.

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23 We suspect that domestic regulation agencies would prohibit alliances that would involve of more than one MNO of a country. Members of these alliances could then collude on their domestic retail prices as well, thereby weakening competition.

24 In Europe, the Standard Terms for International Roaming Agreement (STIRA) issued by the GSM Association provide guidelines how wholesale prices have to be set. They prescribe that wholesale prices have a validity of at least six months.
3 Retail tariffs and market share

In this section we take as given the choice of the foreign host operator which provides its network for the visiting subscribers abroad and characterize the equilibrium retail tariffs, market shares, and the operators’ retail profits.

Perceived marginal costs of roaming services: The marginal cost of the reselling operator $x_i$, which we denote as $c_{xi}$, equal the wholesale price of its host operator. For example, if operator $Ai$ offers to its subscribers roaming services in country $B$ which are provided by operator $Bj$ then the perceived marginal costs of operator $Ai$ are $c_{Ai} = a_{Bj}$.

Since we assume that both competing operators in one country set their prices simultaneously, the optimal retail tariff maximizes each operator’s retail profit for a given tariff of the domestic competitor. Per subscriber, an operator earns $(p_{xi} - c_{xi})q(p_{xi}) + F_{xi} - C_F$. The retail equilibrium tariff can be more easily derived by solving for the optimal retail per call price and the optimal net surplus $(p_{xi}, w_{xi})$ instead of the optimal fixed fee.\footnote{Since the fixed fee depends linearly on the net surplus, it can be easily retrieved using the identity $F_{xi} = v(p_{xi}) - w_{xi}$.
}When charging the per call price $p_{xi}$, offering the net surplus $w_{xi}$ and incurring the perceived marginal cost of $c_{xi}$, an operator earns the following retail profit per customer

$$\pi^R(p_{xi}, w_{xi}, c_{xi}) \equiv q(p_{xi})(p_{xi} - c_{xi}) + v(p_{xi}) - w_{xi} - C_F$$

(3)

The retail profits $\Pi^R_{xi}$ are defined as follows:

$$\Pi^R_{xi} = \Pi^R(p_{xi}, w_{xi}, w_{xj}, c_{xi}) \equiv n(w_{xi}, w_{xj})\pi^R(p_{xi}, w_{xi}, c_{xi})$$

(4)

Thus the retail profit (4) of operator $x_i$ depends only on the net surplus of its competitor’s tariff, not on its competitor’s retail per call price.

Since the per call price does not enter the market share, the usage price is chosen solely as to maximize the per customer profit (3). The availability of two-part retail tariffs renders it optimal to set the per call price equal to perceived marginal costs, that is $p^*_xi = c_{xi}$.\footnote{This finding is by now well understood. See e.g. Laffont, Rey, and Tirole (1998a); Armstrong (2002). This claim is formally proved in Lemma 1. We offer a proof here mainly to keep our paper self-contained.} Intuitively, when the usage price equals the perceived marginal costs, any dead-weight loss (from the viewpoint of the reselling operator) is avoided and the surplus is maximized since $q(p_{xi})(p_{xi} - c_{xi}) + v(p_{xi}) \leq v(c_{xi})$.\footnote{If $q'(c_{xi}) = 0$, then $p^*_xi = c_{xi}$ is not a strict maximizer of $\pi^R(p_{xi}, a_{xi}, c_{xi})$, and its maximum is also attained by other per call prices. However, this does neither change retail profits nor the best response of the retail competitor. As all retail per call prices that attain the maximum retail profits are economically equivalent, we treat them as one equivalence class.} The difference between the maximal surplus $v(c_{xi})$ and the desired net surplus $w_{xi}$ is then transferred between the subscriber and the operator via the implicitly determined fixed fee without causing any inefficiencies. The maximal per customer retail profit is

$$\pi^{R*}(w_{xi}, c_{xi}) = v(c_{xi}) - w_{xi} - C_F$$

(5)

The optimal level of net surplus can be determined explicitly by use of the corresponding first order condition as follows:
\[ w^*(c_{x_i}, w_{x_j}) = \frac{1}{2}[v(c_{x_i}) + w_{x_j} - C_F - \frac{1}{2\sigma}] \tag{6} \]

The following Lemma characterizes the retail equilibrium:

**Lemma 1** A retail equilibrium always exists. If the difference between perceived marginal costs is not too big, namely \(|v(c_{x_i}) - v(c_{x_j})| \leq \frac{3}{2\sigma}\), the retail equilibrium is uniquely characterized by (7)-(9). If instead \(v(c_{x_i}) - v(c_{x_j}) > \frac{1}{2\sigma}\), then there exists a unique equilibrium in weakly undominated strategies\(^{28}\) where operator \(x_i\) serves the whole market and offers \(w^*_{x_i} = \frac{1}{2\sigma} + v(c_{x_i}) - C_F\), while its competitor sets \(w^*_{x_j} = v(c_{x_j}) - C_F\).

**Proof.** Suppose that \(|v(c_{x_i}) - v(c_{x_j})| < \frac{3}{2\sigma}\). We first show that (6) indeed maximizes retail profits given \(w^*_{x_j}\). Since \(\frac{\partial \Pi^R}{\partial w_{x_i}}(p_{x_i}, w_{x_i}, w_{x_j}, c_{x_i}) = n_{x_i}q'(p_{x_i})(p_{x_i} - c_{x_i})\), \(\Pi^R(p_{x_i}, w_{x_i}, c_{x_i}) - \Pi^R(c_{x_i}, w_{x_i}, c_{x_i}) = n_{x_i}\int_{p_{x_i}}^{p_{c_{x_i}}} q'(p)(p - c_{x_i})dp \leq 0\) with strict inequality whenever \(n_{x_i} > 0\) and \(q(c_{x_i}) \neq q(p_{x_i})\). Thus \(p^*_{x_i} = c_{x_i}\) maximizes \(\Pi^R_{x_i}\) independently of \(w_{x_i}\) and \(w_{x_j}\). We have \(\frac{\partial \Pi^R}{\partial w_{x_i}}(c_{x_i}, w_{x_i}, w^*_{x_j}, c_{x_i}) = 2\sigma (w^*_{x_i} - w_{x_i})\) so that \(\Pi^R(c_{x_i}, w^*_{x_i}, w^*_{x_j}, c_{x_i}) > \Pi^R(c_{x_i}, w_{x_i}, w^*_{x_j}, c_{x_i})\).

Solving simultaneously the reaction functions (6) for both operators yields the equations below. Being a system of linear equations, the solution is unique. The condition \(|v(c_{x_i}) - v(c_{x_j})| < \frac{3}{2\sigma}\) assures that the market share stays between zero and one.

The case \(|v(c_{x_i}) - v(c_{x_j})| \geq \frac{3}{2\sigma}\) is treated in appendix A.1.

\[ w^*(c_{x_i}, c_{x_j}) = \frac{2}{3}v(c_{x_i}) + \frac{1}{3}v(c_{x_j}) - \frac{1}{2\sigma} - C_F \tag{7} \]
\[ n^*(c_{x_i}, c_{x_j}) = \frac{1}{2} + \frac{\sigma}{3}[v(c_{x_i}) - v(c_{x_j})] \tag{8} \]
\[ \Pi^R^*(c_{x_i}, c_{x_j}) = \frac{(n^*(c_{x_i}, c_{x_j}))^2}{\sigma} \tag{9} \]

The previous characterization of the retail equilibrium offers two interesting insights. Firstly, equation (8) confirms the intuition that MNOs with a lower perceived marginal cost than their domestic competitor will also achieve a bigger market share in equilibrium. The equilibrium market share depends only on the difference in surplus \(v(c_{x_i}) - v(c_{x_j})\) that both operators generate on the retail level using optimal per call prices. Since the optimal per customer profits increase in the market share, the difference in equilibrium net surplus \(w_{x_i} - w_{x_j}\) is only one third of the difference in surplus.

Secondly, equation (9) shows that the retail profits depend on the perceived marginal costs only through the equilibrium share. If wholesale roaming prices of both competitors in one country are reduced such that the market share is left unchanged, then the

\(^{28}\)See Palfrey and Srivastava (1991) for a definition of the undominated Nash Equilibrium concept. An undominated NE may not consist of strategies that are weakly dominated. A strategy is weakly dominated if there exists another strategy that yields for any strategy of the remaining agents at least the same payoff as the dominated one, and yields a strictly higher payoff for at least one strategy of the remaining agents.
retail profits also stay constant. Any surplus gains are passed on to the customers. Thus, concerning the retail profits, operators are indifferent against a rise of both competitors' perceived marginal costs that leaves the market share unchanged. This profit neutrality is reminiscent of results found by Laffont, Rey, and Tirole (1998a).29

However, an increase of the perceived marginal cost of operator $x_i$ while holding that of operator $x_j$ fixed decreases operator $x_i$'s market share and its retail profits.

\[
\frac{\partial \Pi^*_{R}(c_{x_i}, c_{x_j})}{\partial c_{x_i}} = -\frac{2}{3} n^*_x q(c_{x_i}) \tag{10}
\]

\[
\frac{\partial \Pi^*_{R}(c_{x_i}, c_{x_j})}{\partial c_{x_j}} = \frac{2}{3} n^*_x q(c_{x_j}) \tag{11}
\]

By the envelope theorem, changes of the own retail tariff which are caused by an increase in the perceived per call cost have no marginal effects on retail profits. Consequently, in case both operators have a strictly positive market share, an increase in the own perceived marginal cost (10) affects the retail profit through two channels. Firstly, a higher per call cost directly decreases the retail profit by $-n^*_x q(c_{x_i})$. However, the loss from a higher perceived marginal cost is partially compensated by softer competition on the retail market. Competitor $x_j$ anticipates that operator $x_i$ passes a higher wholesale price on to the customers and optimally decreases its own net surplus by $dw_{x_j}/dc_{x_i} = -\frac{1}{3}q(c_{x_i})$. This increases the retail profit of operator $x_i$ by $\frac{1}{3} n^*_x q(c_{x_i})$. Taken together, the negative marginal effect of an increase in the perceived cost outweighs the positive effect of softer competition. So each operator prefers to pay as low wholesale prices as possible.

According to equation (11), operator $x_i$ benefits from an increase in the perceived marginal cost of its domestic competitor $x_j$. By (6), an increase in $c_{x_j}$ induces operator $x_j$ to offer less attractive retail tariffs. This in turn increases operator $x_i$'s market share and thereby leads to an increase in profits.

Lemma 1 also describes the theoretical possibility of a corner equilibrium if the difference in perceived per call costs is too big. As long as the competitor stays out of the market, a marginal increase in own per call costs triggers no strategic effect of softer competition. So there remains solely the direct negative effect of a higher perceived cost.

4 Wholesale prices without international alliances

In this section, which serves as a benchmark, we assume that operators compete in a standard Bertrand way to serve as host operator. Operators cannot commit to channel their roaming traffic to a particular network.

In stage 1 each operator $x_i$ offers (simultaneous with its domestic competitor $x_j$) to act as host operator for subscribers of country $y$ at the wholesale price $a_{x_i}$. For simplicity we assume that each operator then selects one foreign operator that subsequently serves as host operator for roaming calls from abroad.

29Laffont, Rey, and Tirole (1998a) analyze the effect of two-part tariffs in section 8. They also find that increase of perceived marginal costs that leaves the equilibrium market shares unchanged has no effect on equilibrium retail profits. This result hinges on the fact that $\partial n(w_\omega)/\partial w_\omega$ is constant.
In case the foreign operator $yj$ selects operator $xi$ as host operator, from section (2), the latter earns $n_{yj}q(a_{xi})[a_{xi} - c]$ from roaming subscribers of operator $yj$. As shown in the previous section, it is optimal for any operator of country $y$ to buy roaming services from the foreign operator which offers the lowest wholesale price. Therefore any operator optimally undercuts the offered wholesale prices of its domestic competitor as long as the margin $a_{xi} - c$ is strictly positive. By the usual Bertrand reasoning, any operator offers roaming services at wholesale price $a_{xi} = c$ in equilibrium. The following conclusion summarizes the resulting equilibrium wholesale prices:

Lemma 2 If international alliances are not feasible, in equilibrium the wholesale price equals the real cost of providing a roaming call $c$.

Proof. In the text. ■

5 Wholesale prices under international alliances

In this section we take the following bilateral cooperations as exogenously given: operator $A0$ collaborates with $B0$ and $A1$ forms an international alliance with $B1$. More precisely, $Ai$ is host operator for the subscribers of $Bi$ and conversely $Bi$ is host operator for subscribers of $Ai$. The most important characteristic of this alliance is that both members commit to buy roaming services only from the foreign partner network, even in case another foreign operator offers cheaper wholesale prices for roaming services.

We assume that members of an alliance set wholesale prices cooperatively to maximize joint profits. For simplicity, we impose that both partners must agree on one wholesale price that applies for roaming calls in both directions: $a_{A1} = a_{B1} \equiv a_i$.\footnote{Due to our symmetry assumptions, both members of an alliance have identical preferences on the wholesale price $a_{xi}$. Assuming symmetric bargaining power, they would deliberately choose $a_{Ai} = a_{Bi}$ even if they were allowed to set possibly differing wholesale prices $(a_{A1}, a_{B1})$.} We later consider richer sets of agreements in section 8.2.

The negotiated wholesale prices are public knowledge. Hence the ensuing retail equilibrium tariffs are as described in section 3, treating the own wholesale price as a perceived marginal cost: $c_{xi} = a_i$. In particular, operators anticipate the strategic effect on their domestic competitors' retail tariffs.

Operator $xi$'s overall profit comes from reselling roaming calls to subscribers in its home country $x$ and from selling roaming services to operator $yi$. Due to reciprocal wholesale prices, symmetric costs and demand across countries all members of one alliance receive equal market shares $n_{Ai}^* = n_{Bi}^* \equiv n_i^*$ and equal retail profits $\Pi_{Ai}^R = \Pi_{Bi}^R \equiv \Pi_i^R$. Therefore, the total profits of alliance $i$'s members are as follows:

$$\Pi_i = \Pi(a_i, a_j) \equiv n_i^*(a_i, a_j) \left[ \pi_W(a_i) + \pi_R^*(a_i, a_j) \right]$$

where

$$\pi_W(a_i) \equiv q(a_i)[a_i - c]$$

is the per customer wholesale profit of operator $xi$.\footnote{Note that in case each operator applies the optimal retail tariffs as determined in section (3) and would offer subscribers to choose among foreign operators, then it would be also optimal for subscribers to choose the cheapest foreign network.}
The following Lemma establishes that cornered-market configurations cannot be an equilibrium:

**Lemma 3** In any equilibrium in weakly undominated strategies\(^{32}\) both alliances have a positive market share: \(n^*(a^*_i, a^*_j) \in (0, 1)\).

**Proof.** See appendix. \(\blacksquare\)

We prove that any such equilibrium would entail wholesale prices not lower than marginal costs. In addition, a corner equilibrium would necessarily involve one alliance that sets high wholesale prices, therefore attracts no customers and consequently earns zero profits. But then it would be a better strategy to match the competing alliance’s wholesale price, which guarantees positive retail and nonnegative wholesale profits. Hence no alliance will ever set a wholesale price that leads to its exclusion from the market.

Since in any equilibrium both operators will achieve a positive market share, we focus on interior equilibria, i.e. on wholesale prices that lead to a shared retail market (i.e. \(|v(a_0) - v(a_1)| < \frac{3}{2}\)) in the further presentation. Building on the results of section 3, the marginal profit generated by an increase in the wholesale price of an alliance is:

\[
\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = q(a_i) \left[ \left( \frac{1}{3} - \epsilon(a_i) \right) n_i^* - \frac{\sigma}{3} W(a_i) \right]
\]

where \(\epsilon(p) \equiv \frac{-q(p)c}{q(p)}\) is the markup elasticity of per customer demand.\(^{33}\)

If the retail equilibrium tariffs were not affected by a change of the wholesale price, an increase of the wholesale profit, that would be achieved by raising the wholesale price would be completely offset by a reduction of the retail profit. However, there arise indirect effects because a change of the wholesale price also affects the retail tariffs.

Once the wholesale price has been fixed within an alliance, each member chooses the tariff that maximizes its retail profits, not taking into account the effects on the wholesale profits that foreign members of the alliance earn by providing roaming services. In particular, by section 3, the wholesale price will be passed on to customers directly. The indirect effect of a marginal increase in \(c\) through the usage price on the per customer wholesale profit is \((a_i - c) q'(a_i)\).\(^{34}\) In addition, the retail equilibrium net surplus \(w^*_i\) depends on the value \(v(a_i)\) implied by the wholesale price. Therefore,

\(^{32}\)This refinement is needed to rule out unplausible equilibria in case case demand is constant below \(c\). In this case, the following class of corner equilibria exists: \(a^*_i\) satisfies \(\pi^W(a^*_i) = v(c) - v(a^*_i) < -\frac{10}{3}\), \(a^*_j\) satisfies \(v(a^*_j) < v(c) - \frac{3}{2}\). Clearly, \(n^*(a^*_i, a^*_j) = 1\) and \(\pi^W(a^*_i) = v(a^*_i) - \frac{1}{3\sigma} - v(a^*_j) + \pi^W(a^*_j) = v(c) - \frac{1}{3\sigma} - v(a^*_j) > \frac{1}{2}\). In addition, any deviation of alliance \(j\) yields at most profits 0. In the deviation price is \(\hat{a}_j\) such that \(v(\hat{a}_j) > v(a^*_i) - \frac{1}{3\sigma}\) so that alliance \(j\) achieves a positive share, then \(\pi^W(\hat{a}_j) \leq n^*(\hat{a}_j, a^*_j) \left[\frac{n^*(\hat{a}_j, a^*_j)}{\sigma} + \pi^W(\hat{a}_j)\right] \leq 0\) since \(\pi^W(\hat{a}_j) < -\frac{10}{3} + \frac{3}{2\sigma} < \frac{n^*(\hat{a}_j, a^*_j)}{\sigma}\). This class of equilibria is unplausible since it requires operator \(i\) to use a weakly dominated strategy on \(\mathbb{R}\) and operator \(j\) to use a weakly dominated strategy on \(a_i \in (-\infty, 0]\).

\(^{33}\)Note that the demand elasticity in markup terms is closely connected to the price elasticity of demand which is defined as \(\eta(p) \equiv \frac{-q'(p)}{q(p)}\). The following relationship holds: \(\epsilon(p) = \eta(p) \frac{(p-c)}{p} < \eta(p)\). In case of \(c = 0\), the markup elasticity coincides with the price elasticity of per customer demand.

\(^{34}\)Similar to section 3, while the envelope theorem states that an increase in the retail per call price has no marginal effect on retail profits, it has a negative effect on wholesale profits.
slightly increasing the wholesale price reduces the equilibrium retail market share by
\[ \frac{d\pi^*_i}{da_i} = -\frac{1}{2} \sigma q(a_i) \] and thereby affects the wholesale profit by
\[ \frac{d\pi^*_i}{da_i} \pi^W(a_i). \]

There is also a strategic effect since the domestic competitor in the retail market observes the negotiated wholesale prices. It reacts on higher wholesale prices by offering itself less attractive tariffs as explained in section 3. Inducing softer competition renders increasing wholesale prices attractive from the viewpoint of an alliance. The marginal profit due to softer competition is
\[ \frac{1}{3} \sigma q(a_i) \pi^R_i. \]

The following mild technical assumption suffices to guarantee existence and uniqueness:

**Assumption 2** The markup elasticity of per customer demand \(\epsilon(p)\) is non-decreasing for all prices above marginal costs whenever \(\epsilon(p) \leq 1\).

Assumption 2 is satisfied by many commonly used demand functions, including constant demand, linear demand or constant (price) elasticity demand.\(^{35}\)

Analyzing the marginal profit (13) allows to restrict the relevant range of candidate equilibrium wholesale prices as follows:

**Lemma 4** In any interior equilibrium, the wholesale price \(a_i^*\)
i) never lies below the marginal costs: \(a_i^* \geq c_i\),
ii) is low enough to ensure positive per customer demand: \(q(a_i^*) > 0\),
iii) ensures that the per customer demand markup-elasticity is low: \(\epsilon(a_i^*) < \frac{1}{3}\),
iv) is low enough to satisfy \(v(c) < v(a_i^*) + \frac{3}{2\sigma}\) if Assumption 2 holds.

**Proof.** See appendix. \(\blacksquare\)

Lemma 4 summarizes some intuitive and economically important insights. The first part states that in any equilibrium an alliance will never set wholesale prices below the marginal cost for two reasons: Firstly, a low wholesale price fuels competition on the retail market. Secondly, wholesale prices below costs induce subscribers to place calls inefficiently often and losses on the wholesale level are even aggravated by an inefficiently high market share. Thus, setting the wholesale price equal to marginal cost clearly dominates prices below marginal costs.

Part ii) states that alliances never set the wholesale price so high that roaming demand is completely choked off. Given that the rival alliance charges a wholesale price that covers at least its marginal costs, by setting the own wholesale price equal to marginal costs, an alliance could increase its retail profits while maintaining a wholesale profit of zero.

Part iii) of Lemma 4 states that alliances always operate at wholesale prices where demand is quite inelastic with respect to the markup. Equation (13) directly reveals that if \(\epsilon(a_i) > \frac{1}{3}\), the adverse effect of increasing the dead-weight loss would dominate the gains from softer competition.\(^{36}\)

---

\(^{35}\)Examples are constant demand \(q(p) = q\ \forall p\), constant elasticity demand \(q(p) = ap^{-\gamma}\ a, \gamma > 0\), or demand of the form \(q(p) = \max\{a - bp^\gamma, 0\}\) with \(a, b, \gamma > 0\).

\(^{36}\)It is interesting to compare this insight with standard results from one-stage models of oligopolistic consumer choice models where firms compete in linear prices such as Anderson, de Palma, and Nesterov (1995). In their model, firms set prices such that \(\epsilon(a_i) < 1\), hence potentially operate in slightly more elastic regions. This comes from the fact, that in their model, an increase in the price really increases the markup, while in our model it only leads to gains via softening the competition.
The most surprising result of Lemma 4 is its last part. It is never optimal to choose a wholesale price that would excessively reduce the value $v(a_i)$, compared to that generated by a usage price that equals the true marginal costs, $v(c)$. Intuitively, setting a high wholesale price means that the wholesale profits per customer are high. But then it would be more profitable to marginally decrease the wholesale price in order to expand the market share. This result has an important implication for equilibrium existence: As the equilibrium prices are not too far away from the marginal costs, deviating from any equilibrium by setting the wholesale price equal to marginal costs does not suffice to corner the market. Deviations that allow to corner the market thus require prices below the true marginal costs and are therefore less attractive than smaller deviations. Thus, in contrast to Laffont, Rey, and Tirole (1998a), local equilibria are always robust to big deviations if Assumption 2 holds.

For future reference, we define $\mathcal{E}$ as the relevant set of wholesale prices that potentially might be chosen in equilibrium according to Lemma 4.\textsuperscript{37} Assumptions 1 and 2 on customer demand guarantee that $\mathcal{E}$ is an interval nonempty\textsuperscript{38} with lowest element $c$.

Setting marginal profits (13) to zero and rearranging, we obtain the Lerner condition

$$\frac{a_i^* - c}{a_i^*} = \frac{1}{3 \left[ \eta_q(a_i^*) + \eta_n(a_i^*) \right]}$$

(14)

where $\eta_q(a_i) \equiv -\frac{a_i q'(a_i)}{q(a_i)}$ is the price elasticity of per customer demand and $\eta_n(a_i) \equiv -\frac{dn_i}{da_i} n_i$ is the price elasticity of the equilibrium retail market share. In a symmetric equilibrium, each alliance achieves a market share of $n_i^* = \frac{1}{2}$ and the price elasticity of the market share simplifies to $\eta_n(a_i) \equiv \frac{2}{3} \sigma a_i q(a_i)$. We now state our main result:

**Proposition 5** The wholesale prices of any interior equilibrium are characterized by equation (14) and are strictly above marginal costs: $a_i^* > c$. If Assumption 2 holds, then a symmetric equilibrium exists with both alliances setting the wholesale price $a_i^* = a_1^* = a^*$. This unique interior equilibrium entails an equilibrium per customer wholesale profit of $\pi_{W^*} = \frac{1-3\epsilon(a^*)}{2\sigma}$.

**Proof.** See the appendix. \hfill \blacksquare

Besides existence and uniqueness, proposition 5 confirms that alliances will set higher wholesale prices for roaming calls than would be socially optimal. Assumption 2 assures existence and uniqueness but is not needed to derive that a strictly positive markup on the wholesale level necessarily occurs.

The intuition of this proposition is as follows: By setting high wholesale prices, operators credibly commit to offer less attractive retail tariffs and thereby soften competition on the retail price setting stage. Accepting a high unilateral wholesale price for own customers would not be profitable as shown in section 4. However, setting high wholesale prices bilaterally in an alliance is attractive: Within an international alliance, each operator acts as host operator and fully benefits from high wholesale prices by providing roaming calls for the foreign operator’s subscribers. After wholesale

\textsuperscript{37}Formally, $\mathcal{E} = \{p \in \mathbb{R} | (p < \frac{3}{2} \wedge p \geq c \wedge q(p) > 0 \wedge v(p) > v(c) - \frac{3}{2\sigma})\}$

\textsuperscript{38}Since we assume that the per customer demand is continuously differentiable on $\mathbb{R}$ and therefore bounded around $c$ and that $q(c) > 0$, the markup elasticity $\epsilon(p)$ approaches zero for wholesale prices close to $c$. 

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prices have been set in an alliance, an operator cannot affect the retail market share of members in the foreign country any more. Hence wholesale profits will not affect the decision of the own retail tariff for domestic subscribers. However, being committed to pay high wholesale prices when buying roaming services for own subscribers softens competition on the home retail market.

Looking at the marginal effects of an increase in wholesale prices when they are close to marginal costs is helpful to understand why there is always a positive wholesale markup in equilibrium. Evaluating (13) at \( a_i = c \), yields
\[
\left( \frac{\partial \Pi_i}{\partial a_i} \right)_{c,a_j} = \frac{1}{3} \frac{q^*_i q(c)}{q^*_i + q(c)}>0
\]
and illustrates that there is a positive first-order effect on the joint profit of an alliance. In contrast, losses caused by the operators from not taking into account the wholesale profits when setting retail prices are of second-order at \( a_i = c \). The reason is that when the wholesale markup is small, so is the indirect marginal effect on the wholesale profit. Thus, taking the wholesale profit not into account when setting retail prices leads only to a minor distortion for small wholesale margins. Taken together, for wholesale prices close to marginal costs, the positive effect from softening competition dominates. Hence it is always better to set the wholesale price slightly above the true marginal cost, until the marginal effects are equalized.

The role of wholesale prices differs from that of access-prices in Laffont, Rey, and Tirole (1998a). In their model of network interconnection, the level of the access price does not influence the market share since it equally applies to both domestic competitors. Therefore, even the industry monopoly profits can be attained provided the retail equilibrium exists. In our model, taking \( a_j \) as given and increasing the bilateral wholesale price of alliance \( i \) decreases its market share. The danger of losing too much market share keeps wholesale prices below the level that maximizes industry profits.

The next proposition summarizes some comparative statics:

**Proposition 6** Suppose that Assumption 2 holds.

i) The equilibrium wholesale price \( a^* \) is decreasing in the degree of competition on the retail market \( \sigma \).

ii) Suppose that the per customer demand function \( \bar{q} \) is more elastic than \( q \): \( \eta_{\bar{q}}(p) > \eta_q(p) \forall p \in E \). Denote the corresponding equilibrium wholesale prices by \( \bar{a}^* \) and \( a^* \). If the per customer demand \( \bar{q} \) is weakly higher than \( q \) at the equilibrium price \( a^* \) (i.e. \( \bar{q}(a^*) \geq q(a^*) \)), then \( \bar{a}^* < a^* \).

**Proof.** See appendix. ■

Proposition 6 confirms that the comparative statics are as expected. Part i) states that an increase in the degree of retail competition reduces the wholesale equilibrium prices. If the taste differences of customers are small, then the negative effect of losing market share when increasing the wholesale price is strong relative to the competition softening effect. Hence alliances find it optimal to set a small wholesale markup.

Part ii) compares differences in the elasticity of demand. When demand is more elastic, then the dead-weight loss invoked by setting the wholesale price above marginal costs becomes more pronounced.\(^{39}\) In addition, the marginal gains from reduced com-
petition which are proportional to demand, diminish quicker as prices are increased. Taken together, more elastic demand serves to discipline alliances.

**Examples.** The results of this section can be illustrated by some common demand functions that admit explicit solutions. First, we assume that the per customer demand \( q \) is constant: \( q(p) = \bar{q} \). The elasticity of the retail market share becomes \( \eta_i(a_i) = \frac{\sigma_i}{3\bar{q}} \) and the equilibrium wholesale price can be determined explicitly by solving condition (14): \( a^*_q = c + \frac{1}{2\bar{q}} \). This formula confirms that the equilibrium price is decreasing in the degree of competition \( \sigma \).

Another example that admits an explicit solution is the commonly used constant elasticity demand \( \bar{q}(p) = \frac{A}{p} \). Using this specification, the equilibrium wholesale price is \( a^*_q = c + \frac{c}{2+2\sigma A} \). If \( A \geq (\bar{q} + \frac{1}{2\bar{q}}) \) then \( \bar{q}(a^*_q) \geq \bar{q} \) and the hypothesis of proposition 6, part ii) is satisfied. Indeed, for \( A = (\bar{q} + \frac{1}{2\bar{q}}) \), we get \( a^*_q = c + \frac{c}{3+2\sigma c} < a^*_q \).

It remains to assess the effects of alliances. Like in section 4 every operator achieves a market share of \( \frac{1}{2} \) in the domestic retail market. Since the retail equilibrium profit \( \Pi^R = \frac{(n^*_c)^2}{\sigma} \) depends only on the market share but not on the absolute level of retail prices, it is equally high with and without alliances. However, compared to section 4, operators additionally earn a strictly positive wholesale margin which makes them better off in total.

Subscribers are unambiguously worse off once alliances have formed because of two reasons: First, the overall surplus generated by roaming calls decreases when retail per call prices are above true marginal costs. In addition, by the preceding paragraph, in equilibrium alliances earn higher profits and hence a smaller part of the surplus (which is smaller compared to section 4) is retained with the subscribers.

Note that strategic effects could not be achieved if operators \( Ai \) and \( Bi \) merged instead of forming an alliance. A merged operator \( i \) would possess a network in both countries. It would therefore set the retail prices in each country as to maximize the sum of wholesale and retail profits of both countries. Setting a (virtual) wholesale price \( a_i \neq c \) within a merged international operator would be meaningless, as the retail tariff in each country is set to maximize the joint profit generated in both countries. The joint equilibrium profit of a merged operator that competes against one alliance can be written as \( 2\Pi^R(c,a_j) \). It is easy to confirm that the industry profit that obtains if there are two merged operators \( i \) and \( j \) equals the industry profit in case alliances are unfeasible.

**Policy Intervention:** We now analyze the effects of imposing a price cap. In 2007, the European Commission introduced a price cap both at the retail and the wholesale level. Prior to this decision, there was a debate whether a single regulatory intervention in one of these markets might be sufficient. In our model, imposing a binding price cap above the true marginal cost solely at the *wholesale level* clearly increases both welfare and consumer surplus but reduces industry profits. However, it turns out that *solely* restricting the *retail usage* price is likely to have a detrimental effect on consumer welfare.

To see this, consider the effects of a fixed retail price cap \( \bar{p} \). Remember that each effects as a higher level of \( \sigma \). This can be seen analyzing condition (32) and considering \( \bar{q}(p) = Aq(p) \) for some constant \( A \) which yields the same equilibrium condition as considering \( \sigma = A\sigma \). In principle we could have normalized \( \sigma = 1 \) and set \( \bar{q}(p) = \frac{1}{2}q(p) \) but for expositional reasons we have not done so.
operator optimally sets the retail usage price $p_i$ as to maximize the surplus which is generated. If the wholesale price $a_i$ exceeds the price cap, then the optimal choice is to set the usage price as high as possible, namely $p^*_i = \bar{p}$. The maximized surplus generated on the retail stage when respecting the price cap $\bar{p}$ is therefore:

$$
\pi(a_i) \equiv \begin{cases} 
  v(\bar{p}) + q(\bar{p}) (\bar{p} - a_i) & \text{if } a_i > \bar{p} \\
  v(a_i) & \text{if } a_i \leq \bar{p}
\end{cases} \quad (15)
$$

Clearly, restricting the usage price to lie below $\bar{p}$ reduces the surplus created at the retail level whenever $a_i > \bar{p}$ and the demand is decreasing at $\bar{p}$.

Remember that all results concerning the retail equilibrium depend on the wholesale prices only through the surplus which is created at the retail level. Therefore, they extend to the case when a price cap is in place if we replace $v(a_i)$ by the function $\pi(a_i)$. In particular, if the wholesale prices of the competing alliances are close enough, namely $|\pi(a_0) - \pi(a_1)| < \frac{\sigma}{2}$, then both operators achieve a positive market share determined by $\frac{1}{2} + \frac{\sigma}{3} |\pi(a_i) - \pi(a_j)|$ which parallels equation (8). In this case the equilibrium level of net surplus $w^*_i$ conceded to consumers reads as follows:

$$
\bar{w}^*_i = \frac{2}{3} \pi(a_i) + \frac{1}{3} \pi(a_j) - C_F - \frac{1}{2\sigma} \quad (16)
$$

Since the equilibrium per customer retail profit is $\pi^{R*}_i = \pi(a_i) - \pi^*_i - C_F$, in case of symmetric wholesale prices operators earn $\pi^{R*}_i = \frac{1}{2\sigma}$, which is the same as in section 3. Thus we can determine the effect of a price cap for given symmetric wholesale prices as follows:

**Lemma 7** Suppose that both alliances set wholesale price $a$ in case of no intervention and wholesale price $\bar{a}$ after a price cap $\bar{p}$ has been introduced. Denote the resulting retail equilibrium net surplus by $w^*(a)$ and $\bar{w}^*(\bar{a})$. The difference in consumer surplus between both regimes equals the difference of surplus that is generated on the retail level: $w^*(a) - \bar{w}^*(\bar{a}) = v(a) - \pi(\bar{a})$.

In particular, if the wholesale price is exogenously fixed at $a$, (i.e. $a = \bar{a}$) and the consumer demand is decreasing at $a$ (i.e. $q'(a) < 0$), then introducing a binding price cap $\bar{p} < a$ reduces the consumer surplus.

**Proof.** By the same reasoning as in the proof of Lemma 1, in case of symmetric wholesale prices the first order conditions are sufficient for retail profit maximization. Solving the system of first order conditions yields (16). Comparing equation (7) with $a_i = a_j = a$ and (16) with $\bar{a}_i = \bar{a}_j = \bar{a}$ yields the result.

If $a = \bar{a}$, $q'(a) < 0$ and $\bar{p} < a$, then $v(a) > \pi(a) = \pi(\bar{a})$ so that $w^*(a) > \bar{w}^*(\bar{a})$ by the first part of the Lemma. ■

The last part of the previous Lemma implies that even if a regulator could impose a cap on the usage retail prices and fix the wholesale prices, the consumer surplus would be generally reduced.

However, imposing a price cap on the retail level also influences the equilibrium wholesale prices. For wholesale prices above $\bar{p}$ and for an interior configuration ($0 < \pi^*_i < 1$), the total marginal profits of an operator are:
Let us denote the wholesale per customer profit in case of a binding cap by \( \pi_i^W \equiv q(\bar{p})(a_i - c) \). Independently of the customer demand function, the requirement that in any symmetric equilibrium marginal profits are zero yields \( \pi_i^W = \frac{1}{2\sigma} \). Comparing to the unrestricted equilibrium profits \( \pi_i^W = \frac{1 - 3\epsilon(a^*)}{2\sigma} \) derived in Proposition 5 reveals that the per customer profit unambiguously increases after the introduction of a cap.

In the proof of the next Proposition we confirm that even if the price cap is set in the most efficient way, namely \( \bar{p} = c \), the efficiency gains from lower usage prices fall short of the difference in equilibrium wholesale profits. Therefore, introducing a binding price cap tends to lower consumer surplus:

\[ \frac{\partial \Pi^*}{\partial a_i}(a,i,a_j) = \frac{q(\bar{p})}{3} [\pi_i^* - \sigma q(\bar{p})(a_i - c)] \]

**Proposition 8** Denote by \( a^* \) the equilibrium wholesale price according to proposition 5. Suppose that Assumption 2 holds and that demand is decreasing at \( a^* \): \( q'(a^*) < 0 \). Then introducing a retail per call price cap \( \bar{p} \leq a^* \) decreases consumer surplus and increases industry profits. If \( \bar{p} < a^* \) and the price cap is not set below the true marginal cost it increases total welfare. If the price cap is sufficiently close to the unrestricted equilibrium wholesale price (i.e. \( q(\bar{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^* - c)} \)) then the equilibrium wholesale price increases.

**Proof.** See appendix.

If the mild conditions of Proposition 8 are satisfied, restricting the retail per call price increases total welfare. Since the market is always covered, the retail per call price uniquely determines total welfare. By hypothesis, the retail price cap reduces this price and therefore increases total welfare as long as it does not undercut the true marginal cost.

Two counteracting effects determine how a price cap influences the wholesale equilibrium price. A retail price cap prevents operators from passing through high wholesale prices to the subscribers. Therefore, increasing the wholesale price does not aggravate the dead-weight loss, which renders higher wholesale prices more attractive. On the other hand, the cap on the retail price guarantees that each subscriber places at least \( q(\bar{p}) \) calls. This increases the wholesale profits per customer and renders subscribers more valuable, thereby inducing alliances to set lower wholesale prices. Whenever the condition \( q(\bar{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^* - c)} \) holds, the first effect dominates and higher wholesale prices obtain. Note that for \( \epsilon(a^*) > 0 \), by continuity of \( q \), a price cap which is set close enough to \( a^* \) satisfies this condition and thus increases the wholesale price.

Our results suggest that in order to protect subscribers, price caps should preferably be imposed on the wholesale level. This might explain why national regulation authorities that can usually only intervene on the retail level have mostly chosen not to regulate roaming prices prior to the intervention of the European Commission.
6 Endogenous formation of alliances

We now endogenize the choice of MNOs to form alliances. Alliances may not be formed of MNOs within the same country, for example due to legal constraints.\footnote{Suppose all operators could jointly fix the wholesale prices of all roaming calls \( a \) to maximize joint profits (for example by a multilateral agreement). Then, under the assumption that the market is always fully covered, all operators would agree on a wholesale price that to maximizes joint \emph{wholesale} profits, i.e. that is defined by the Lerner formula: \( a^* - c = \frac{1}{q(a^*)} \).} Therefore any alliance consists of exactly one MNO with home country \( A \) and another with home country \( B \). As before, joining an alliance means committing to buy roaming services from the foreign alliance member at the negotiated mutual wholesale price, even though another foreign operator may offer lower prices.

Formally, we introduce a formation stage that takes place before wholesale prices are set. In this stage any operator may either announce that it is willing to join an alliance or remain silent. In order to circumvent coordination failures, we assume that first both operators in country \( A \) declare their intentions.\footnote{The equilibrium that we characterize in this section is also an equilibrium if all operators announce simultaneous whether they want to participate in an alliance. The simultaneous setup yields additionally other equilibria, with one or zero alliances being formed. However, both of these alternative equilibria lead to lower profits and therefore operators would coordinate on the more profitable equilibrium with two alliances if possible.} The MNOs of country \( B \) observe these declarations before announcing themselves their disposition to engage in an alliance. Due to our symmetry assumptions, operators are indifferent with which of the two foreign operators to form an alliance. Therefore we assume without loss of generality that if all operators announce to join an alliance, operators with the same position in the retail market are matched. If two operators in country \( x \) are interested in forming an alliance, while only one operator in country \( y \), then one of the two operators in country \( x \) is randomly chosen to participate in the alliance. However, the process how alliances are formed plays only a minor role as long after the formation of alliances all members retain equal bargaining power and symmetric preferences concerning the wholesale price.

Our main line of reasoning does not hinge on foreclosure. Therefore we assume that joining an alliance does not preclude any MNO from selling roaming services to foreign operators that do not pertain to this alliance.\footnote{In addition, domestic regulation authorities might prohibit alliances that force members not to sell to outsiders as this behavior might be perceived illegal.} More precisely, after having possibly joined an alliance and negotiated on internal wholesale prices \( a_i \), every MNO may post a wholesale price \( \tilde{a}_{xi} \) that applies to foreign operators that have not joined the same alliance. Thus, even if one alliance has been forged, there remains competition to act as host operator.

By a similar Bertrand reasoning as in section 4, operators that have not joined an alliance will buy roaming services at a wholesale price equal to true marginal costs in equilibrium. Intuitively, suppose that \( xi \) and \( yi \) have formed an alliance and fixed wholesale price \( a_i \) while operators \( xj \) and \( yj \) remain without alliance. The retail profits of operator \( xi \) are unaffected by the wholesale price that it charges to deliver roaming services outside the alliance, since it is obliged to buy roaming services at price \( a_i \) and the retail pricing decision of operator \( xj \) is independent of its wholesale profits. This renders it profitable for operator \( xi \) to undercut its competitor’s wholesale price
whenever $\tilde{a}_{xj} > c$: When not undercutting, then operator $xi$ will sell roaming services only to operator $yi$ that achieves a market share of $n^*(a_i, \tilde{a}_{xj})$. By undercutting $\tilde{a}_{xj}$ slightly, the perceived marginal cost of operator $yj$ and hence the retail market shares stay almost constant. But operator $xi$ then additionally earns strictly positive wholesale profits from selling to $yj$. Since operator $xj$ also sells on the wholesale market at most to the foreign operator $yj$, it clearly undercut $xi$ for any $\tilde{a}_{xi} > c$. Hence in any equilibrium, $\tilde{a}_{xi} = \tilde{a}_{xj} = c$. Taken together, MNOs that do not join an alliance anticipate being offered roaming services at the true marginal cost but earning zero wholesale profits.

Forming an alliance has two effects: First, members of alliance $i$ may coordinate on a wholesale price that possibly differs from the true marginal costs. Second, if competing operators $j$ have also formed an alliance, they anticipate the equilibrium wholesale price $a^*_i$ and set their wholesale price accordingly. Let $a^*(a_j)$ denote the wholesale price that an alliance sets to maximize its profit, if it expects its competing operators in both countries to purchase roaming services at wholesale price $a_j$. The following Lemma establishes that optimal wholesale prices are complements on the relevant range $E$:

**Lemma 9** Suppose that assumption 2 holds.

i) For any $a_j \in E$ there exists a unique $a^*(a_j) \in E$ that strictly maximizes $\Pi(a_i, a_j)$ in $\mathbb{R}$.

ii) For any $a_j \in E$, the profit maximizing wholesale price $a^*(a_j)$ is strictly increasing in $a_j$.

iii) If competing operators purchase roaming services at marginal costs $c$, the optimal wholesale price within an alliance lies above: $a^*(c) > c$.

**Proof.** See appendix. ■

The preceeding Lemma implies that each operator prefers creating an additional alliance to staying alone.\(^{43}\) If operators $j$ do not form an alliance, then members of alliance $i$ anticipate that non-members will procure roaming services at marginal costs and therefore set the wholesale price $a_i = a^*(c)$. By Lemma 9, it is still attractive to form an alliance since $\Pi(a^*(c), c) > \Pi(c, c).$\(^{44}\) If operators $j$ enter an alliance, then $\Pi(a^*, a^*) > \Pi(c, a^*) > \Pi(c, a^*(c))$ where the first inequality comes from Lemma 9 and the second reflects that the total profit is increasing in the competitors’ prices.\(^{45}\) Hence operators $i$ prefer to form a second alliance instead of staying alone. In both cases, creating an aditional alliance allows its members to commit to higher wholesale prices than true marginal costs. Higher own prices induce competitors raise their wholesale prices if possible which aditionally increases own profits.

The following proposition confirms that two alliances emerge in the unique equilibrium:

**Proposition 10** Suppose that assumption 2 holds. Then a unique subgame perfect equilibrium exists with two competing alliances being formed. In every country, the market

\(^{43}\)Note however, that announcing to form an alliance is not dominant for operators in country $B$. Whenever only one MNO in country $A$ announced to form an alliance, remaining silent yields a higher payoff as long as the domestic competitor forms an alliance. This might also explain why domestic competitors have not complained against the formation of international alliances.

\(^{44}\)We use the notation of section 5.

\(^{45}\)Formally, $\frac{\partial \Pi}{\partial a_j}(a_i, a_j) = \frac{1}{\theta}q(a_j) \left[ 2n^*_i + \sigma \pi^w(a_i) \right]$
is equally split between both alliances. Both alliances set the equilibrium wholesale price characterized by Proposition 5.

Proof. See appendix. ■

Proposition 10 is based on the following intuition: If the announcement of any operator $Bi$ affects the total number of alliances that are created, given the announcements of operators in country $A$ and of the competitor $Bj$, then $Bi$ optimally declares to form another alliance. Hence in any equilibrium as many MNOs in country $B$ announce to form an alliance as have already done so in country $A$. But this implies that any operator in country $A$ can increase the number of alliances by announcing to form an alliance. By the preceding paragraph creating additional alliances increases an operator’s profit so that both operators in country $A$ announce to form an alliance in the unique equilibrium.

7 The role of host network selection

This section serves to analyze the competitive impact of recent technological developments that have improved the home operators’ control over the choice of foreign host networks for roaming. As mentioned in the introduction, by 2006 roughly 80% of the European roaming traffic was already actively directed to preferred networks abroad. In this section we derive that the possibility of traffic direction increases the competitive pressure in the wholesale market. In addition, we point out why the importance of international alliances has increased in light of these technological developments. While alliances are welfare reducing when the host network can be selected sufficiently well, they are without bite if the host network is randomly determined.

We consider here the polar case of operators having no control on which foreign network their subscribers log in. Comparing the outcome to the results of the base model with perfect control allows to understand how the technology of network selection affects decisions on the pricing and the formation of alliances. For sake of completeness, appendix B extends the results of this section to intermediate levels of imperfect control.

We assume that operators cannot discriminate the per call price on the retail market contingent on which foreign network is used. If price discrimination was feasible and subscribers could actively choose their host network abroad, they would always choose the cheapest network. Hence their home operator could perfectly control the network selection by setting the price of the preferred foreign network lower than that of the not-desired network. The outcome would then be economically equivalent to our base model.

When buying roaming calls from foreign MNOs on the wholesale market, operator $xi$’s perceived marginal cost is:

$$c_{xi} = \frac{1}{2} (a_{y0} + a_{y1})$$  (17)

Since operators cannot discriminate the retail prices according to which host network provides the roaming services, the per call price equals the perceived marginal cost:

---

\[\text{Salsas and Koboldt (2004) offer a more extensive treatment of recent technological developments.}\]
$p^*_x = c_x$. The equilibrium net surplus, market share and the retail equilibrium profits remain as established in Lemma 1.

We now turn to the wholesale market.

**No international alliances.** In absence of alliances the total wholesale demand of operator $x_i$ (where the superscript NA means “no alliance”) is:

$$Q^{NA}_{x_i} = Q^{NA}(a_{x_i}, a_{x_j}) \equiv \frac{1}{2} q \left( \frac{1}{2} (a_{x_0} + a_{x_1}) \right)$$

The demand does not depend on the actual market share of the reselling operators, since for all price combinations, both foreign operators purchase half of their traffic at operator $x_i$. The overall profit of operator $x_i$ is:

$$\Pi^{NA}_{x_i} = \Pi^{NA}(a_{x_i}, a_{x_j}) \equiv \Pi^{Rs}(c_{x_i}, c_{x_j}) + (a_{x_i} - c) Q^{NA}(a_{x_i}, a_{x_j})$$

(18)

Similar to section 4, in equilibrium each operator takes the foreign wholesale prices and therefore its retail profits as given. Therefore operator $x_i$ sets its wholesale price in order to maximize its wholesale profits $(a_{x_i} - c) Q^{NA}(a_{x_i}, a_{x_j})$. We make the following mild technical assumption in order to state our first result:

**Assumption 3** The markup elasticity of per customer demand $\epsilon(p)$ is increasing for all prices above marginal costs whenever $q(p) > 0$ and there exists some $\tilde{p} > c$ with $\epsilon(\tilde{p}) = 2$.

**Lemma 11** Suppose that assumption 3 holds. If operators cannot select the host networks, there exists a unique symmetric equilibrium with wholesale price $a^{NA*}$, characterized by

$$\frac{a^{NA*} - c}{a^{NA*}} = \frac{2}{\eta_q(a^{NA*})}$$

(19)

where $\eta_q(\cdot)$ is the price elasticity of per customer demand.

**Proof.** The wholesale price $a_{x_i}$ does not influence operator $x_i$’s retail profits. Hence, $a_{x_i}$ is chosen to maximize $\frac{1}{2} (a_{x_i} - c) q \left( \frac{1}{2} (a_{x_i} + a_{x_j}) \right)$. Rearranging the resulting first order condition yields condition (19). Rewriting the marginal profit in terms of markup-elasticity and evaluating at $a_{x_j} = a_{x_i}$ yields $\frac{\partial \Pi^{NA}}{\partial a_{x_i}} = \frac{1}{2} q (a_{x_i}) \left[ 1 - \frac{1}{2} \epsilon(a_{x_i}) \right]$. Thus the first order condition is satisfied at $\tilde{p}$ which exists and is unique by Assumption (3).

The profit is strictly quasiconcave since $\epsilon'(p) > 0$ whenever $q(p) > 0$ by assumption.

By Lemma 11, if operators cannot influence which foreign network subscribers use to place roaming calls, the resulting equilibrium wholesale price is extremely high. Unilaterally increasing the wholesale price $x_i$ causes a negative externality on the rival, since the wholesale demand of operator $x_j$ is reduced while only the margin of operator $x_i$ increases. As operators do not take this externality into account, the resulting equilibrium price even exceeds the monopoly price.

**Two international alliances.** Similar to section 5, we now analyze the equilibrium outcome after operators with same location have formed two competing alliances. We omit the country index for brevity.
We restrict operators to sell roaming calls on the wholesale market to all foreign operators for the same price \( a_i \) that is negotiated within an alliance.\textsuperscript{47} Otherwise, we maintain all assumptions of the base model but assume that alliance members cannot commit to direct their subscribers to the partner network. Thus, the only virtue of alliances that remains is to set the wholesale price cooperatively instead of competitively.

If both alliances have negotiated wholesale prices \( a_i \) and \( a_j \), the equilibrium wholesale demand for roaming calls of operator \( i \) is \( Q_i = Q(a_i, a_j) \equiv \frac{1}{2} q \left( \frac{1}{2} (a_0 + a_1) \right) \). The profits of each operator in alliance \( i \) are:

\[
\Pi_i = \Pi(a_i, a_j) \equiv \Pi^{R^*}(c_i, c_j) + \frac{1}{2} (a_i - c) q \left( \frac{1}{2} (a_0 + a_1) \right) \tag{20}
\]

Comparing equations (20) and (18) immediately reveals that the same profits obtain for equal wholesale prices. Thus the following Lemma confirms that we should expect the same equilibrium prices:

**Lemma 12** Suppose that assumption 3 holds and operators cannot select the host network of their subscribers. The formation of two alliances does not affect the wholesale equilibrium price, that remains characterized by (19). Ceterus paribus, the equilibrium wholesale price under random selection of the host network lies above that of perfect network selection given by Proposition 5.

**Proof.** The proof of existence and uniqueness parallels that of Lemma (11), since the same objective function is maximized. For proposition 18 of appendix B we prove that the equilibrium price decreases with the quality of network selection. In particular, the equilibrium wholesale price under no control exceeds that of perfect control. \( \blacksquare \)

Intuitively, there are two reasons why equilibrium prices are higher if the host network is selected randomly. Firstly, compared to the base model, an alliance’s retail market share is insensitive to increases of the wholesale price. This is because the perceived marginal costs \( c_i \) of operators within alliance \( i \) and those of the rival alliance \( j \) both depend equally on the own wholesale price \( a_i \). Secondly, without control of the roaming traffic, members of alliance \( j \) that have to procure half of their subscribers’ roaming calls from alliance \( i \). Therefore, raising the wholesale price \( a_i \) may increase the wholesale profit generated from sales to operators of the competing alliance.

The insight that the presence of alliances does not affect the wholesale prices without network control is at first glance surprising. One might be tempted to conject that alliances mitigate the problem of double marginalization as in Carter and Wright

\textsuperscript{47}Even though this restriction might seem very restrictive at first glance, it allows to better compare the results with those of the base model. When allowing MNOs to discriminate between members of the alliance and non-members, then the wholesale price \( \tilde{a}_i \) that applies to non-members will be set extremely high and in many cases there is no equilibrium. Intuitively, as foreign operator \( j \) that is not in alliance \( i \) has to half of its roaming calls from operator \( i \), setting a high \( \tilde{a}_i \) increases the perceived marginal costs of operator \( j \) and therefore increases the retail market share of alliance \( i \).
However, because competition on the retail market is in two part tariffs, double marginalization is not an issue since no deadweight loss is caused on the retail level. Since increasing the wholesale price unilaterally increases the perceived marginal costs of both domestic competitors, it does not affect the retail profits. Therefore members of an alliance cannot increase their profits by coordinating on a wholesale price that differs from the individually optimal level. So there is no point in forming an alliance, as with or without alliances the subscribers are divided evenly among the foreign networks. Compared to the base model, international alliances are unattractive without the technology to direct subscribers to foreign partner networks. Our model therefore provides an explanation why in Europe international roaming alliances were formed mainly after a powerful network selection technologies have become available.

8 Extensions

The base model is constructed to point out our main result of potentially harmful alliances while keeping our model tractable. We relax the assumption of homogenous customers in section 8.1 in order to show that this generalization does not change our main results qualitatively. In section 8.2 we vary the admissible set of pricing instruments.

8.1 Heterogeneous consumers

Our main result of this section is that heterogeneous consumers lead to unambiguously lower profits in equilibrium. However, alliances still serve to raise equilibrium profits since the equilibrium profits decrease only gradually in the degree of heterogeneity. As in section 5 we assume that operators of both countries with same position in their home market have formed alliances. As all results are valid for both countries, we omit the country index for brevity of notation. In this section, we focus on candidate symmetric equilibria that satisfy the necessary first order conditions of profit maximization.

Retail demand structure. In contrast to our main setup, there are two types of consumers indicated by \( \theta_k \) with \( k \in \{ L, H \} \) and \( \theta_L < \theta_H \). We assume that the consumer’s type is observable by the MNOs but will discuss later the implications of relaxing this assumption. A consumer of type \( \theta_k \) values roaming calls according to \( v_k(p) = \theta_k v(p) \) with \( v(p) \) remaining defined as in section 2. Likewise, \( u_k(q) \) denotes the utility that a subscriber of type \( \theta_k \) obtains from consuming \( q \) roaming calls. Subscribers still have quasilinear preferences so that the demand of an \( \theta_k \) subscriber is given by \( q_k(p) = \theta_k q(p) \). The measure of subscribers remains normalized to 1 in every country. A proportion \( \beta \) of these are light users with type \( \theta_L \) and relatively low demand. Likewise, the remaining fraction of \( 1 - \beta \) are heavy users characterized by \( \theta_H \). Without loss

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48 In contrast to our model, they assume that there is a monop list in each country and that the monopolists set linear tariffs both on the wholesale and retail market. They find that if operators cooperatively set wholesale prices to maximize their profits, then both consumer surplus and profits exceed the uncooperative outcome. This result obtains as cooperation allows to circumvent the double-marginalization problem.

49 In a model of network interconnection, Dessein (2003) uses a similar setup.

50 Note that due to our specification, \( u_k(q) \neq \theta_k u(q) \) in general.
of generality, we normalize $\theta_L < 1 < \theta_H$ such that $\beta \theta_L + (1 - \beta) \theta_H \equiv 1$. This normalization allows to interpret $q(p)$ as the mean demand per consumer at per call prices $p$. All consumers have the same degree of differentiation $\sigma$ and the location of the consumers is stochastically independent from their type. For future reference, we define the heterogeneity of consumers as the variance of their type: $\rho \equiv \beta (\theta_L - 1)^2 + (1 - \beta) (\theta_H - 1)^2$. If $\rho = 0$, we are back in the base model with homogeneous consumers.

**Retail pricing structure.** Similar to section 3, operator $i$ sets the retail per call price $p_{ki}$ and the fixed fee $F_{ki}$ for a type $\theta_k$ subscriber. We equivalently express the problem in terms of quantity $q_{ki} \equiv q_k(p_{ki})$ and net surplus $w_{ki} \equiv u_k(q_{ki}) - q_{ki}p_{ki} - F_{ki}$. As before, the fixed component $F_{ki}$ can be easily recovered for any level of quantity and net surplus.

**Wholesale pricing structure.** MNOs still charge a linear wholesale price to foreign operators. MNOs cannot discriminate the wholesale prices according to which type of customers the roaming calls are sold finally.

**Retail profits.** The retail profits of operator $i$ are now

$$\Pi_i^R = \beta n_{Li} \pi_{Li}^R + (1 - \beta) n_{Hi} \pi_{Hi}^R$$

with $\pi_{ki}^R = \pi_k^R(w_{ki}, q_{ki}, c_k) \equiv u_k(q_{ki}) - w_{ki} - q_{ki}c_k - C_F$ being the per customer retail profit and $n_{ki} = n_k(w_{ki}, w_{kj}) \equiv \frac{1}{2} + \sigma(w_{ki} - w_{kj})$ being the market share in segment $k \in \{L, H\}$.

**Retail equilibrium.** The surplus generated when offering quantity $q$ to a customer of type $\theta_k$ is defined as follows: $s_k(q, c) \equiv u_k(q) - qc$. By the same reasoning as in section 3, it is optimal to offer subscribers of type $\theta_k$ the quantity $q_{FB}^k$ that maximizes the surplus which is generated at perceived marginal costs $c_i$.\footnote{In contrast to section 3, we express the problem in terms of the per customer quantity $q_{ki}$ of the price $p_{ki}$ since this makes it easier to consider more complicated pricing structures when discussing second-degree price discrimination.}

$$q_{FB}^k(c_i) \equiv \arg \max_q \{u_k(q) - qc\}$$

By definition, we have $u_k(q_{FB}^k) = \theta_k v(c_i)$. Solving for the equilibrium net surplus and market share yields

$$w_{ki}^* = -\frac{1}{2\sigma} + \theta_k \left( \frac{2}{3} v(c_i) + \frac{1}{3} v(c_j) \right)$$

$$n_{ki}^* = \frac{1}{2} + \frac{\theta_k \sigma}{3} (v(c_i) - v(c_j))$$

If both competitors have equal perceived marginal costs, then each of them controls a market share of $n_{ki}^* = \frac{1}{2}$ in the candidate retail equilibrium. This implies $\pi_{ki}^{R*} = \frac{1}{2\sigma}$ and therefore the per customer profits are constant across both segments. Inserting the optimal tariffs in (21) and rearranging yields that for perceived marginal costs $(c_i, c_j)$, the retail equilibrium profits are:

$$\Pi_i^{R*} = \Pi_i^{R*}(c_i, c_j) \equiv \frac{\sigma \rho}{9} (v_i - v_j)^2 + \frac{1}{\sigma} \left( \frac{1}{2} + \sigma (v_i - v_j) \right)^2$$

\footnote{The quantity $q_{FB}^k$ equals the quantity that would be chosen by a subscriber when the per call price equals the perceived marginal costs.}
where \( v_i \equiv v(c_i) \) as before. Comparing this equation to (9) reveals that consumer heterogeneity does not affect the retail profits if \( c_i = c_j \). However, for \( c_i \neq c_j \) both operators earn higher retail profits compared to section 3. This can be best understood by noting that given the perceived marginal costs \((c_0, c_1)\) the average market share of both operators \( n_i^* = \beta n_{iL}^* + (1 - \beta) n_{iH}^* \) equals the market share that obtains in case of homogenous subscribers. For \( c_i \neq c_j \), \( n_i^* \neq n_{iH}^* \) so that both operators are better off since the retail profits are convex in the market share.

A marginal change in perceived marginal costs affects the retail profits as follows:

\[
\frac{\partial \Pi_i}{\partial c_i}(c_i, c_j) \equiv \frac{\sigma \rho}{9} (v_i - v_j)^2 + \frac{1}{\sigma} \left( \frac{1}{2} + \sigma (v_i - v_j) \right)^2
\]

(24)

It is useful to define the total equilibrium demand for roaming calls of operator \( i \) as follows:

\[
Q_i(c_i, c_j) = \beta n_{iL}^* F_i + (1 - \beta) n_{iH}^* F_i
\]

Using the previous results and simplifying yields:

\[
Q_i = Q(c_i, c_j) \equiv q(c_i) \left[ \frac{1}{2} + \frac{\sigma}{3} (v(c_i) - v(c_j)) (1 + \rho) \right]
\]

The previous equation shows that consumer heterogeneity renders the mean quantity more sensible to differences in the perceived costs. The reason is that according to (22), an operator that faces higher costs also offers less attractive tariffs. This difference increases in the subscriber’s type. Since the level of differentiation \( \sigma \) is independent of the type, the market shares in the heavy user segment are always less balanced than in the light user segment.

**Wholesale equilibrium.** As in section 5, we now consider two competing alliances. The negotiated wholesale prices equal the perceived marginal costs when setting the retail offers. The profits per member of the alliance remain defined as in (12), using the retail equilibrium profits (24). For small differences between \( a_0 \) and \( a_1 \), the marginal profits are:

\[
\frac{\partial \Pi_i}{\partial a_i}(a_i, a_j) = \left[ \frac{1}{3} - \epsilon(a_i) \right] Q(a_i, a_j) - \frac{\sigma}{3} (a_i - c) (1 + \rho) q(a_i)^2
\]

We can characterize the symmetric equilibrium wholesale price in a compact way, after defining the equilibrium share of roaming calls (in contrast to the market share of subscribers) as follows: \( \tilde{n}_i^* = \frac{1}{2} + \frac{\sigma}{2} (v(c_i) - v(c_j)) (1 + \rho) \). Note that the additional factor \( 1 + \rho \) indicates that the equilibrium share of roaming calls \( \tilde{n}_i^* \) reacts more sensitively to differences in wholesale prices compared to the equilibrium share of subscribers \( n_i^* \). Rearranging the first order condition, yields:

\[
\frac{a^* - c}{a^*} = \frac{1}{3 \left[ \eta_i(a^*) + \eta_{\tilde{n}_i^*}(a^*) \right]}
\]

(25)

with \( \eta_i(a_i) \) being the price elasticity of the mean per customer demand and \( \eta_{\tilde{n}_i^*}(a_i) \equiv - \frac{d \tilde{n}_i^*}{da_i} \frac{d a_i}{\tilde{n}_i^*} \) being the price elasticity of the equilibrium share of calls. In particular, in a

\[v(a_0) - v(a_1)\]
symmetric equilibrium the share of roaming calls is \( \tilde{n}_i^* = \frac{1}{2} \) and \( \eta_{\tilde{n}_i}(a_i) = \frac{2\sigma}{3} (1 + \rho) a_i q(a_i) \). Now we can identify the effect of consumer heterogeneity on the candidate equilibrium wholesale price:

**Proposition 13** Suppose that 25 uniquely characterizes the equilibrium wholesale price. Then an increase in consumer heterogeneity \( \rho \), holding everything else constant, reduces the symmetric wholesale equilibrium price.

**Proof.**

In any equilibrium, condition (25) must be satisfied. Observe that for \( \tilde{n}_i^* = \frac{1}{2} \) \( \eta_{\tilde{n}_i}(a_i) = \frac{2\sigma}{3} (1 + \rho) a_i q(a_i) \) is increasing in \( \rho \). By the same reasoning as in the proof of proposition 6, \( 3\frac{a_i - c}{a_i} [\eta_{\theta}(a_i) + \eta_{\tilde{n}_i^*}(a_i)] - 1 \) is increasing in \( a_i \). Applying the implicit function theorem on condition (25) yields then \( \frac{da_i^*}{d\rho} < 0 \).

Intuitively, consumer heterogeneity renders reducing the wholesale price more profitable since this allows to attract disproportionately many heavy users.

**Non observable customer types:** We now argue that even when customer types are unobservable for the MNOs, the results of this section are likely to remain intact. Since customers’ type \( \theta_k \) is now unobservable to the operators, they have to elicit this information by offering incentive compatible contracts. Without loss of generality, we consider direct revelation mechanisms. We slightly modify the retail pricing structure and allow operators to offer quantity \( q_{Li} \) at tariff \( T_{Li} \) for light users and quantity \( q_{Hi} \) at tariff \( T_{Hi} \) for heavy users.\(^{54}\)

When joining operator \( i \), heavy users select the package which is designed for them if:

\[
 w_{Hi} \geq w_{Li} + \Delta u(q_{Li}) \tag{26}
\]

where \( \Delta u(q) \equiv u_H(q) - u_L(q) \). Likewise, the incentive constraint for light users is:

\[
 w_{Li} \geq w_{Hi} - \Delta u(q_{Hi}) \tag{27}
\]

It turns out that in any symmetric equilibrium, the incentive constraints \( 26 \) and \( 27 \) are slack. This at first glance surprising result is in line with the observation of Armstrong and Vickers (2001) and Rochet and Stole (2002), that private information of consumers may not cause any quantity distortions in competitive environments.\(^{55}\)

To confirm that the sorting conditions are satisfied, we insert the retail equilibrium

\[^{54}\text{Due to the consumer heterogeneity and in contrast to section 3, operators can now generally do better than offering two part tariffs. Therefore, we allow operators in this section to set the quantity for each segment instead of the price per call. However, it turns out that the optimal quantity for each type of customer would equal the quantity that this customer would consume if the per call price was set to the perceived marginal costs.}\]

\[^{55}\text{They also discuss the sensitivity of this result with respect to crucial assumptions like symmetry.}\]
tariffs\textsuperscript{56} into (26) and (27), yielding the following requirement for $v(c_j) + v(c_i)$:\textsuperscript{57}
\begin{equation}
 s_L(q_L^{FB}(c_i), c_i) - s_L(q_H^{FB}(c_i), c_i) \geq \frac{1}{3} (\theta_H - \theta_L) (v(c_j) - v(c_i)) \geq s_H(q_L^{FB}(c_i), c_i) - s_H(q_H^{FB}(c_i), c_i)
\end{equation}

By definition of $q_L^{FB}(c_i)$ and $q_H^{FB}(c_i)$ the left hand side is non-negative while the right hand side is non-positive. Therefore, if $c_0$ and $c_1$ are close enough together, (26) and (27) are not binding for both operators. In particular, this is true in any symmetric candidate equilibrium where $c_0 = c_1$. This means that any wholesale prices $a_0^* = a_1^* = a^*$ that satisfy (25) for $n_i^* = \frac{1}{2}$ also locally maximize the operators’ profits in case they cannot observe the subscribers’ type. Whenever these prices are also global maximizers, they form an equilibrium even if operators have to resort to second degree price differentiation on the retail market.

8.2 Allowing for per customer wholesale fees

So far, we have assumed that operators can only charge linear prices on the wholesale level. This assumption reflects roughly the wholesale price structure that is used in practice at the moment. However, in this section we show that two-part tariffs on the upstream level render alliances even more profitable. More precisely, we assume that operators may both charge a per call wholesale-price $a_{xi}$ as above and a fee $\phi_{xi}$ that has to be paid for any foreign customer that visits the network.\textsuperscript{58} As in section 5, we assume that operators with same position have formed alliances. We focus on country $A$ (by symmetry the same results hold for country $B$) and omit the country index for brevity. The retail profit of operator $i$ conditional on alliance $i$ having agreed on $a_i$ and $\phi_i$ reads as follows:
\begin{equation}
 \Pi^R_i = \Pi^R(p_i, w_i, w_j, a_i, \phi_i) \equiv n(w_i, w_j)(q(p_i)(p_i - a_i) + v(p_i) - w_i - \phi_i - C_F)
\end{equation}

The per customer customer fee enters as perceived fixed costs and therefore renders customers less attractive for the operator in the retail market. The optimal retail per call price remains equal to the wholesale per call price of the alliance. By the same reasoning as in section 3 we still have $\pi_i^{R*} = \frac{n_i^*}{\sigma}, \quad \frac{d\pi_i^{R*}}{da_i} = -\frac{2q(c_i)}{3}, \quad \frac{d\pi_i^{R*}}{d\phi_i} = -\frac{q(c_i)}{3}$ and similar computations yield $\frac{d\pi_i^{R*}}{d\phi_i} = -\frac{2}{3}$ and $\frac{d\pi_i^{R*}}{d\phi_i} = -\frac{1}{3}$.

Using these results for the retail market, the modified first order conditions which characterize the optimal per call wholesale price $\tilde{a}_i^*$ and the optimal per customer wholesale fee $\phi_i^*$ are as follows:
\begin{equation}
 \frac{\sigma}{3} \tilde{\pi}_i^{R*} = \frac{\sigma}{3} \tilde{\pi}_i^{W*} + n_i^* \epsilon(\tilde{a}_i^*)
\end{equation}

\textsuperscript{56}Whenever the optimal net surplus given by equation (22) satisfies (28), then it is indeed a maximizer of retail profits. By the same reasoning as in the proof of Lemma 1, setting the efficient quantity $q_L^{FB}(c_i)$ and net surplus as given in (22) maximizes retail profit in any segment. If this solution yields a maximum of the unrestricted problem, it remains the solution of the restricted problem, as long as conditions (26) and (27) are satisfied.
\textsuperscript{57}The first inequality comes from the upward incentive constraint and the second from the downward incentive constraint.
\textsuperscript{58}Note that this pricing structure differs from two-part tariffs used for example as franchise fees. In our setup, the fixed fee $\phi_{xi}$ is paid for any customer. In contrast, a franchise fee is paid only once.
\[ \pi_{i}^{R*} = \tilde{\pi}_{i}^{W*} \]

where

\[ \tilde{\pi}_{i}^{W*} = q(a_{i}^*) (\tilde{a}_{i}^* - c) + \phi_{i}^* \]

denotes the per customer wholesale profit and \( \epsilon(\cdot) \) denotes the per customer demand elasticity in terms of markup.

Inserting condition (31) into condition (30) yields \( n_{i}^* \epsilon(\tilde{a}_{i}^*) = 0 \) which for \( n_{i}^* \neq 0 \) is only satisfied for \( \tilde{a}_{i}^* = c \). Hence as long as operator \( i \) expects to achieve a strictly positive retail market share, it is optimal to set the wholesale per call price equal to the true marginal costs.

The following proposition uses this insight to establish that if operators have the possibility to charge wholesale per customer fees, equilibrium profits will be higher and consumers will be worse off.

**Proposition 14** Suppose operators have formed two competing alliances. If each alliance can negotiate both on wholesale per call prices and on per customer fees, there exists a unique symmetric equilibrium. In this equilibrium wholesale per call prices are set at the true marginal costs and wholesale profits are \( \tilde{\pi}_{i}^{W*} = \phi_{i}^* = \frac{1}{2}\sigma \). Compared to any symmetric equilibrium without per customer fees, each operator’s wholesale profit is higher. Let \( a^* \) denote the symmetric equilibrium wholesale price that obtains under linear wholesale prices and is characterized by Proposition (5). Then introducing per customer wholesale fees increases total welfare by \(-\int_{c}^{a^*} (x - c) q'(x) dx\).

**Proof.**

First note that in any symmetric equilibrium, each operator has market share \( n_{i}^* = \frac{1}{2} \) and hence earns retail profits of \( \pi_{i}^{R*} = \frac{1}{2}\sigma \). Inserting these values and \( \tilde{a}_{i}^* = c \) into equation (31) yields \( \phi_{i}^* = \frac{1}{2}\sigma \). Furthermore, this critical point is a maximum, since \( \frac{\partial^2 \Pi}{\partial a_i^2} (\phi_i, \phi_j) = -\frac{1}{3} - \frac{\sigma}{9} < 0 \) for \( (\phi_i, \phi_j) \) such that \( n_i^* \in (0, 1) \). It can be easily verified that \( \Pi(\phi_i, \phi_j) \leq \Pi(\phi_j - \frac{3}{2\sigma}, \phi_j) \) for all \( \phi_i < \phi_j - \frac{3}{2\sigma} \), so that cornering the market is never optimal. If wholesale per customer fees are not feasible, rearranging equation (13) yields:

\[ \pi^{W}(a_{i}^*) = \pi^{R*}(a_{i}^*, a_{j}) - 3\sigma n_{i}^* \epsilon(a_{i}^*) \]

Inserting market share \( n_{i}^* = \frac{1}{2} \), this simplifies to \( \pi^{W}(a_{i}^*) = \frac{1}{2\sigma} - \frac{3}{2\sigma} \epsilon(a_{i}^*) < \frac{1}{2\sigma} = \tilde{\pi}_{i}^{W*} \).

As by assumption the whole market is covered in both cases, the difference in welfare is: \( \Delta WF = u(q(c)) - cq(c) - u(q(a_{i}^*)) + cq(a_{i}^*) = -\int_{c}^{a_{i}^*} (x - c) q'(x) dx = -\int_{c}^{a_{i}^*} (x - c) q'(x) dx > 0 \).

Intuitively, increasing the per customer fee allows to soften retail competition more efficiently than by increasing the wholesale per call price within an alliance. Introducing per customer fees reduces the per customer retail profits for any net surplus and retail per call price. However, by raising the per customer fee and setting the wholesale price equal to the true marginal costs, members of an alliance leave the retail per call price at the socially optimal level and therefore avoid deadweight loss.
9 Conclusion

This paper presents a tractable model of international roaming in which operators compete both on a wholesale and retail market simultaneously. We have accounted for recent technological developments and based our analysis on the assumption that MNOs may determine which foreign network their subscribers use to place roaming calls. We have shown that operators have incentives to form alliances that mutually provide roaming services at inefficiently high wholesale prices which translate to high retail prices. We have also shown that the ban of mutual roaming agreements might bring down roaming prices. Our suggestion might have constituted an easier approach than the price cap on roaming prices which was introduced by the European Parliament in 2007.

A Appendix - Proofs of Lemmas & Propositions

A.1 Proof of Lemma 1:

(We omit the country index for brevity). We show that whenever \( v(c_i) - v(c_j) \geq \frac{3}{2\sigma} \) there exists a unique equilibrium in pure weakly undominated strategies, which entails \( n_i^* = 1 \) and \( n_j^* = 0 \).

We first establish that any such corner equilibrium necessarily involves \( p_i^* = c_i, w_i^* = \frac{1}{2\sigma} - C_F + v(c_j), p_j^* = c_j \) and \( w_j^* = v(c_j) - C_F \). Define \( \tilde{w}_i \) such that given \( w_j, v(c_i), v(c_j) \), operator \( i \) just serves the whole market: \( \frac{1}{2} + \sigma (\tilde{w}_i - w_j) = 1 \). Note that whenever \( n_i^* = 1 \) then necessarily \( w_i^* = \tilde{w}_i \) as setting \( w_i > \tilde{w}_i(w_j) \) would yield strictly lower profits and \( w_i^* < \tilde{w}_i \) would contradict \( n_i^* = 1 \).

We now show that whenever \( n_j^* = 0 \), then necessarily \( w_j^* = v(c_j) - C_F \) and \( p_j^* = c_j \): Any strategy \( (w_j, p_j) \) with \( \pi^R(w_j, p_j) < 0 \) is weakly dominated by \( p_j = c_j \) and \( w_j = w_j^* \). Any strategy with \( \pi^R(w_j, p_j) \geq 0 \) and \( p_j \neq c_j \) is weakly dominated by choosing \( p_j = c_j \) and while leaving \( w_j \) constant as this increases profits per customer. Hence we have \( p_j^* = c_j \) in any equilibrium. It remains to show that whenever \( n_j^* = 0 \), then \( w_j^* = v(c_j) - C_F \). Suppose to the contrary that \( w_j < v(c_j) - C_F \). By the preceding discussion, necessarily \( w_i = \tilde{w}_i(w_j) \). Then player \( j \) could achieve a strictly positive market share and per customer profit by deviating to \( w_j + \frac{v(c_j) - C_F - w_j}{2} \), which contradicts equilibrium.

We now show that a unique corner equilibrium arises iff \( v(c_i) - v(c_j) \geq \frac{3}{2\sigma} \): (If-Existence) Given, \( w_j^* = v(c_j) - C_F \) and \( w_i^* = \frac{1}{2\sigma} - C_F + v(c_j) \), it can be directly verified that \( \frac{\partial \Pi^R}{\partial w_i}(w_i, w_j^*, c_i) > 0 \) for \( w_i < w_i^* \) and \( \frac{\partial \Pi^R}{\partial w_j}(w_j, w_i^*, c_i) < 0 \) for \( w_j > w_j^* \) which together with the preceding paragraphs confirms that \( w_i^* \) and \( w_j^* \) are mutually profit maximizing. (If-Uniqueness): There exists no interior equilibrium since inserting \( v(c_i) - v(c_j) \geq \frac{3}{2\sigma} \) into (8) yields \( n_i^* \geq 1 \) which is not interior. By the reasoning above, there is only one corner equilibrium in weakly undominated strategies. (Only-if): Suppose that \( 0 \leq v(c_i) - v(c_j) \leq \frac{3}{2\sigma} \): For \( w_j^* = v(c_j) - C_F \) as required in any corner equilibrium, the best response of player \( i \) is \( w_i^* < \tilde{w}_i \) which implies \( n_i^* < 1 \) and therefore causes a contradiction.
A.2 Proof of Lemma 3:

Note first that any trembling hand perfect equilibrium contains no weakly dominated strategy (see e.g. Mas-Colell, Whinston, and Green (1995), p. 259).

Suppose to the contrary that \( n^*(a^*_i, a^*_j) = 1 \) which implies \( \Pi(a^*_i, a^*_j) = 0 \). Define the highest wholesale price that allows to corner the market \( \bar{a}_i \) implicitly by \( v(\bar{a}_i) = v(a^*_j) + \frac{\partial}{\partial a_i} \). We show that any trembling-hand perfect equilibrium requires \( a^*_i \geq c \) since any \( a_i < c \) is weakly dominated by \( a^*_i = c \). Whenever \( a^*_i \) is such that \( \bar{a}_i < c \), then for \( a_i \in (\bar{a}_i, c) \), by equation (13), \( \frac{\partial}{\partial a_i} (a_i, a_j) = q(a_i) \left( \frac{1}{3} - \epsilon(a_i) \right) n^*_i - \frac{\sigma}{3} \pi^W (a_i) \) > 0 since \( \pi^W (a_i) < 0 \) and \( \epsilon(a_i) \leq 0 \). For \( a_i < \bar{a}_i \), \( \frac{\partial}{\partial a_i} (a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0 \). Thus for \( \bar{a}_i < c \) and for any \( a_i < c \), \( \Pi(c, a^*_j) \geq \Pi(a_i, a^*_j) \). If \( \bar{a}_i \geq c \), then \( \Pi(c, a^*_j) \geq \Pi(a_i, a^*_j) \).

Since \( a^*_i \geq c \), the corner equilibrium involves \( \Pi(a^*_i, a^*_j) \geq \Pi^R(a^*_i, a^*_ j) \geq \frac{1}{\sigma} \). Then deviating to \( a_j = a^*_i \) yields \( \Pi(a^*_i, a^*_j) \geq \frac{1}{\sigma} \) contradicting optimality of \( a^*_j \).

A.3 Proof of Lemma 4:

We prove the following properties that together imply the claim.

For all \((a_i, a_j)\) s.t. \( n^*(a_i, a_j) \in (0, 1) \) the following inequalities hold:

i) If \( a_i < c \) then \( \frac{\partial}{\partial a_i} (a_i, a_j) \geq 0 \).

ii) If \( a_i = 0 \) then \( \Pi(c, a_j) > \Pi(a_i, a_j) \).

iii) If \( a_i > c \) then \( q(a_i) > 0 \) and \( \epsilon(a_i) \geq \frac{1}{3} \) then \( \frac{\partial}{\partial a_i} (a_i, a_j) < 0 \).

iv) If assumption 2 holds and \( a_i > c \), then \( v(a_i) < v(c) = \frac{3}{2} \) then \( \frac{\partial}{\partial a_i} (a_i, a_j) < 0 \).

Part i)

By assumption 1, \( q(a_i) \geq q(c) > 0 \) which implies that \( \pi^W (a_i) < 0 \) for \( a_i < c \) and thus by equation (13), \( \frac{\partial}{\partial a_i} (a_i, a_j) > 0 \).

Part ii)

Any \( a_i \) with \( q(a_i) = 0 \) implies that \( a_i > c \) and \( q'(a_i) = 0 \) by assumption 1. As \( q'(a'_i) = 0 \) for \( a'_i \geq a_i \), we have \( v(a_j) \geq v(a_i) \) and hence \( n^*(a_i, a_j) \leq \frac{1}{3} \). In addition, \( q(a_i) = 0 \) implies \( q(a_i) (a_i - c) = 0 \). Hence \( \Pi(a_i, a_j) = \frac{1}{3} n^*(a_i, a_j)^2 < \frac{1}{3} n^*(c, a_j)^2 \geq \Pi(c, a_j) \) holds which contradicts \( a_i \) being optimal. To see that \( \Pi(c, a_j) \geq \frac{1}{3} n^*(c, a_j)^2 \), distinguish two cases: if \( v(c) - v(a_i) < \frac{3}{2} \), then \( \Pi(c, a_j) \geq \Pi^R(c, a_j) \) by Lemma 1. If \( v(c) - v(a_j) > \frac{3}{2} \), then by the same Lemma \( \pi^W (c) > \frac{1}{3} \) and hence \( \Pi(c, a_j) > \frac{1}{3} n^*(c, a_j)^2 \).

Part iii)

Since \( \epsilon(a_i) \geq \frac{1}{3} \) and \( q(a_i)(a_i - c) > 0 \), \( \frac{\partial}{\partial a_i} (a_i, a_j) = q(a_i) \left( -\frac{\sigma}{3} q(a_i)(a_i - c) + n^*(a_i, a_j) \left( \frac{1}{3} - \epsilon(a_i) \right) \right) < 0 \).

Part iv)

If \( \epsilon(a_i) \geq \frac{1}{3} \) then by part iii) the claim follows. If \( \epsilon(a_i) < \frac{1}{3} \) then by assumption 2, for all \( a_i \in [c, a_i] \), \( \epsilon(a_i) < \epsilon(a_i) \). By definition \( v'(p) = -q(p) \) and the condition \( v(c) - v(a_i) < \frac{3}{2} \) is equivalent to \( \int_c^{a_i} q(a)da < \frac{3}{2}. \) By assumption 2, \( \epsilon(\bar{a}_i) \geq 0 \) for \( \bar{a}_i \in [c, a_i] \) and thus \( \pi^W (a_i) = \int_{a_i}^{c} (1 - \epsilon(a)) q(a)da \geq (1 - \epsilon(a_i)) \int_{a_i}^{c} q(a)da. \) Therefore, \( \int_{a_i}^{c} q(a)da \geq \frac{3}{2} \) implies \( \pi^W (a_i) \geq (1 - \epsilon(a_i)) \frac{3}{2}. \) From (13) we have \( \frac{\partial}{\partial a_i} (a_i, a_j) \leq \left[ \frac{1}{1} - \epsilon(a_i) - \frac{\sigma}{3} \pi^W (a_i) \right] q(a_i) \leq \left[ \frac{1}{3} - \epsilon(a_i) - \frac{1}{2} (1 - \epsilon(a_i)) \right] q(a_i) = \frac{1}{2} \left[ -\frac{1}{3} - \epsilon(a_i) \right] q(a_i) < 0 \) where the first inequality is because \( \left( \frac{1}{3} - \epsilon(a_i) \right) n^*_i \geq \frac{1}{3} - \epsilon(a_i) \).

A.4 Proof of Proposition 5

We first prove the following auxiliary Lemma:
Lemma 15 If assumption 2 holds, then:

i) $\pi^W(a_i)$ is concave on $\mathcal{E}$ in $a_i$.

ii) Given $a_j \in \mathcal{E}$, any $a_i \in \mathcal{E}$ that satisfies the first order necessary conditions for being a local maximum of $\Pi(a_i, a_j)$ in $\mathcal{E}$ strictly maximizes $\Pi(a_i, a_j)$ in $\mathcal{E}$.

Proof.

Part i) $\frac{\partial \pi^W}{\partial p}(p) = (p-c)q'(p)+q(p) = q(p)(1-\epsilon(p))$. Hence $\frac{\partial^2 \pi^W}{\partial p^2}(p) = q'(p)(1-\epsilon(p)) - q(p)\epsilon'(p) < 0$ as $\epsilon'(p) > 0$ by assumption 2 and $1 - \epsilon(p) > 0$ for $p \in \mathcal{E}$.

Part ii) Suppose that $a_i, a'_i \in \mathcal{E}$, $a_i \neq a'_i$. By definition of $\mathcal{E}$, $\forall a_i, a_j \in \mathcal{E}$, since $|v(c) - v(a_i)| < \frac{3\sigma}{2}$ we have $n^*(a_i, a_j) \in (0, 1)$. We show that $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) \geq 0$ implies $\Pi(a_i, a_j) > \Pi(a'_i, a_j)$.

By (13), $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) = q(a_i)\left[\left(\frac{1}{3} - \epsilon(a_i)\right)n^*_i - \frac{\pi W(a_i)}{3}\right]$ on $\mathcal{E}$. For brevity, define $\varphi(a_i, a_j) \equiv (1 - 3\epsilon(a_i))n^*_i - \sigma\pi W(a_i)$ and note that $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) = \frac{1}{3}q(a_i)\varphi(a_i, a_j)$. Then $\frac{\partial \varphi}{\partial a_i}(a_i, a_j) = -2\sigma q(a_i)\left[\frac{2}{3} - \epsilon(a_i)\right] - 3\epsilon(a_i)n^*_i < 0$ as $\sigma > 0$, $\epsilon(a_i) < \frac{1}{3}$ and $\epsilon'(a_i) \geq 0$ by assumption 2.

We first show that $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) \leq 0$ implies $\frac{\partial \Pi}{\partial a_i}(a'_i) < 0$ for $a'_i > a_i$: By (13) and $q(a_i) > 0$ in $\mathcal{E}$, $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) \leq 0$ implies $\varphi(a_i, a_j) \leq 0$. But $\frac{\partial \varphi}{\partial a_i}(a_i, a_j) < 0$ and together with $q(a_i) > 0$ we get $\frac{\partial \Pi}{\partial a_i}(a'_i) < 0$.

We next show that $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) \geq 0$ implies $\frac{\partial \Pi}{\partial a_i}(a'_i) > 0$ for $a'_i < a_i$: $\frac{\partial \Pi}{\partial a_i}(a_i - a'_i) \geq 0$ implies $\varphi(a_i, a_j) \geq 0$ and by $\frac{\partial \varphi}{\partial a_i}(a_i, a_j) < 0$ we get $\varphi(a_i, a_j) > 0$. Together with $q(a_i) > 0$ we get $\frac{\partial \Pi}{\partial a_i}(a'_i) > 0$.

Proof of Proposition 5.

We first show existence of a symmetric equilibrium $a^*_0 = a^*_1 = a^*$ and consequently $n^*_0 = n^*_1 = \frac{1}{2}$. By Lemma 4 this equilibrium involves $a^* \in \mathcal{E}$. Define $\psi(p) \equiv (1 - 3\epsilon(p)) - 2\sigma\pi W(p)$. Note that by assumption 2, and as $\frac{\partial \pi^W}{\partial p}(p) > 0$ for $\epsilon(p) < 1$, we have $\psi'(p) < 0$ in the interior of $\mathcal{E}$. Using (13) and $n^*_1 = \frac{1}{2}$, the necessary first order condition is $\psi'(a^*) = 0$. Note that $\psi(1/2) > 0$. Distinguish two cases:

- There exists some $p \in \mathbb{R}$ such that $\psi(p) = -1/3$. Define $\tilde{p} = \min\{p \in \mathcal{P} | \psi(p) = -1/3\}$.

Then $\psi(\tilde{p}) = -2\sigma\pi W(\tilde{p}) < 0$. By continuity there must exist a unique $\hat{a} \in \mathbb{R}^+$ such that $\psi(\hat{a}) = 0$.

- There does not exist some $p \in \mathcal{P}$ such that $\psi(p) = 1/3$. Then for all $p \geq c$, $p \in \mathcal{E}$ and hence $\psi'(p) < 0$. In addition, using $\epsilon(p) < 1/3 \forall p \geq c$ implies that $\lim_{p \to \infty} \pi W(p) = \infty$. Hence $\lim_{p \to \infty} \psi(p) = -\infty$ and again there exists a unique $\hat{a} \in \mathcal{E}$ such that $\psi(\hat{a}) = 0$.

By Lemma 4, $\psi(\hat{a}) = 0$ and $q(\hat{a}) > 0$ can only be satisfied for $\hat{a} \in \mathcal{E}$ and since $\psi'(a) < 0$ in the interval $\mathcal{E}$, $\hat{a}$ is unique in $\mathcal{E}$.

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50 This property is usually denoted strict pseudoconcaocity and is stronger than strict quasiconcavity.

51 If $q(p) = 0$ for some $p$, then there must exist $\bar{p} < p$ with $\epsilon(\bar{p}) = \frac{1}{3}$.

61 Integrating up $\int_{\frac{1}{3}}^{1/3} (\pi - \psi(p)) dp = \epsilon(p) \int_{\frac{1}{3}}^{1/3} \psi(p) dp < 0$ $\forall p > c$, we get

$$\pi(p) \geq \pi(p) \left[\frac{1}{3} - \frac{1}{3}\right]^2$$

which goes to infinity as $p \to \infty$.  

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It remains to show that the candidate \( \hat{a} \) is indeed a symmetric equilibrium. By definition of \( \psi \), \( a_i = \hat{a} \) satisfies the necessary first order condition when \( a_j = \hat{a} \). By Lemma 15, the first order conditions are also sufficient for being a global maximum on \( \mathcal{E} \). By the proof of Lemma 4, \( a_i = \hat{a} \) remains a maximizer on the set of all \( a_i \in \mathbb{R} \) such that \( n(a_i, a_j) \in (0, 1) \). Setting \( a_i \) high enough so that \( n_i = 0 \) cannot be optimal either, as this gives zero profits.

It remains to show that \( \Pi(\hat{a}, \hat{a}) \geq \Pi(\tilde{a}_i, \tilde{a}) \) for \( \tilde{a}_i \) such that \( n(\tilde{a}_i, \hat{a}) = 1 \). Since \( \hat{a} \in \mathcal{E} \), the inequality \( v(c) < v(\hat{a}) + \frac{3}{2\sigma} \) holds. Cornering the market requires \( v(\tilde{a}_i) \geq v(\hat{a}) + \frac{3}{2\sigma} \), and thus \( \tilde{a}_i < c \). For any \( a_i < c \) such that \( v(a_i) > v(a_j) + \frac{3}{2\sigma} \), marginal profits are 

\[
\frac{\partial \Pi}{\partial a}(a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0 \quad \text{since} \quad \epsilon(a_i) \leq 0. \quad \text{Thus} \quad \Pi(a_i, \hat{a}) \leq \Pi(c, \hat{a}) < \Pi(\hat{a}, \hat{a}).
\]

Uniqueness:

There is no other symmetric equilibrium since any interior equilibrium must belong to \( \mathcal{E} \) and since the necessary first order condition is uniquely satisfied in \( \mathcal{E} \) at \( \hat{a} \) by the previous discussion.

We now show that no asymmetric equilibrium exists:

Suppose to the contrary that an asymmetric equilibrium with \( a_i^* > a_j^* \) and hence \( n_i^* < n_j^* \) exists. By assumption 2 \( a_i^* > a_j^* \) implies \( \epsilon(a_i^*) \geq \epsilon(a_j^*) \). By Lemma 3, this equilibrium must involve a strictly positive market share for both alliances and a strictly positive per customer demand. The necessary first order conditions are:

\[
\left( \frac{1}{3} - \epsilon(a_i^*) \right) n_i^* - \frac{\sigma}{3} \pi^W(a_i^*) = 0 \\
\left( \frac{1}{3} - \epsilon(a_j^*) \right) n_j^* - \frac{\sigma}{3} \pi^W(a_j^*) = 0
\]

But \( \epsilon(a_i^*) \geq \epsilon(a_j^*) \) and \( n_i^* < n_j^* \) implies \( \left( \frac{1}{3} - \epsilon(a_i^*) \right) n_i^* < \left( \frac{1}{3} - \epsilon(a_j^*) \right) n_j^* \). Furthermore, by Lemma 4, \( a_i^*, a_j^* \in \mathcal{E} \). Hence \( \frac{1}{3} \geq \epsilon(a_i^*) \geq \epsilon(a_j^*) \) and thus \( \pi^W(a_i^*) > \pi^W(a_j^*) \). Taken together this implies \( \left( \frac{1}{3} - \epsilon(a_i^*) \right) n_i^* - \frac{\sigma}{3} \pi^W(a_i^*) < n_j^* \left( \frac{1}{3} - \epsilon(a_j^*) \right) - \frac{\sigma}{3} \pi^W(a_j^*) = 0 \) which contradicts the first order necessary conditions.

Finally, we show that \( a^* > c \). Any equilibrium involves \( n^*(a_i^*, a_j^*) \in (0, 1) \). The necessary first order condition (13) for \( a^*_i = c \) is \( n^*(c, a_j^*)2(c) = 0 \) which is never true as \( q(c) > 0 \) by assumption 1.

Rearranging the equilibrium condition \( \psi(a^*) = 0 \) yields the equilibrium per customer profits.

\[\blacksquare\]

### A.5 Proof of Proposition 6

Rewriting condition 14 for a symmetric equilibrium yields

\[2\sigma q(a^*) (a^* - c) + 3\epsilon(a^*) - 1 = 0 \quad (32)\]

Part i) Applying the implicit function theorem on this condition, the claim is true if \( \frac{\partial}{\partial a} (2\sigma q(a^*) (a^* - c) + 3\epsilon(a^*)) > 0 \). By assumption 2, \( \epsilon'(a^*) > 0 \). In addition, \( \frac{\partial}{\partial a} q(a^*) (a^* - c) > 0 \) since \( \epsilon(a^*) < \frac{1}{3} \) which completes the proof.

Part ii) Consider any pair of demand functions \( q \) and \( \tilde{q} \) with \( \eta_q(a) > \eta_{\tilde{q}}(a) \forall a \in P \) and \( \tilde{q}(a^*) \geq q(a^*) \). Since \( \epsilon(a) = \eta_q(a) \frac{\alpha - \epsilon}{\alpha} \), \( \eta_q(a) > \eta_{\tilde{q}}(a) \) implies \( \epsilon_q(a) > \epsilon_{\tilde{q}}(a) \) for
We show that the equilibrium wholesale price $\tilde{a}^*$ that corresponds to per customer demand $q$ is higher than the equilibrium price $a^*$ for demand $q$. By the proof of proposition 5, the function $\psi_2(q) \equiv 2\sigma q(a)(a-c) + 3\epsilon q(a) - 1$ is increasing in $a$ for $a \in \mathcal{E}$ and $\psi_2(c) = -1$. Define $\psi(q)$ likewise for demand $q$. To show that $\tilde{a}^* \in (c, a^*)$, just note that $\psi_2(\tilde{a}^*) = 0$ where the first inequality comes from the hypothesis $\tilde{q}(a^*) \geq q(a^*)$ and $\epsilon q(a) - \epsilon q(a) > 0$ and the last equality is the equilibrium condition of $a^*$ being an equilibrium for demand $q$. Since $\psi(q) > 0$, by continuity there exists an $\tilde{a}^* < a^*$ such that $\psi_2(\tilde{a}^*) = 0$. This equilibrium candidate is indeed an equilibrium for demand $q$ by the proof of proposition 5.

### A.6 Proof of Proposition 8

We first prove the following auxiliary Lemma:

**Lemma 16** If assumption 2 holds and $q' (a^*) < 0$, then $v(c) - v(a^*) < \frac{1}{2\sigma}$, where $a^*$ is the equilibrium wholesale price defined by Proposition 5.

**Proof.**

The equilibrium condition $\frac{\partial \Pi}{\partial a_i}(a^*, a^*) = 0$ yields $\pi^{W*} \equiv q(a^*)(a^* - c) = \frac{1-3\epsilon(a^*)}{2\sigma}$. Assumption 2 implies that $v(c) - v(p) \leq \frac{\pi^{W}(p)}{1-\epsilon(p)}$ for any $p \in \mathcal{E}$. Putting these results together yields $v(c) - v(a^*) \leq \frac{\pi^{W}(a^*)}{1-\epsilon(\tilde{a}^*)} = \frac{1-3\epsilon(a^*)}{2\sigma(1-\epsilon(\tilde{a}^*))} < \frac{1}{2\sigma}$ where the last inequality is due to $\frac{1}{3} \geq \epsilon(a^*) > 0$.

We now show existence of a unique symmetric equilibrium. Denote the wholesale price that obtains after the retail price cap has been introduced by $\overline{\pi}^*$ and the equilibrium net surplus as $\overline{w}^*$. By the same reasoning as in Lemma 15, the first order condition is sufficient for a (local) maximum. Similar to the proof of Proposition 5, we define $\overline{\psi}(a) \equiv \frac{\epsilon}{q(p)} \frac{\partial \Pi}{\partial a_i}(a, a) = 1 - 2\sigma \overline{\pi}^*(a - c)$. We claim that that wholesale prices $a_0 = a_1 = \overline{a}^*$ with $\overline{a}^*$ being uniquely characterized by $\overline{\psi}(\overline{a}^*) = 0$ support an equilibrium. By definition of $\overline{\psi}$, the equilibrium price $\overline{\pi}^*$ locally strictly maximizes both alliances’ profits.

Next we show that $\overline{\Pi}(a_i, \overline{a}^*)$ is strictly quasiconcave in $a_i$ if both alliances have a positive market share: Define $\overline{\pi}^{W}(a_i, a_j) \equiv \frac{1}{2} + \frac{\sigma}{3} [\overline{\pi}(a_i) - \overline{\pi}(a_j)]$ using the generalized value $\overline{\psi}(\cdot)$ of (15). For $a_i \geq \overline{p}$, $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) = \frac{\epsilon}{q(\overline{p})} \left[\overline{\pi}^{W}(a_i, \overline{a}^*) - \frac{1}{3}\overline{\pi}^{W}(a_i)\right]$ with $\overline{\pi}^{W}(a_i) \equiv q(\overline{p})(a_i - c)$. Since $\frac{\partial \overline{\Pi}}{\partial a_i}(\overline{\pi}^*, \overline{a}^*) = 0$ and $\overline{\pi}^*(a_i, \overline{a}^*)$ decreases in $a_i$ while $\overline{\pi}^{W}(a_i)$ increases in $a_i$, we have $(\overline{a}^* - a_i) \frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) > 0$ for $a_i > \overline{p}$ and $a_i \neq \overline{a}^*$. For $a_i < \overline{p}$, $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) = \frac{\epsilon}{q(\overline{p})} \left[(1-3\epsilon(a_i))\overline{\pi}^{W}(a_i, \overline{a}^*) - 3\overline{\pi}^{W}(a_i)\right]$ which differs from 13 only by the market share $\overline{\pi}^{W}(a_i, \overline{a}^*)$ instead of $n^*(a_i, a_i)$. We show below that $\overline{\pi}(\overline{\pi}^*) < v(a^*)$ which implies $\overline{\pi}^*(a_i, \overline{a}^*) > n^*(a_i, a_i)$. We show below that $\overline{\pi}(\overline{\pi}^*) < v(a^*)$, which implies $\overline{\pi}^*(a_i, \overline{a}^*) > n^*(a_i, a_i)$ for $a_i < \overline{p}$. Since by hypothesis $\overline{p} \leq a^*$, we have $\frac{\partial \overline{\Pi}}{\partial a_i}(a_i, \overline{a}^*) > \frac{\partial \overline{\Pi}}{\partial a_i}(a_i, a_i) > 0$ where the last inequality is due to Lemma (15).

It remains to prove that drastic deviations in order to corner the market are unprofitable. We first show that given $\overline{p} < a^*$, any deviation wholesale price $\tilde{a}_i$ to corner the market requires that $\tilde{a}_i < c$ or equivalently $u(\tilde{a}_i) > v(c)$. To derive a lower bound for $\tilde{a}^*$, note that $\tilde{v}^* = v(\overline{p}) - q(\overline{p})(\overline{a}^* - \overline{p}) = v(c) - \pi^{W^*} - \int_{\overline{p}}^{c} \epsilon(p)q(p)dp$ with $\pi^{W^*} \equiv q(\overline{p})(\overline{a}^* - c)$. The equilibrium condition $\overline{\psi}(\overline{\pi}^*) = 0$ implies $\overline{\pi}^{W^*} = \frac{1}{2\sigma}$. Besides,
Thus \( \bar{\sigma} > a^* \in \mathcal{E} \) guarantees that 
\[
\int_{\bar{\sigma}}^{a^*} \epsilon(p)q(p)dp \leq \epsilon(\bar{\sigma}) (v(c) - v(\bar{\sigma})) \leq \frac{1}{3} (v(c) - v(a^*)) < \frac{1}{6\sigma},
\]
where the last inequality is due to Lemma 16. Taken together, \( \bar{\sigma} > v(c) - \frac{1}{6\sigma} \). Cornering the market requires 
\[
v(\bar{a}_i) \geq \bar{\sigma} + \frac{3}{2\sigma} > v(c) + \frac{5}{6\sigma} > v(c).
\]
For any \( a_i < c \) such that \( v(a_i) > v(\bar{a}) + v(a_j) \), marginal profits are 
\[
\frac{\partial \Pi}{\partial a_i}(a_i, a_j) = -q(a_i)\epsilon(a_i) \geq 0 \text{ since } \epsilon(a_i) \leq 0.
\]
Thus \( \Pi(\bar{a}_i, a^*) \leq \Pi(c, a^*) < \Pi(\bar{a}_i, a^*) \).

The preceding two paragraphs establish that there is no profitable deviation, which completes the proof of existence.

We now show that \( \bar{\sigma} < v(a^*) \), which by help of Lemma 7 suffices to prove that any binding price cap reduces the consumer surplus. The condition \( v(\bar{p}) - q(\bar{p}) (\bar{\sigma} - \bar{p}) < v(a^*) \) can be rewritten as \( v(\bar{p}) + q(\bar{p}) (\bar{p} - c) - \bar{\pi}^{\sigma} < v(a^*) \) and is satisfied if \( v(c) - \bar{\pi}^{\sigma} < v(a^*) \), \( v(\bar{p}) + q(\bar{p}) (\bar{p} - c) \leq v(c) \). Reordering this condition and using \( \bar{\pi}^{\sigma} = \frac{1}{\sigma} \) yields \( v(c) - v(a^*) < \frac{1}{\sigma} \bar{\pi}^{\sigma} \) which is true by Lemma 16.

If \( \bar{p} < a^* \), then clearly \( v(\bar{p}) + q(\bar{p}) (\bar{p} - c) > v(a^*) + q(a^*) (a^* - c) \) and total welfare increases.

Comparing \( \bar{\psi}(a) \) to \( \psi(a) \) defined in the proof of Proposition 5 yields \( \bar{\psi}(a) - \psi(a) = 3\epsilon(a) + 2\sigma (q(a) - q(\bar{p}) (a - c) \). Therefore, the condition \( \bar{\psi}(a^*) > \psi(a^*) = 0 \) holds by the hypothesis \( q(\bar{p}) - q(a^*) < \frac{3\epsilon(a^*)}{2\sigma(a^* - c)} \). Since \( \bar{\psi}(a) = -\sigma q(\bar{p}) < 0, \bar{\psi}(a^*) > 0 \) implies \( \bar{\psi}(a^*) = 0 \) for \( \bar{\sigma} > a^* \).

### A.7 Proof of Lemma 9

**Part i)**

Note that for all \( a_i, a_j \in \mathcal{E} \), \( n^*(a_i, a_j) \in (0, 1) \) by definition of \( \mathcal{E} \).

We show that for any \( a_j \in \mathcal{E} \) there exists a unique \( \hat{a} \in \mathcal{E} \) such that \( \varphi(\hat{a}, a_j) = (1 - 3\epsilon(a_i)) n^*(\hat{a}, a_j) - \sigma \bar{\pi}^{\sigma}(a_i) = 0 \). Since \( \frac{\partial \varphi(a_i, a_j)}{\partial a_i} = \frac{1}{3} q(a_i) \varphi(a_i, a_j) \) and by Lemma 15, part ii), \( a_i = \hat{a} \) strictly maximizes \( \Pi(a_i, a_j) \) in \( \mathcal{E} \). By Lemma 4, \( \hat{a} \) remains a strict maximizer in \( \mathbb{R} \).

Note that \( \varphi(c, a_j) = n^*(c, a_j) > 0 \). Distinguishing two cases:

- There exists some \( p \in \mathbb{R} \) such that \( \epsilon(p) = \frac{1}{3} \). Define \( \bar{p} = \min \{ p \in \mathbb{R} | \epsilon(p) = \frac{1}{3} \} \).

  Then \( \varphi(\bar{p}, a_j) = -\sigma \bar{\pi}^{\sigma}(\bar{p}) < 0 \). By continuity and by \( \frac{\partial \varphi(a_i, a_j)}{\partial a_i} < 0 \) for \( a_i, a_j \in \mathcal{E} \), there must exist a unique \( \hat{a} \) (which is necessarily in \( \mathcal{E} \)) such that \( \varphi(\hat{a}, a_j) = 0 \).

- There does not exist some \( p \in \mathbb{R} \) such that \( \epsilon(p) = \frac{1}{3} \). Then for all \( a_i \geq c \), \( a_i \in \mathcal{E} \) and hence \( \frac{\partial \varphi(a_i, a_j)}{\partial a_i} < 0 \). In addition, using \( \epsilon(a_i) < \frac{1}{3} \forall a_i \geq c \) implies that \( \lim_{a_i \to \infty} \bar{\pi}^{\sigma}(a_i) = \infty \). Hence \( \lim_{a_i \to \infty} \varphi(a_i) = -\infty \) and again there exists a unique \( \hat{a} \in \mathcal{E} \) such that \( \varphi(\hat{a}, a_j) = 0 \).

**Part ii)**

Any profit maximizing wholesale price \( a^*(a_j) \) involves \( \frac{\partial \Pi}{\partial a_i}(a^*(a_j), a_j) = 0 \). By Lemma 15, part ii), any critical point is also a strict maximum which implies \( \frac{\partial^2 \Pi}{\partial a_i \partial a_j}(a^*(a_j), a_j) < 0 \). Therefore, by the implicit function theorem, the claim is true if \( \frac{\partial^2 \Pi}{\partial a_i \partial a_j}(a^*(a_j), a_j) > 0 \). Differentiating (13) with respect to \( a_j \) yields \( \frac{\partial^2 \Pi}{\partial a_i \partial a_j}(a^*(a_j), a_j) = \frac{2}{5} \Pi(q(a_j)^2 (\frac{1}{3} - \epsilon(a_i)) > 0 \).

**Part iii)**

Follows immediately from Lemma 4 and condition (14)

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A.8 Proof of Proposition 10

We first show that operators in country $B$ announce to form an alliance if this increases the total number of alliances. Suppose that operator $Bi$ does not announce to form an alliance, but all other operators do so. Then one alliance is formed and $Bi$ as well as one operator in country $A$, say $Ai$, remain without alliance. The remaining alliance $j$ anticipates that $Ai$ and $Bi$ will provide and purchase roaming services at the true marginal costs and sets the intra-alliance wholesale price $a_j = a^*(c)$ that maximizes the profits of its members. According to Lemma 9, this price uniquely exists and exceeds the true marginal costs: $a^*(c) > c$. For operator $Bi$ this yields profits $\Pi^R(c, a^*(c))$ (using notation of section 3). Since own profits increase in the competitors’ wholesale prices, the following string of inequalities holds by Lemma 9: $\Pi^R(c, a^*(c)) = \Pi(c, a^*(c)) < \Pi(c, a^*) < \Pi(a^*, a^*)$ with $a^*(c) < a^*$ and $a^*$ defined by proposition 5. Hence operator $Bi$ has higher profits if it announces to form an alliance, too. Lemma 9 also implies that $\Pi(a^*(c), c) > \Pi(c, c)$ which makes announcing an alliance a best response for $Bi$ if only one operator in $A$ has announced to form an alliance and $Bj$ declares to stay alone.\(^{62}\) Given the best response of operators of country, $B$, the same reasoning for operators in country $A$ yields the result.

B Appendix - Continuous model of network selection

We assume that at most the proportion $\tilde{\gamma} \in [0.5, 1]$ of roaming calls can be directed to a particular foreign network.\(^{63}\) This bound on the proportion reflects the fact that the restriction does not come from capacity constraints (which would render an absolute constraint more plausible) but rather from an unreliable technology that cannot guarantee that a subscriber registers in the preferred network. We have analyzed the polar case of perfect network selection ($\gamma = 1$) in the base model. In section 7 we have presented the other extreme of no control ($\tilde{\gamma} = 0.5$), meaning that each foreign network hosts a travelling subscriber equally likely. As in section 7, operators cannot discriminate on the retail market according to which foreign network is used abroad.

For clarity, we present the results from the viewpoint of operators with home network in country $A$. When buying roaming calls from foreign MNOs on the wholesale market, operator $Ai$ may decide to buy proportion $\gamma_{Ai}$ from operator $B0$ and proportion $1 - \gamma_{Ai}$ from operator $B1$. Operator $Ai$’s perceived marginal costs are:

$$c_{Ai} = \gamma_{Ai} a_{B0} + (1 - \gamma_{Ai}) a_{B1} \quad (33)$$

Since operators cannot discriminate the retail prices according to which host network provides the roaming services, the per call price equals the perceived marginal costs:

\(^{62}\)Note that if only one operator in $A$ and $Bj$ declares to form an alliance, $Bi$ strictly prefers to stay alone as only one alliance can be created and $\Pi^R(c, a^*(c)) > \Pi(a^*(c), c)$.

\(^{63}\)This specification is equivalent to the following assumption: Operators can direct their subscribers to the desired foreign network only with probability $\tilde{\gamma} \in [0, 1]$. The remaining subscribers are assigned randomly to the host networks. Then one immediately sees that $\bar{\gamma} = \tilde{\gamma} + \frac{1}{2} (1 - \tilde{\gamma}) = \frac{1}{2} (1 + \tilde{\gamma})$. See also Salsas and Koboldt (2004), section 3.5 for a slightly different assumption.
\[ p_{Ai}^* = c_{Ai}. \] The equilibrium net surplus, market shares and the retail equilibrium profits remain as established in Lemma 1.

We now turn to the wholesale market.

**No international alliances.** As discussed in sections 3 and 4, operators prefer to buy roaming calls from the cheapest foreign operator.

\[
\gamma_{Ai}^* = \begin{cases} 
\bar{\gamma} & \text{if } a_{B0} < a_{B1} \\
1 - \bar{\gamma} & \text{if } a_{B0} > a_{B1}
\end{cases}
\]

We define the optimized perceived marginal cost of operator \( Ai \) as the cheapest possible mean cost for roaming calls, given the posted prices of foreign operators:

\[
c_{Ai}^* = c^* (a_{B0}, a_{B1}) \equiv \bar{\gamma} \min\{a_{B0}, a_{B1}\} + (1 - \bar{\gamma}) \max\{a_{B0}, a_{B1}\}
\]

The main implication of imperfect host network selection is that operators may generate positive demand even when not offering the cheapest wholesale price. We assume for simplicity that foreign operators divide the traffic evenly among both domestic networks if these offer equal wholesale prices. Using the results of the retail equilibrium, in absence of alliances the total wholesale demand of operator \( Ai \) (where the superscript \( NA \) means “no alliance”) is:

\[
Q_{NA}^{Ai} = Q^{NA}(a_{Ai}, a_{Aj}) \equiv \begin{cases} 
\bar{\gamma} q ((1 - \bar{\gamma}) a_{Aj} + \bar{\gamma} a_{Ai}) & \text{if } a_{Ai} < a_{Aj} \\
\frac{1}{2} q(a_{Ai}) & \text{if } a_{Ai} = a_{Aj} \\
(1 - \bar{\gamma}) q ((1 - \bar{\gamma}) a_{Ai} + \bar{\gamma} a_{Aj}) & \text{if } a_{Ai} > a_{Aj}
\end{cases}
\]

Note that the demand is independent of the actual market share of the reselling operators, since for all price combinations, both foreign operators purchase the same part of their traffic at operator \( Ai \).

The overall profit of operator \( Ai \) is therefore:

\[
\Pi_{NA}^{Ai} = \Pi^{NA}(a_{Ai}, a_{Aj}) \equiv \Pi^{R*}(c_{Ai}, c_{Aj}) + (a_{Ai} - c) Q^{NA}(a_{Ai}, a_{Aj})
\]

Similar to section 4, in equilibrium each operator takes the foreign wholesale prices and therefore its retail profits as given. Therefore operator \( Ai \) sets its wholesale price in order to maximize its wholesale profit \((a_{Ai} - c) Q^{NA}(a_{Ai}, a_{Aj})\). Under the technical assumption 3 of section 7, no pure strategy equilibrium obtains for \( \bar{\gamma} \in (0.5, 1) \):

**Lemma 17** Suppose that assumption 3 holds. For \( \bar{\gamma} \in (0.5, 1) \), there is no pure strategy equilibrium.

**Proof.** We first show that there is no symmetric equilibrium. Suppose to the contrary that \( a_{A0}^* = a_{A1}^* \). If \( a_{A0}^* = c \), then increasing the own price increases wholesale profits. If \( a_{A0}^* > c \), then undercutting slightly increases profits.

We now show that there is no asymmetric equilibrium. Let \( p^* \) denote the maximizer of \((p - c) q(p)\).\(^{64}\) Suppose to the contrary w.l.o.g. that \( a_{A0}^* \neq a_{A1}^* \). Then there exists an operator \( Ai \) such that \( a_{Ai}^* \neq p^* \). But then there exists an \( \hat{\alpha}_{Ai} \) such that

\(^{64}\)Which exists by assumption 3.
sign($\hat{a}_{Ai} - a_{Aj}$) = sign($a^*_{Ai} - a_{Aj}$) and $|\hat{a}_{Ai} - p^*| < |a^*_{Ai} - p^*|$. By assumption 3, this implies that $(\hat{a}_{Ai} - c) Q^{NA}(\hat{a}_{Ai}, a^*_{Aj}) > (a^*_{Ai} - c) Q^{NA}(a^*_{Ai}, a^*_{Aj})$ and therefore contradicts equilibrium.

By symmetry and the usual Bertrand reasoning, there cannot exist an equilibrium in which both operators set different wholesale prices. However, under imperfect network selection the fully competitive equilibrium of section 4 vanishes and there is no other equilibrium in which both operators set higher wholesale prices. Intuitively, there is no equilibrium with $a^*_{A0} = a^*_{A1} = c$ because deviating upwards generates strictly positive wholesale profits.

**Two international alliances.** Similar to section 5, we now analyze the equilibrium outcome after operators with same location have formed two competing alliances. We omit the country index for brevity.

We maintain all assumptions of the base model but assume that each member can at most commit that a proportion $\bar{\gamma}$ of its subscribers uses the foreign partner network to place roaming calls. Furthermore, we restrict operators to sell roaming calls on the wholesale market to all foreign operators for the same price $a_i$ that is negotiated within an alliance.

If both alliances have negotiated the wholesale prices $a_i$ and $a_j$, the equilibrium wholesale demand for roaming calls of operator $i$ is

$$Q_i = Q(a_i, a_j) \equiv \bar{\gamma} n_i^* q (\bar{\gamma} a_i + (1 - \bar{\gamma}) a_j) + (1 - \bar{\gamma}) (1 - n_i^*) q (\bar{\gamma} a_j + (1 - \bar{\gamma}) a_i)$$

where

$$n_i^* = \frac{1}{2} + \frac{\sigma}{3} [v (\bar{\gamma} a_i + (1 - \bar{\gamma}) a_j) - v (\bar{\gamma} a_j + (1 - \bar{\gamma}) a_i)]$$

is the equilibrium retail market share. The profit of each operator in alliance $i$ is:

$$\Pi_i = \Pi(a_i, a_j) \equiv \Pi^{Rs}(c_i, c_j) + (a_i - c) [\bar{\gamma} n_i^* q (c_i) + (1 - \bar{\gamma}) (1 - n_i^*) q (c_j)] \quad (34)$$

If both firms realize a strictly positive market share, the marginal profit with respect to the own wholesale price is:

$$\frac{\partial \Pi_i}{\partial a_i} (a_i, a_j) = Q(a_i, a_j) + \frac{dn_i^*}{da_i} \left[ \frac{2 n_i^*}{\sigma} + (a_i - c) (\bar{\gamma} q(c_i) + (1 - \bar{\gamma}) q(c_j)) \right]$$

$$+ (a_i - c) \left[ \bar{\gamma}^2 n_i^* q' (c_i) + (1 - \bar{\gamma})^2 (1 - n_i^*) q' (c_j) \right] \quad (35)$$

with

$$\frac{dn_i^*}{da_i} = \frac{\sigma}{3} ((1 - \bar{\gamma}) q(c_j) - \bar{\gamma} q(c_i))$$

Considering a symmetric equilibrium with $a_i^* = a_j^* = a^*$ and therefore $c_i^* = c_j^* = a^*$ as well as $n_i^* = \frac{1}{2}$ yields the following characterization:
\[
\frac{a^* - c}{a^*} = \frac{1 - \frac{2}{3} (2\Bar{\gamma} - 1)}{[(\gamma^2 + (1 - \Bar{\gamma})^2) \eta_q(a^*) + (2\Bar{\gamma} - 1)^2 \eta_n(a^*)]}
\]

(36)

where \(\eta_q(\cdot)\) is the price elasticity of the per customer demand and \(\eta_n(a^*) \equiv \frac{2}{3} \sigma a^* q(a^*)\) is the price elasticity of the retail market share for \(a_j = a_i = a^*\) in case of perfect traffic direction.\(^{65}\)

Comparing (36) with the equilibrium characterization (14) of the base model reveals that for the same wholesale price \(a_i\), the right hand side of (36) is always larger than that of (14) since \(1 - \frac{2}{3} (2\Bar{\gamma} - 1) \geq \frac{1}{3}, \gamma^2 + (1 - \gamma)^2 \leq 1\) and \(2\Bar{\gamma} - 1 \leq 1\) hold. These observations allow to establish that imperfect traffic steering leads to higher equilibrium wholesale prices:

**Proposition 18** Suppose that assumption 2 holds. Then the equilibrium wholesale price \(a^*\) in any symmetric equilibrium is decreasing in the quality of the traffic steering technology \((\Bar{\gamma})\).

**Proof.**

Using (35) with \(a_i = a_j\) and \(\frac{dn^*_i}{da_i}|_{a_i=a_j} = \sigma q(a_i) (1 - 2\Bar{\gamma})\) and reordering, yields the first order condition

\[
1 - \frac{2}{3} (2\Bar{\gamma} - 1) [1 + (2\Bar{\gamma} - 1) \sigma (a^* - c) q(a^*)] - \epsilon(a^*) (\gamma^2 + (1 - \gamma)^2) = 0
\]

As the the middle term is strictly negative for \(\Bar{\gamma} > 0.5\) and 0 for \(\Bar{\gamma} = 0.5\), it follows that \(\epsilon(a^*) (\gamma^2 + (1 - \gamma)^2) < 1\). Applying the implicit function theorem yields

\[
\frac{da^*}{d\gamma} = \frac{2 [1 + 2\sigma q(a^*) (a^* - c)] + 2\epsilon(a^*) (2\Bar{\gamma} - 1)}{-(2\Bar{\gamma} - 1)^2 \sigma q(a^*) (1 - (\gamma^2 + (1 - \gamma)^2) \epsilon(a^*)) - \frac{3}{2} (\gamma^2 + (1 - \gamma)^2) \epsilon'(a^*)}
\]

Clearly, the denominator of the right hand side is strictly negative since \(1 - (\gamma^2 + (1 - \gamma)^2) \epsilon(a^*) > 0\) and \(\epsilon'(a^*) \geq 0\) by assumption 2. The the numerator is strictly positive. Taken together \(\frac{da^*}{d\gamma} < 0\).

Intuitively, there are two channels that cause a higher equilibrium price when network selection is imperfect \((\Bar{\gamma} < 1)\). Firstly, compared to the base model \((\Bar{\gamma} = 1)\), the retail market share is less sensitive to increases of the wholesale price. This is because the perceived marginal costs \(c_i\) of operators within alliance \(i\) depend less on the own wholesale price \(a_i\) while the perceived marginal costs of operators of the rival alliance \(j\) depend partly on \(a_i\). Secondly, under imperfect traffic direction, operators of alliance \(j\) have to procure a proportion \(1 - \Bar{\gamma}\) of their subscribers’ roaming calls from alliance \(i\). When selling to non-alliance operators, the alliance does not take lower retail profits that are implied by a higher wholesale price into account, which renders a high wholesale price more attractive.

\(^{65}\)Both \(\eta_q(\cdot)\) and \(\eta_n(a^*)\) are defined as in section 5.
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