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Taxing Carbon under Market Incompleteness

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Taxing Carbon under Market Incompleteness

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Abstract

This paper is the first attempt, to the best of our knowledge, to study the impact of a carbon tax by means of a heterogeneous agents model. The objectives of the paper are two: i) To assess how the results of a representative agent model compare to those coming from a model accounting for heterogeneity across agents when evaluating aggregate economic and environmental impacts of a carbon tax; ii) To assess the distributional implications of a carbon tax (and equivalent cap) and how they can be mitigated through different recycling schemes or allocations.

Keywords: Carbon tax, double dividend, heterogeneous agents model

JEL codes: Q58, Q54, E2
1 Introduction

As the debates over the occurrence of climate change and its anthropogenic causes are finally settled (IPCC, 2007) the reality of a global convergence towards some form of regulation of carbon emissions seems unavoidable, although the timing and the form of this process are still open questions. The recent IMF (2012) report on *Fiscal Policy to Mitigate Climate Change* represents a sign of how the idea of using environmental taxation to contrast the current budget deficits is gaining momentum. It has been long argued that the imposition of environmental taxation does not necessarily imply welfare losses for the economy, even when environmental implications on welfare are not included in the analysis. Indeed, under specific circumstances, environmental taxation might also lead to a less distorted tax system, therefore partly or entirely compensating its costs: Sandmo (1975) suggested for the first time that revenues from Pigouvian taxation might be used to mitigate other distortionary taxes, thus reducing the cost of the environmental policy or even improving the non environmental welfare with respect to the no policy case.\(^1\) This is indeed a resurrection of the carbon tax discourse in the political debate, which seemed definitely closed with the adoption of the emission trading scheme by the European Commission and within the *Kyoto Protocol*.

There exists a series of concerns related to the introduction of as such environmental measures, ranging from the potential impact on businesses of countries adopting climate regulation in the face of international competition, the combined effect with economic shocks and cycles, and the potential distributional impacts on production costs and employment. Several studies have concentrated on the first concern, namely the impact on international competitiveness of firms located in countries that have adopted a unilateral climate policy, see for example sectoral impact analysis as in Alexeeva-Talebi et al. (2012) and in Aldy and Pizer (2011). A second, more recent strand of literature, has worked within the macroeconomics business cycle framework in order to assess the impact of pricing emissions in the presence of macroeconomic shocks. Specific attention has been devoted to the study of the optimal level of climate policy and to the performance of alternative economic instruments (carbon taxes versus quantity instruments with different allocation rules), by using Real Business Cycle models, both in a single sector (Fischer and Springborn, 2011, and Heutel, 2012) and in a multi-sector frameworks (Dissou and Karnizova, 2012). Heutel (2012) finds that optimal emissions are pro-cyclical and that the optimal emission policy should respond accordingly to economic fluctuations and cycles. Different policy tools are evaluated in Fischer and Springborn (2011), where the authors find that a cap system would achieve a given emission reduction with a slightly higher welfare cost than the tax, but it would ensure that the cut is achieved without lag, result-

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\(^1\)See Bovenberg and Goulder (1996) for a detailed discussion on the counteracting effects of tax interaction and revenue recycling.
ing in higher welfare if these additional reductions are valued; the cap system also features a lower level of labor variance than all other policies considered. These studies all make several simplifying assumptions. Because they are based only on a representative-agent framework, they are not suited to say anything about distributional implications of the policy. In addition they cannot explore the advantages of taxes or tradeable permits over command and control policies in the presence of heterogeneous abatement costs.

In the present paper we employ a standard incomplete markets model with heterogeneous agents - in the spirit of Aiyagari (1994) and Huggett (1993) - and we contrast the results obtained with a representative agent model with those derived through the inclusion of various idiosyncratic characteristics of agents. This allows us to address a set of crucial questions. First, whether the introduction of heterogeneity across agents - under different forms - is a relevant issue per se when analyzing the impact of a carbon tax on some aggregate indicator, as for example total emissions reductions, costs of a policy or ranking among alternative policy instruments. In Krusell and Smith (2007) it is indeed argued that in many of those instances when the interest is placed on aggregate variables, then a representative model can perform well enough. By means of our model we can test whether this is the case in the context of a policy putting a price on carbon, either through a tax or a cap mechanism.

A second crucial question is whether a climate policy has relevant distributional implications and what are the potential remedies through revenues recycling schemes to alleviate these impacts. Obviously, without a model where heterogeneity across agents is not spelled out this exercise would simply be impossible. By means of our model we are able to assess the level of regressivity of emission taxes and to test the double dividend hypothesis discussed in Bovenberg and Goulder (1996) and Bovenberg and van der Ploeg (1994).

Finally, we contrast our findings under a carbon tax with those derived by applying an emission trading scheme and we test the distributional impacts of alternative allocation schemes.

In the next section of the paper we lay out the model structure, while in section three we discuss the calibration methodology. The fourth section describes a preliminary set of results, while the last section concludes the paper by discussing future research developments.

2 The model

Time is discrete, indexed by \( t \in \{0, 1, \ldots, \infty\} \). There exists a continuum of \textit{ex-ante} identical and infinitely lived households, with total mass equal to one. Households own both factors of production, capital and labor. Firms, directly owned by the households, produce a single homogeneous final good competitively, via a constant-returns-to-scale production
function, using capital, labor and energy. The final good can be used for consumption and investment, and is assumed to be the numéraire. Asset markets are incomplete: households are allowed to invest in physical capital only, and we assume that capital holdings cannot be negative. Hence, households cannot fully insure themselves against idiosyncratic shocks to their income. The next Sections will describe the model components more in detail. The recursive equilibrium is formally defined in the Appendix.

2.1 Households

2.1.1 Capital income

As in Angeletos (2007) and Covas (2006), each household owns a single private firm. Firms employ labor and purchase energy in competitive markets but use the capital stock accumulated by the respective owner. Let us denote $e_{it}$ the amount of energy used: we assume that emissions at the firm level are proportional to the use of $e_{it}$ and units of emissions are chosen such that the quantity of emissions is equal to $e_{it}$. At the beginning of each period, households obtain from the government an endowment of emissions permits denoted $\bar{e}(k_{it})$ that possibly depends on the stock of installed capital, i.e. on the current size of the firm; emissions permits can be traded on a competitive secondary market.² There is no secondary market for physical capital, so households can invest capital only in the firm they own. The capital income of a generic household $i$, excluding the non-depreciated capital stock installed in the firm, is given by the firm’s earnings net of factor costs:

$$\pi_{it} = \kappa \phi_{it} \left( k_{it}^{\alpha} n_{it}^{1-\alpha} \right)^{1-\gamma_{it}} e_{it}^{\gamma_{it}} - (1 + \tau_N) w_t n_{it} - (1 + \tau_E) p_t e_{it} - z_t [e_{it} - \bar{e}(k_{it})],$$  

where $k_{it}$ denotes the stock of capital in place at the beginning of period $t$, $n_{it}$ the amount of labor hired, $w_t$ the wage rate, $p_t$ the price of energy, $\tau_E$ the carbon tax, when we assume a price mechanism is adopted, while $z_t$ is the price of emission permits and $\bar{e}(k_{it})$ is the amount of allocated permits when we consider a quantity mechanism is in place; $\tau_N$ is the payroll tax, $\kappa$ represents Total Factor Productivity, $\phi_{it}$ is an idiosyncratic shock to plant-level productivity, and $\gamma_{it}$ an idiosyncratic shock to the share of energy in gross output. Both idiosyncratic shocks are realized at the beginning of period $t$, after capital is installed but before labor $n_{it}$ and $e_{it}$ are chosen.

The log of productivity follows a stationary discrete Markov process, characterized by a transition matrix $\pi_{\phi}$, which evolves independently across households. We assume that $\mathbb{E}(\phi) = 1$. The share of energy in gross output follows an independent discrete Markov process too, characterized by a transition matrix $\pi_{\gamma}$; the shares of capital and

²If $\bar{e}$ were sufficiently generous, no household would ever need to buy additional permits on the secondary market, and this would drive their equilibrium price to zero.
labor in gross value added will be characterized by the parameter $\alpha \in (0, 1)$. Note that
the previous formulation can accommodate also the case in which $\gamma_i$ reduces to a time-
invariant household’s characteristic: in this case, $\pi_\gamma$ collapses to an identity matrix, and
the corresponding Markov process becomes an absorbing chain.

Since $n_{it}$ and $e_{it}$ affect only $\pi_{it}$ in period $t$, and since they are chosen after $k_{it}$ has been
installed, $\bar{e}_{it}$ has been obtained, and $\phi_{it}$ and $\gamma_{it}$ have been observed, the optimal $n_{it}$ and
$e_{it}$ maximize $\pi_{it}$ state by state. In other words, the firms solve the following maximization
problem in each period:

$$\max_{\{n_{it}, e_{it}\}} \ q_{it} - (1 + \tau_N) w_t n_{it} - [(1 + \tau_E) p_t + z_t] e_{it} + z_t \bar{e} (k_{it}),$$

(2)

where $q_{it} \equiv \kappa \phi_{it} (k^\alpha_{it} n_{it}^{1-\alpha})^{1-\gamma_{it}} \tilde{e}_{it}^{\gamma_{it}}$ denotes the individual firm’s output.

The individual factor demands and the firm’s earnings are linear in $k_{it}$, because of
constant returns to scale:

$$n_{it} = \Xi n_{it} k_{it},$$

(3)

$$e_{it} = \Xi e_{it} k_{it},$$

(4)

$$\pi_{it} = r_{it} k_{it} + z_t \bar{e} (k_{it}),$$

(5)

where:

$$\Xi_{it} \equiv \gamma_{it} (1 - \alpha) (1 - \gamma_{it}) \kappa \phi_{it} n_{it}^{\alpha(1-\gamma_{it})} e_{it}^{\gamma_{it}},$$

(6)

$$\kappa_{it} \equiv \Xi_{it} n_{it},$$

(7)

$$r_{it} \equiv \alpha (1 - \gamma_{it}) \kappa \phi_{it} n_{it}^{(1-\alpha)(1-\gamma_{it})} e_{it}^{\gamma_{it}},$$

(8)

and:

$$\Xi_{it} \equiv \gamma_{it} (1 - \alpha) (1 - \gamma_{it}) \kappa \phi_{it} n_{it}^{\alpha(1-\gamma_{it})} e_{it}^{\gamma_{it}}.$$  

(9)

2.1.2 Labor income

At the beginning of each period, households receive a fixed time endowment, normalized
to unity, whose productivity on the labor market is affected by an exogenous and
idiosyncratic shock, denoted $\varepsilon_{it}$; this shock is modeled as a finite-state Markov process,
characterized by a transition matrix $\pi_{\varepsilon}$, which evolves independently across households.

We furthermore assume that $E(\varepsilon) = 1$. After the realization of labor productivity, the
household optimally allocates its time endowment between labor and leisure; for the sake
of simplicity, we assume that the labor productivity shocks does not directly affect the
utility function.
2.1.3 The optimization problem

Households’ preferences over stochastic consumption and leisure streams are given by:

\[ u_{it} = \mathbb{E}_t \left[ \sum_{j=t}^{\infty} \beta^j \left( \frac{c_{it}^{1-\mu} - 1}{1 - \mu} - \xi l_{it}^{1+\eta} \frac{1+\eta}{1+\eta} \right) \right], \tag{10} \]

where \( c_{it} \) is the consumption level, \( l_{it} \in [0, 1] \) the share of time devoted to labor, \( \beta \in (0, 1) \) the intertemporal discount factor, \( \mu > 0 \) the reciprocal of the elasticity of intertemporal substitution, and \( \eta > 0 \) a parameter equal to the inverse of the Frisch elasticity of labor supply.

The stock of physical capital evolves over time according to the following accumulation equation:

\[ k_{it+1} = (1 - \delta) k_{it} + y_{it} - T(y_{it}) + G_t - c_{it}, \tag{11} \]

where:

\[ y_{it} = \pi_{it} + w_{it} \varepsilon_{it}, \]

while \( T(\cdot) \) denotes the (possibly non-linear) tax function, \( G_t \geq 0 \) the per-capita government lump-sum transfers, and \( \delta \in [0, 1] \) a physical depreciation rate. As already mentioned, households also face a borrowing constraint: \( k_{it+1} \geq 0 \).

We can now put all the elements together; for given sequences of factor prices and transfers, the dynamic optimization problem of a generic household is as follows:

\[
\max_{\{c_{it}, l_{it}, k_{i,j+1}\}_{j=t}^{\infty}} \mathbb{E}_t \left\{ \sum_{j=t}^{\infty} \beta^j \left[ \frac{c_{it}^{1-\mu} - 1}{1 - \mu} - \xi l_{it}^{1+\eta} \frac{1+\eta}{1+\eta} \right] \right\}, \tag{12}
\]

s.t. \( k_{it+1} = (1 - \delta) k_{it} + y_{it} - T(y_{it}) + G_t - c_{it} \),
\( l_{it} \in [0, 1] \),
\( k_{it+1} \geq 0 \).

The first order conditions can be combined to obtain the following inequalities:\(^3\)

\[ \xi l_{it}^\mu \leq c_{it}^{1-\mu} (1 - T_{y_{it}}) w_{it} \varepsilon_{it}, \tag{14} \]
\[ c_{it}^{1-\mu} \geq \beta \mathbb{E}_t \left\{ c_{i,t+1}^{-\mu} \left[ 1 - \delta + \left( 1 - T_{y_{i,t+1}} \right) \left( r_{i,t+1} + z_{t+1} \bar{e}_{k_{i,t+1}} \right) \right] \right\}. \tag{15} \]

\(^3\)In equation (14) we are anticipating an equilibrium outcome: given our utility function, it turns out that in equilibrium \( l_{it} \) remains always strictly positive, but not necessarily strictly below one. Hence, the marginal benefit in terms of utility of an additional hour of work can be greater (not lower) than the additional cost.
2.2 Aggregate variables

Energy is imported from abroad, at a given international price $p_t = \bar{p}$, and its supply is perfectly elastic. In other words, our economy can be characterized as a small open economy in the international market for energy; however, it should be remembered that households do not have access to international financial markets, and can only invest in physical capital. This implies that trade is balance by assumption: energy imports are financed via final good exports. Note furthermore that the carbon tax acts effectively as a sales tax on energy imports.

At the beginning of each period, the government issues a total amount of emissions permits equal to $\bar{M}_t$; an amount equal to $\bar{E}_t \leq \bar{M}_t$ is immediately distributed to the households for free, while the remaining permits are supplied to the secondary market by the government itself. Apart from this, the government plays a minimalist role, collecting tax revenues, selling permits, and paying everything back to the households via lump-sum sum transfers (capital letters denote aggregate variables):

$$G_t = T_t + \tau_N w_t N_t + \tau_E \bar{p} E_t + z_t \left( \bar{M}_t - \bar{E}_t \right).$$

(16)

3 Calibration

The parameters that characterize households’ preferences are selected in the following way: the intertemporal discount factor and the reciprocal of the elasticity of intertemporal substitution are set to standard values in the literature, $\beta = 0.96$ and $\mu = 2$. Reichling and Whalen (2012) report that the Congressional Budget Office incorporates into its analyses an estimate of the Frisch elasticity of labor supply that ranges from 0.27 to 0.53: we set $\eta = 1.9$ in order to make the model reproduce a Frisch elasticity equal to 0.53, and calibrate $\xi$ so that the average hours worked are 40% of the time endowment. The depreciation rate is set to $\delta = 0.048$, while the share of capital in value added, $\alpha$, is assumed to be 0.33. Finally, we normalize $\kappa$ to one.

In our benchmark parametrization, we assume that the share of energy in gross output, $\gamma_i$, is a time-invariant household characteristic: more precisely, we divide the overall population into four technology types, characterized by their factor intensity and mass. Using sectoral data for the U.S. over the 1970-2005 period taken from the EU-KLEMS data-set, described in O’Mahony and Timmer (2009), we construct the long-run shares of energy in gross output for four aggregate sectors, namely Agriculture, Services, High Energy-intensive Industrial Sectors, and Low Energy-intensive industrial Sectors: the corresponding shares are 6%, 2%, 22%, and 4%, respectively. The mass of each type is calibrated in order to replicate in steady state the observed sectoral shares in total gross output, equal to 2%, 60%, 25%, and 13%, respectively. The price $\bar{p}$ is calibrated in order to reproduce the overall share of energy in gross output, equal to 7.3%.
The payroll tax, $\tau_N$, amounts to 15% of wages, which is broadly in line with the average Social Security Payroll tax rate in the U.S. over the 2000-2011 period, as reported by the OECD. Following Conesa and Krueger (2006), we use a flexible functional form for the income tax function $T$ that is theoretically motivated by the equal sacrifice principle, as discussed in Gouveia and Strauss (1994), and encompasses a wide range of progressive, proportional and regressive tax schedules:

$$T(y) = a_0 \left[ y - \left( y^{-a_1} + a_2 \right)^{-\frac{1}{a_1}} \right],$$

(17)

where $a_0 \geq 0$, $a_1 \geq 0$, and $a_2 \geq 0$. Gouveia and Strauss (1994) estimate this tax function for the U.S., obtaining values of $a_0 = 0.258$ and $a_1 = 0.768$. The parameter $a_2$ is calibrated so that total tax revenues, as described in (16), amount to 27% of GDP, a share in line with U.S. recent data.

For the sake of simplicity, the allocation rule for emission permits is assumed to take the general form:

$$\bar{e}(k) = e_0 + e_1 k,$$

(18)

with $e_0 \geq 0$ and $e_1 \geq 0$, to be chosen on the basis of the allocation formula (see discussion in the following section).

The log of the individual labor productivity is assumed to follow an auto-regressive process of the form:

$$\log \varepsilon_{t+1} = \bar{\varepsilon} + \rho_{\varepsilon} \log \varepsilon_t + \epsilon_{\varepsilon,t+1},$$

$$\epsilon_{\varepsilon,t} \sim N \left( 0, \sigma_{\varepsilon}^2 \right).$$

(19)

Borrowing the estimates provided in Karahan and Ozkan (2013), Table 3, we set $\rho_{\varepsilon} = 0.98$ and $\sigma_{\varepsilon} = 0.11$; the aggregate labor endowment in steady state is normalized to one, and this implies, as already mentioned, that $E(\varepsilon) = 1$: we set the parameter $\bar{\varepsilon}$ accordingly. This process is approximated with a 4-state discrete Markov chain computed

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4Note that if $a_1 \to 0$, then $T(y) \to a_0 y$, i.e. the tax schedule collapses to a pure proportional system.

5These estimates are for tax year 1989, the last year reported in Gouveia and Strauss (1994). We are currently not aware of any more recent estimates.

6Two somehow conflicting views on the nature of idiosyncratic income processes have emerged in the literature: as discussed in Guvenen (2009), one view holds that individuals are subject to large and very persistent shocks, while facing similar life-cycle income profiles. The alternative view holds that individuals are subject to income shocks with low persistence, while facing individual-specific income profiles. See also Carroll (1997) for a detailed discussion. Given that currently the jury still seems to be out, our choice of a very persistent labor income process is mainly driven by comparability with the existing literature and numerical convenience.

7The results of Karahan and Ozkan (2013) reported above have been obtained using data on annual earnings: hence, to correctly match this empirical evidence to our model we should take the endogeneity of labor supply into account, significantly complicating the calibration process. However, Karahan and Ozkan (2013) also report that using data on average hourly wages does not significantly change their results: we consider these findings reassuring, and introduce this shortcut for the sake of simplicity.
using Rouwenhorst’s method, as suggested in Kopecky and Suen (2010).

The log of plant-level productivity follows a similar process:

\[
\log \phi_{t+1} = \bar{\phi} + \rho_\phi \log \phi_t + \epsilon_{\phi,t+1},
\]

\[
\epsilon_{\phi,t} \sim N(0, \sigma^2_\phi).
\]

Abraham and White (2006), using a database that covers the entire U.S. manufacturing sector from 1976 until 1999, estimate plant-level TFP for a large number of plants using a specification similar to (20); borrowing their estimates, we set \( \rho_\phi = 0.40 \) and \( \sigma_\phi = 0.45 \). As already mentioned, we impose that \( \mathbb{E}(\phi) = 1 \) and set the parameter \( \bar{\epsilon} \) accordingly. As before, this process is approximated with a 4-state discrete Markov chain computed using Rouwenhorst’s method.

The parameter constellation is summarized in Table 1.

As far as the solution method is concerned, our approach is fairly standard. At the household level, we have to solve a stochastic dynamic optimization problem with occasionally binding constraints: this is done using fixed point iteration on the Euler equation, as discussed in Rendhal (2007). At the aggregate level, we compute the ergodic distribution using the binning approach described in Young (2010).\(^8\)

### 4 Results

In order to assess the implications on aggregate results of different types of modelization of agents heterogeneity, we perform a set of experiments by considering three different models. The first is our benchmark setup, labeled from now on \textit{Heterogeneous Agents Multi Technologies}. The second is a single-sector, or single-technology, version of the benchmark setup, obtained by simply assuming a single \( \gamma \), equal to 0.073, and labeled

\[^8\]To solve for the policy functions, we discretize the state space using 1000 uniformly-spaced nodes over the \([0, 40]\) interval, and employing linear interpolation to evaluate the functions at points that are not on the grid. The same grid is used to compute the stationary distribution. A further increase of the number of nodes does not substantially change the results.
Table 2: Summary of steady-state properties.

**Heterogeneous Agents Single Technology.** The third is a Representative Agent model, where no aggregate nor idiosyncratic shocks are considered: this model replicates perfectly the single-technology model described above, sharing the same parametrization but for the volatility of idiosyncratic shocks.

Table 2 summarizes the main steady-state properties of the three models under our benchmark parametrization. In particular, we report the GDP level, the capital stock, hours worked, energy consumption (the aggregate one and the sectoral ones when applicable), the government balance, and the Gini index for wealth. The baseline output of the representative agent model is not entirely replicating that of the other two models we are introducing. This derives from the fact that we are assuming the same parametrization but a different modelization. In particular, the role of precautionary savings in the stochastic heterogeneous agents models explains the differences among capital stocks and GDP levels across models. As we care about deviations from the benchmark, or no carbon policy scenario, across the three model versions, it is more important for us to maintain the same parametrization while changing the model structure. This allows us to impute differences due to changes to the sole model structure. Figure 1 represents the density of households over asset holdings for the benchmark Heterogeneous Agents Multi Technologies model.

### 4.1 Description of experiments

With each version of the model we perform a set of experiments in order to analyze implications of different policy measures; we repeat each of these experiments for different carbon tax rates, ranging from 10% to 50% of the energy input price, and computing the corresponding stationary equilibrium. Table 3 summarizes the different experiments and labels them appropriately. The first experiment simply implies the imposition in the economy of a carbon tax. The increased revenues are then transferred to households through a lump-sum transfer. Experiments from two to four, instead, assume that the government budget is kept equal to the case without any carbon tax and revenues from the carbon tax are instead recycled in different ways. In particular, recycling scheme 1

9 Defined as $GDP = Y + \tau_N wN + \tau_E \bar{p} \bar{E} + z (\bar{M} - \bar{E})$, i.e. factor income plus indirect taxes plus revenues from permits auction.
(RS1) implies a reduction in payroll taxes, while scheme 2 (RS2) implies a proportional reduction in income taxes, i.e. a reduction in the parameter $a_0$. Finally, scheme 3 (RS3) implies an increase in the degree of progressivity of the tax schedule, i.e. an increase in the parameter $a_1$: since the degree of progressivity has a negative effect on the incentive to accumulate capital, the increase in $a_1$ causes a drop in GDP and income-tax revenues that counterbalances the increase in carbon-tax revenues.

The last two experiments described in Table 3 assume the imposition of a different market based instruments aiming at curbing emissions, namely a cap and trade system. In particular, we consider a cap on emissions that is consistent with the emission level achieved under the carbon tax scenarios, so that the two systems are completely equal in environmental terms, and a fully functioning emission permits market, without allowing for banking of permits. In the first experiment emissions are grandfathered to firms on the basis of an equal per capita principle ($e_0 = \bar{M}$ and $e_1 = 0$ in equation 18). The second experiment assumes that a cap is imposed that is proportional to capital ($e_0 = 0$ and $e_1 > 0$ in equation 18). Since (future) output is a control variable of the firm and the allocation of permits creates a subsidy to output, this creates an incentive to reduce emissions through conservation. In order to keep the overall emissions in line with the corresponding level of abatement under the other scenarios, we solve for the value of $e_1$ - the parameter controlling for the number of permits the government issues in proportion to capital - that generates an amount of emissions equal to the desired level.

\footnote{For the sake of analytical simplicity, we take the capital stock (a state variable) as a proxy for output.}
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TAX</strong></td>
<td>Carbon Tax (10%-50%) &lt;br&gt;Lump-sum transfer of revenues to household</td>
</tr>
<tr>
<td><strong>TAX_RC1</strong></td>
<td>Carbon Tax (10%-50%) &lt;br&gt;Government Budget kept constant &lt;br&gt;Recycling Scheme 1 (lower payroll tax rate)</td>
</tr>
<tr>
<td><strong>TAX_RC2</strong></td>
<td>Carbon Tax (10%-50%) &lt;br&gt;Government Budget kept constant &lt;br&gt;Recycling Scheme 2 (lower average tax rate on income)</td>
</tr>
<tr>
<td><strong>TAX_RC3</strong></td>
<td>Carbon Tax (10%-50%) &lt;br&gt;Government Budget kept constant &lt;br&gt;Recycling Scheme 3 (higher progressivity of income tax)</td>
</tr>
<tr>
<td><strong>CAP_EPC</strong></td>
<td>Cap on emission consistent with emission reductions as in Tax &lt;br&gt;Permits grandfathered on the basis of “equal per capita” principle</td>
</tr>
<tr>
<td><strong>CAP_OUT</strong></td>
<td>Cap on emission consistent with emission reductions as in Tax &lt;br&gt;Permits allocated on the basis of output-based with updating</td>
</tr>
</tbody>
</table>

Table 3: Description of experiments.

### 4.2 Carbon Taxes

Let us start with the first research question we set out to address, namely comparing the aggregate environmental and economic effects of imposing a carbon tax as evaluated with different models. The use of a representative agent model approximates well the result of the heterogeneous agents model, in terms of both environmental and economic implications (see GDP, government budget, and energy/emissions changes shown in Figure 2). This is true but for the case where we introduce different types of technologies in addition to heterogeneous agents. In the latter case, the economy reacts more swiftly by dramatically reducing production based on the more energy intensive technologies. Households endowed with more energy intensive technologies reduce the scale of their operations, leave their capital to depreciate, and increase their labor supply in favor of households characterized by energy efficient technologies. This has an overall effect on emissions reduction which is mirrored in the almost null effect on the government budget. The multi-technology model allows agents to abandon production rather than paying the tax. This has an obvious overall mitigating effect on the negative economic impact of the tax on GDP, which is almost halved for a carbon tax of 50% with respect to the cases where a single technology is considered.

Now, what model is giving us the correct answer? The multi-technology model more realistically depicts some features of reality, as we would clearly expect a change in production modes and technology types as a reaction to the introduction of a carbon tax. What our multi-technology heterogeneous agents model is not realistically representing, though, are the costs of this technological transition, that would inevitably be larger than
zero due to lock-in effects and other frictions.\textsuperscript{11}

### 4.3 Recycling schemes

The next exercise entails changing recycling scheme (TAX\_RS 1-3) and studying the effect on the distribution of wealth and on total policy costs (measured as changes in per capita GDP).

Figure 3 reports the distributional implications, measured by the change in the Gini index for wealth, under the different schemes and computed using the two versions of the heterogeneous agents model. The first striking feature is that the model choice matters way more than the recycling scheme adopted. With the multi-technology model a large set of households abandon production and supply their labor force to those household/firms that are less reliant on carbon. This comes at a clear cost in terms of wealth distribution that can be hardly moderated by recycling the revenues. On the other hand, if we consider the single technology model, under RS3, which implies carbon revenues are used to increase the progressivity of the income tax, the Gini index actually decreases, thus implying a more equal distribution of wealth. The same recycling scheme works best according to the multi-technology model too, although, as we have seen, imposing a carbon tax always has a deteriorating effect on equality.

Figures 4 and 5 report the impacts on GDP per capita and social welfare of the same experiments,\textsuperscript{12} we can now include in our discussion results for the representative agent

\textsuperscript{11}See Acemoglu et al. (2012) for a discussion.

\textsuperscript{12}We assume a Benthamite welfare function, treating all households the same: the welfare level of households in the stationary equilibrium is computed using the value function, and the ergodic distribution
Figure 3: Implication for redistribution of an increasing Carbon Tax under Different Recycling Schemes for the two Heterogeneous Agent models.

Figure 4: Policy Costs of an increasing Carbon Tax under Different Recycling Schemes for the three types of models.
Figure 5: Welfare impacts of an increasing Carbon Tax under Different Recycling Schemes for the three types of models.

model as well. A series of results is robust across different model choices. The recycling scheme that minimizes policy costs, and similarly minimizes welfare losses, robustly across model specifications, is RS2, namely the scheme implying a proportional reduction in income taxes. As just discussed, this very same scheme is the one that performs worst in terms of wealth redistribution consequences (implies larger increases in the Gini coefficient). As we have seen, it is RS3 that implies maximum redistribution, independently of the type of model used. RS3 ranks second in terms of welfare and GDP losses.

As far as the performance of the representative model is concerned, under alternative recycling schemes the heterogeneous agent models, even the one with single technology, start to clearly diverge from the representative one. In particular, under RS2 the heterogeneous agent single technology model finds, for a set of carbon tax values an overall increase in welfare, while this is never the case if we look at the representative agent model. This has important implications when we want to consider the macro economic impacts of a carbon tax under different recycling schemes: the representative model might fail to portray all implications and provide a distorted picture for policy makers to take their decisions. Depending on the policy maker’s priorities this might have important consequences.

is then used to obtain the aggregate welfare level.
4.4 Environmentally Equivalent Caps

We now move to the last series of our experiments, those looking at the impact of different market based instruments aimed at reducing carbon. A long literature in environmental economics, beginning with the seminal paper by Weitzman (1974), has compared price and quantity instruments for regulating emissions. In Fischer and Springborn (2011), the authors set out to study the effectiveness of a tax versus two different allocation schemes in a cap system under macro-economics shocks and find that a cap system would achieve a given emission reduction with a slightly higher welfare cost than the tax, but it would ensure that the cut is achieved without lag, resulting in higher welfare if these additional reductions are valued; the cap system also features a lower level of labor variance than all other policies considered.

In Figure 6 we report policy costs under an increasing carbon tax, TAX, and compare them under two cap scenarios that are environmentally equivalent, CAP_EPC and CAP_OUT. While in the case of CAP_EPC the policy costs are roughly the same as in the case of the TAX scenario, also mirroring differences across model specifications, the EPC_OUT results clearly stand out as GDP per capita impact of the policy are mitigated by and large. This is robust to the different model formulations. The budgetary implications are summarized in Figure 7. Note that, since we are not considering any auctioning of the permits, the government budget always deteriorates under a cap. However, this deterioration is generally minimal under CAP_OUT, given the lower negative impact on GDP of the environmental instrument under this scenario. Furthermore, the comparison
across models shows that the budgetary implications of environmental instruments are quantitatively less relevant in the multi-technology setting, in particular for the TAX and CAP_EPC scenarios: the reallocation of resources across technologies that characterizes this version of our model is able to at least partially counteract the increase in carbon taxes or the tightening of caps as far as the government budget is concerned.

When we look at welfare, reported in Figure 8, a different story emerges, and the choice of model largely impacts on the ranking of different policy options. While, according to the representative agent model the choice of an output based allocation of permits should be preferred, in line with the GDP per capita ranking, this is no longer the case if we look at welfare impacts using heterogeneous agents models. Indeed, the CAP_OUT scenario is clearly performing worse than the other two due to the negative welfare implications of decreasing leisure: as shown in Figure 9, hours worked increase with the policy instrument in the multi-technology setting and decrease in the others under the TAX and CAP_EPC scenarios, while hours worked increase less in the multi-technology setting than in the others under the CAP_OUT scenario. Hence, depending on what the policy objective is, e.g. minimizing GDP costs versus overall welfare implications of a carbon tax, the choice of a cap system might be either dominating or dominated by the other two choices (TAX and CAP_EPC) as well as all three tax systems with recycling schemes. Indeed, it is worthwhile noticing that none of the CAP systems is welfare improving under any level of carbon prices, as we have instead seen for low values of the carbon tax in the case of a carbon tax under RS2. This might change if we were to consider the welfare improving effect deriving from the environmental benefits of such system, although this would be
true for all other policies analyzed here.

4.5 Sensitivity analysis

In order to further explore our finding we have performed an extensive sensitivity analysis on a set of key parameters. In particular, we analyze the effects of: (i) a one percentage point increase in Total Factor Productivity, represented by $\kappa$, (ii) a one percentage point increase in $\bar{p}$, the international price of energy, (iii) a ten percentage point increase in $\sigma_\varepsilon$, the standard deviation of labor income shocks, (iv) a ten percentage point increase in $\sigma_\phi$, the standard deviation of idiosyncratic productivity shocks. The results are summarized in Figure 10.

We find that the sensitivity to key parameters is stable across model specifications and in particular the standard deviation of capital income shocks is the most critical parameter. This is also true for different types of policies. An important exception is the case of a cap system where permits are distributed on an equal per capita basis.

5 Caveats and future research

In the present paper we employ a standard incomplete markets model with heterogeneous agents and we contrast the results obtained with a representative agent model with those derived through the inclusion of idiosyncratic characteristics of agents. We are able to show that, as far as aggregate variables are concerned, the representative model and the
Figure 9: Employment impacts of an increasing climate policy measure: a comparison of Environmentally Equivalent Cap and Tax.

Figure 10: Welfare impacts of an increasing climate policy measure: a comparison of Environmentally Equivalent Cap and Tax.
heterogeneous model with a single technology produce almost identical results, as long as we do not consider tax recycling schemes. Once we do start to consider alternative recycling schemes for the carbon tax revenues, which is obviously an important step forward towards a greater realism of policy modelization, the representative model might deviate largely from projections given by the other two models. When we introduce alternative technologies in the heterogeneous agents model, then aggregate results are very dissimilar even in the most simple set-up, the one with the lump-sum transfer of the carbon tax revenues. In addition the representative model makes it impossible to appreciate redistribution effects of introducing a carbon policy, which is obviously an important political component of designing such policy. Indeed, when considering alternative recycling schemes for the revenues from the carbon tax, we can detect both aggregate as well as important distributional impacts that we would not detect with a representative agent model. This is important because the issue of double dividend and the discussion concerning the potential beneficial effect of using a carbon tax to mitigate distortionary effects of existing income or labor taxes can only be fully tested with a framework as the one presented in the present paper. As in our analysis we find an important and critical trade-off between distribution effect and policy costs of a carbon policy, our next goal is that of devising a recycling scheme rule that maximizes welfare, given a non-deteriorating status of welfare distribution.

There are some important caveats to the current version of the analysis that we are planning to address in future research. First, the modeling of multi- versus single-technology is currently very crude. We aim at integrating realistic frictions (by means of a CES structure governing the substitutability of different technologies). This will allow to get a solution that will lay in between the current two versions (single versus multi-technology) of the model. In addition, we will go beyond the simple comparison of steady states, and study the transition paths from one steady state to another that follow a policy shock. Finally, we should acknowledge that the assumption of equivalence between energy and emissions is rather crude. indeed, there are many possible fuel substitution options that would allow to produce using the same amount of energy but drastically reducing emissions. We are planning to better represent the link between energy and emissions, allowing for technical change in the energy carrier.

References


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A Appendix: the recursive equilibrium

The idiosyncratic stochastic processes are independent across households, and can be jointly represented by a finite-state Markov process, denoted $\sigma \in \Sigma$, where $\Sigma \equiv E \times \Phi \times \Gamma$, and characterized by a transition matrix $\pi = \pi_\varepsilon \otimes \pi_\phi \otimes \pi_\gamma$ such that $\pi (j,i) \geq 0$ stands for the probability that $\sigma_{t+1} = \sigma_j$ if $\sigma_t = \sigma_i$, where, for the sake of notational convenience, $\sigma \equiv \{\varepsilon, \phi, \gamma\}$.

The vector of individual state variables $x_t \equiv \{k_t, \sigma_t\}$ lies in $X = [0, \infty) \times \Sigma$. The distribution of individual states across agents is described by an aggregate state, the probability measure $\lambda_t$. More precisely, $\lambda_t$ is the unconditional probability distribution of the state vector $x_t$, defined over the Borel subset of $X$:

$$\lambda_t (k, q) = \lambda_t (x) = \Pr (k_t = k, \sigma_t = q). \quad (21)$$

For the Law of Large Numbers, $\lambda_t (x)$ can be interpreted as the mass of agents whose individual state vector is equal to $x$. Being $\lambda_t$ a probability measure, the total mass of agents is equal to one.

In a recursive equilibrium, the time-invariant individual policy functions will depend on the exogenous state, $\sigma$, on the beginning of period capital stock, $k$, and on the aggregate distribution $\lambda$. The aggregate wage rate $w$ will depend on the distribution of individual wealth stocks. Hence, the exogenous Markov process for $\sigma$ and the optimal policy function $c (x; \lambda)$ induce a law of motion for the distribution $\lambda_t$:

$$\lambda_{t+1} (x) = \int_X \mathcal{I} (k, k, \sigma) \pi (q, \sigma) d\lambda_t, \quad (22)$$

where:

$$\mathcal{I} (k, k, \sigma) = \begin{cases} 1 & \text{if } k' (x; \lambda_t) = k \\ 0 & \text{otherwise} \end{cases}. \quad (23)$$

Given the absence of aggregate uncertainty, in the long run the economy will reach a stationary equilibrium, i.e. steady state characterized by constant aggregate variables.

Definition 1. A stationary recursive equilibrium is a policy function $c (x; \lambda)$, a wage rate $w$, a price of emissions permits $z$, and a probability distribution $\lambda$ such that:

1. The policy function solves the individual optimization problem (12).

2. The labor market clears:

$$\int_X nk d\lambda = N = \int_X \varepsilon d\lambda.$$
3. The permits market clears:\(^\text{13}\)

\[
\int_X e k d\lambda = E \leq \tilde{M}.
\]

4. The market for the final good clears:

\[
C + K' - (1 - \delta_K) K = GDP = Q - \bar{p}E = Y + \tau_N w N + \tau_E \bar{p} \bar{E} + z \left( \bar{M} - \bar{E} \right).
\]

5. The distribution satisfies the induced law of motion:

\[
\lambda (x) = \int_X I (k, k, \sigma) \pi (q, \sigma) d\lambda, \quad \forall x \in X.
\]

\(^{13}\)Note that \(z > 0\) if this equilibrium condition holds with equality.