Ambiguous Beliefs on Damages and Civil Liability Theories

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Summary

This paper analyzes the meaning of comparing the economic performance of strict liability and negligence rule in a unilateral standard accident model under Knightian uncertainty. It focuses on the cost expectation of major harm on which the injurers form beliefs. It shows first that, when the Court agree with the regulator, whatever the liability regime, the first best level of care is never reached but under both regimes the tortfeasors define the same level of care. Second, when, judge and regulator disagree, it is impossible to discriminate among liability standards because the issue depends on the injurer’s optimism degree.

Keywords: strict liability, negligence rule, ambiguity theory, uncertainty, accident model.

JEL: K0, K32,Q01, Q58

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INTRODUCTION

Originally, in lawmakers’ opinion, civil liability obliges any wrongdoer to compensate another of an illegitimate injury to his/her person or property. However, economists after Coase (1960) in the “The Problem of Social Cost”, Calabresi (1970) and others pointed out that tort law impacts on economic life. For instance, too costly repairs or some ill-adapted liability regime could bring on heavy social costs and threat the Community’s social welfare. Landes and Posner (1987 p.1) summarized clearly the point: "This book explores the hypothesis that the common law of torts is best explained if the judges who created the law through decisions operating as precedents in later cases where trying to promote efficient resource allocation."

According Kaplow and Shavell (1999), Bentham in the XIXth Century conceived the modern analysis of law and Coase in 1960 extended this analysis to probabilistic externalities. Beyond Coase's (1960) article, the sixties and seventies expanded this field with Becker's (1968) article on crime and law enforcement. Calabresi (1970) provided the first systematic work on accident law while Posner (1972) studied economic analysis of law.

Nowadays, tort law occupies a huge place in the economic policy toolbox because besides victims’ losses compensation. Liability is a strong inducement to force potential tortfeasors to take due cares (Brown (1973)). Consequently, the government must persuade the potential injurers to take well-suited prevention measures by enforcing the most effective liability regime. Calabresi (1970), Brown (1973) and Shavell (1980) brought into the economic field the old debate that drives apart law scholars about the respective advantages and disadvantages of strict liability regimes versus negligence rule.

Under negligence, a court can obligate a tortfeasor to repair or compensate the consequences of harm for insufficient care. On the contrary, under strict liability, it is only needed to establish a causal link between the activity and the harm to make the injurer liable
regardless of the level of due care he brought. In practice, strict liability applies to
Environment protection, ultra-hazardous activities and product defaults (Cantu (2001)) while
negligence concerns all the remaining fields. However, despite this demarcation of roles,
determining the most appropriate liability scheme generates wholehearted debates among
economists. Following their view, the regulator should enforce the regime that provides
simultaneously the lowest social cost of accident and the highest care level. In the eighties, the
authors showed that, under conditions, both regimes are equivalent in the minimizing of the
primary accident costs (safety costs plus expected accident losses). This scheme corresponds
to the accident model designed by Brown (1973) and Shavell (1980), (1987). Beyond the
direct costs of the harm itself, the accident costs should comprise its administration, insurance
and preventive costs (Calabresi (1970)).

Proving the equivalence between liability regimes needs some strong suppositions (as
certainty about the level of maximum damage, no consideration for the activity level, etc.).
Weakening them opens the Pandora’s Box of ambivalent results (Cooter (1984)). The actual
debates bear upon the question of knowing whether introducing uncertainty favors either strict
liability or negligence (see Newman and Wright (1990) or Demougin and Fluet (1999) for
contradictory opinion about the influence of liability).

In a seminal paper, Teitelbaum (2007) generalizes the basic unilateral accident model
to Knightian uncertainty. Unilateral accident means that the potential victims cannot act on the
probability of a harm occurrence. This kind of uncertainty (or “real” uncertainty) involves that a
unique probability distribution can no longer represent the agents’ beliefs. This view comes
from Ellsberg (1961) who put into evidence the paradoxical situations to which can lead the
Savage’s axioms (Teitelbaum (2007) for a review of this literature). Teitelbaum assumes that the
injuror is a Choquet expected utility maximizer. Facing ambiguous choices, the injurer forms
beliefs about accident risk and uses a specific kind of capacity called a neoaddititive capacity
that Chateauneuf, Eichberger, and Grant (2007) conceived. Teitelbaum shows that neither strict liability nor negligence is generally efficient when ambiguity is at stake. As a second result, he suggests that negligence is more robust than strict liability.

The present paper puts into question the well-accepted opinion that one can compare liability regimes on the ground of their economic efficiency. To show this, I describe a unilateral accident model that borrows Chateauneuf, Eichberger, and Grant (2007)’s methodology about neoadditive capacity. Here, Knightian uncertainty bears on the range of major damages that form random variables rather than on the probability distribution of accident as Teitelbaum (2007) did it. The issues are that, first, when regulator and judge agree on the first best level of care (welfare maximizing), then, strict liability and negligence are perfect substitutes. This issue complies with the standard model but it diverges from it because the injurer’s equilibrium care level is socially inefficient. Second, releasing the compliance assumption between Court and regulator does not lead to a first-best solution and neither strict liability nor negligence dominates the other one.

This paper organizes as follows. A first part analyzes the question of the assessment of the major damage costs by the injurer. It describes the injurer’s beliefs about the likelihood of uncertain events and the foundations of ambiguity analysis. By assumption, the potential tortfeasor acts to minimize the expected value of a cost function corresponding to the Choquet integral of neocapacities. A second part assumes a perfect compliance between the regulator and the judge concerning the first best level of care. A third part extends this model to the Teitelbaum (2007)’s one. This leads to define a broader uncertainty field that incorporates ambiguity on probability of accident and on the distribution of major damages. A fourth part allows errors from the judges’ side concerning the assessment of due care and issues on an indeterminacy rule concerning efficiency among liability regimes. A fifth part concludes.
I. AMBIGUITY APPLIED TO EXPECTED MAJOR DAMAGE

In most representations of the unilateral accident model, the value of damage is given by the term “given” we consider also the deterministic expression of a decrease in damage according a proportional increase in care (Dari-Mattiacci and Parisi, (2003)). This value could correspond to the average value of several potential events associated with different probable state of nature. However, under Knightian uncertainty, assuming a given level of damage can no longer hold because the agents form beliefs about the likelihood of events, this independently of the level of care. For example, the explosion of a fuel tank could produce a harm equivalent to either $x$ or $y$ thousand Euros, the polluting leakage of groundwater could cost either 500,000 Euros or three millions, and so on. This indeterminacy depends on several concrete factors as, for instance, the moment of the day, the weather, some specific circumstances, etc. Obviously, real conditions influence the extent of damages and increase their randomness. A regulatory agency can officially assess the probabilities about the range of potential maximum damage. The operator knows this information but he can form different estimates that issue on another probability distribution compared the “official” one.

The discrepancy between the “official” probability distribution and the injurer’s beliefs about it constitutes the root of the use of ambiguity theory here. The literature on this theme started from Allais’ criticisms of Savage’s expected utility theory (SEU) in the early fifties and with the Ellsberg’s Paradox formulation in 1961. As a simplified example, let us consider an agent that must select two alternative issues. In the first one, he can choose one action with a known probability (for instance, drawing a blue ball in an urn that contains blue and red balls in a known proportion). In the second alternative, the agent faces a similar choice, but with an unknown probability distribution. These experiments showed that most of people prefer to select the first choice (the urn in which the proportion of red and blue balls is known) rather than the other one. Consequently, people feel aversion for ambiguous choices.
and, they implicitly allocate lower prior probabilities of winning to the second choice. As a result, the sum of probabilities is higher than 1. Schmeidler (1989) systematized ambiguity by applying Choquet’s integral to expected utility theory. Then, ambiguity is the lack of confidence of an agent about the probability distribution of uncertain events. For ambiguity theory, non-additive probability (or “capacity”) represents the agent’s beliefs about the likelihood of these uncertain events. Agents maximize an expected utility function based on capacities. A Choquet’s integral computes this utility. This allows taking into account ambiguity and the formation of beliefs facing uncertainty. More precisely, concave capacity involves optimism (super-additivity) while a convex one entails pessimism (subadditivity) (see Teitelbaum (2007)).

Chateauneuf, Eichenberger and Grant (2007) (CEG in the following) develop the concept of neo-additive capacity. This value is additive on non-extreme outcomes contrary to a capacity and this allows to take into account optimistic and pessimistic attitudes towards uncertainty. This gives theoretical foundation to the empirical evidence that investors do not behave according the of the SEU’s patterns. Camerer and Weber (1992) reviewed and gathered the whole set of significant criticisms brought to standard expected utility theory. Gonzales and Wu (1999) or Abdellaoui (2000) and many others empirical studies showed that, in concrete world, in betting situations, agents tend to overweight probabilities close to zero and underweight probabilities close to 1. This corresponds to the famous S-shaped curve that figures the willingness to bet. Individuals prefer to bet when the probability of winning is low but with high payoff (for national lottery tickets for instance) and are more reluctant to bet when the probability of winning is high but with low gains (see appendix 1).

More precisely, the polluter and the society cannot a priori assess with certainty the exact costs of a major hazard. Let $\mathcal{E}$ be the finite set of states to which correspond catastrophic events included in $\mathcal{A}$ ($\sigma$-algebra of $\mathcal{E}$). Let be a finite set of outcomes $A$, ($A \subset \mathbb{R}$) and a set of
simple functions: $\Phi, \Phi = \{f: \mathcal{E} \to A\}$ from states to outcomes which correspond to simple acts and takes on values: $a_1 \geq a_2 \ldots \geq a_n$. The expected costs of the maximum damages $E_p(a)$ expresses as:

$$E_p(a) = \int_d^\infty a \ p(a) \, da$$

From it, the neoadditive capacity is then for $\alpha, \delta \in [0,1]$ (see appendix 1 for details):

$$\mu(A / p, \delta, \alpha) = \begin{cases} 
\delta \alpha v_0(A) + \delta (1 - \alpha) v_1(A) + (1 - \delta) p(A) & \text{for } \emptyset \subsetneq A \subsetneq \mathcal{E} \\
0 & \text{for } A = \emptyset \\
1 & \text{for } A = \mathcal{E} 
\end{cases}$$

Let us consider that $v_0(A) = \inf(f) = d$ where $d$ is the minimum cost of the maximum damage and $v_1(A) = \sup(f) = \overline{D}$, the highest one. Here, $\delta$ and $\alpha$ represents the weight that the injurer allocates to the extreme events. $\delta$ represents the preference for ambiguity, and $\alpha$ the degree of optimism. Then, for $\emptyset \subsetneq A \subsetneq \mathcal{E}$ the neoadditive capacity is:

$$\mu(\cdot) = \delta \alpha d + \delta (1 - \alpha) \overline{D} + (1 - \delta) p(A)$$

The corresponding Choquet integral builds on the capacities of the cost of a major harm:

$$V_p(A/p, \delta, \alpha) = \delta \alpha d + \delta (1 - \alpha) \overline{D} + (1 - \delta) E_p(a)$$

(4) is the weighted sum of, respectively, the minimum, the maximum and the expectation of the damage value of a major harm. Here, optimism and pessimism refer to the scope of the major accident. Optimism involves high value of $\alpha$ (the lowest damage). A low value of $\alpha$ tends to overweight the highest harm $\overline{D}$. When $\alpha = 0$ (pessimistic feeling), then $V_p(A/p, \delta, 0) (= \delta \overline{D} + (1 - \delta) E_p(a))$ only depends on the injurer’s ambiguity aversion ($\delta$). Furthermore, if $\delta = 0$, then, the capacity reduces to $E_p(a)$ and the injurer feels no ambiguity on the probability distribution, $\mu(A / p, 0, \alpha) = p(A)$. The higher $\delta$ is, the less confident the operator about the likelihood of the probability distribution while, $\delta = 1$ shows an absolute distrust in it:

$$V_p(A / p, 1, \alpha) = \alpha d + (1 - \alpha) \overline{D}$$
This expression corresponds to the Hurwitz criteria weighted by the injurer’s degree of optimism. Before going further, note that the injurer’s Choquet integral of the expected cost can be higher or lesser than the major expected damage cost:

(6) \[ \delta ad + \delta (1 - \alpha)\overline{D} + (1 - \delta)E_p(a) > or \leq E_p(a) \]

when \( \alpha \) is such that:

(7) \[ \frac{\overline{D} - E_p(a)}{\overline{D} - d} > \alpha \geq 0 \text{ for superiority or the reverse for inferiority.} \]

Lemma 1 summarizes this point:

**Lemma 1:**

- **[A]** When \( V_p(A/p, \delta, \alpha) > E_p(a) \) then \( \frac{\overline{D} - E_p(a)}{\overline{D} - d} > \alpha \) and,

- **[B]** When \( V_p(A/p, \delta, \alpha) \leq E_p(a) \) then \( \frac{\overline{D} - E_p(a)}{\overline{D} - d} \leq \alpha \) (with equality for \( V_p(A/p, \delta, \alpha) = E_p(a) \)).

This lemma means that the value of the Choquet integral of expected cost depends on the level of optimism of the injurer. This value compares \( \alpha \) to the ratio \( \frac{\overline{D} - E_p(a)}{\overline{D} - d} \) (where obviously \( \frac{\overline{D} - E_p(a)}{\overline{D} - d} < 1 \)).

**II. COURT AND REGULATOR AGREE ON THE OPTIMAL LEVEL OF CARE**

By assumption, the regulator and the judge use a common framework to determine the first-best level of care for Society. The whole argument assumes that the activity maintains at a constant level. Then:

**Assumption 1:** There is no divergence between the regulator and the Court in the assessment of the first best level of care.

Let \( x \) be the level of care \( (x \geq 0, x \in \mathbb{R}^+) \). The probability \( \pi(x) \) corresponds to the probability of a major accident with, as usual: \( x, \pi'(x) < 0, \pi''(x) \geq 0 \).

In the following, the index “NR” indicates negligence rule, the index “SL”, strict liability, “P” points out the injurer or polluter and “A” the victims.

**Assumption 2:** The regulator is risk neutral.
The regulator (or social planner) minimizes the expected social costs of an accident that expresses as:

\[ ECS(x) = x + E_p(a) \pi(x) \]  

(8)

For the regulator, the major damage cost consists in the expectation \( E_p(a) \) (equation 1). Then, the optimal level of care \( x^* \) deduces from the first order conditions \( ECS' \leq 0 \) and \( \pi'(x^*) \leq -\frac{1}{E_p(a)} \) with equality for for \( x^* > 0 \)

Considering that the injurer conceives the cost of damages as the Choquet integral defined in (4), the accident costs function expresses as:

\[ EC_p(x) = x + V_p(A/p, \delta, \alpha)\pi(x) \]  

(9)

Solving for the first order conditions gives the injurer’s equilibrium level of care \( x^{SL} \):

\[ EC_p(x^{SL})' \leq 0 \implies \pi'(x^{SL}) \leq -\frac{1}{V_p(A/p, \delta, \alpha)} \text{ with equality for for } x^{SL} > 0. \]  

(10)

The following proposition shows the inefficiency of the solution:

**Proposition 1:** Under a strict liability regime, with a Choquet expected cost-minimizer injurer, by lemma 1 and under assumptions 1 and 2:

- When [A] is verified, then the injurer over-invests in care, i.e. \( x^{SL} > x^* \)
- When [B] is verified, then the injurer under-invests in care: \( x^{SL} \leq x^* \).

**Proof:** See appendix 2

Proposition 1 shows that under Knightian uncertainty too much or not enough care depends on the injurer’s degree of optimism. Furthermore, this extends Beard’s results (Beard (1991)) beyond judgement-proofed injurers who can take too much care (rather than under invest as Shavell (1985) and (1987) showed it). For Teitelbaum (2007) strict liability only involves an under-investment in care. Considering negligence, the injurer has the choice of complying by adopting the first best level of care or not. However, if he forms beliefs about the occurrence of a major harm, he will rather conform to his ambiguity aversion and his
optimism/pessimism degree. Then, the degree of compliance is closely linked with this values (See for instance Teitelbaum (2007 proposition 3). Hence, the cost functions expresses as:

\[ EC_{NR \theta} = \begin{cases} 
  x^\mu & \text{if } x^\mu \geq x^* \\
  x^s + V_p(A|p, \delta, \alpha) \pi(x^s) & \text{if } x^s < x^* 
\end{cases} \]

The level of care that the injurer puts into place depends on the parameters that constitute his cost function. As for the above situation, his equilibrium level of care is not optimal as shows it proposition 2:

**Proposition 2:** Under a rule of negligence, with a Choquet expected cost-minimizer injurer, by lemma 1 and under assumptions 1 and 2:

When \([A]\) is verified, the injurer will over-invests in safety. Hence, if \(x^{NR}\) is the optimum level of safety for the injurer, then \(x^{NR} > x^*\).

When \([B]\) is verified, then he under-invests: \(x^{NR} \leq x^*\).

**Proof:** See appendix 2 ■

This result is similar to the strict liability situation. Either the injurer spends too much in care or not enough compared to the optimal level. As a main conclusion, the model shows that when the Court and the regulator perfectly agree about the first best safety level, then, the injurer’s chooses an optimal level of safety that is independent from the responsibility regime as shows it the following theorem:

**Theorem 1:** By lemma 1, proposition 1 and 2 and under assumption 1 and 2, whatever the enforced liability rule (strict liability regime or negligence rule), when:

- \([A]\) is verified, then, \(x^0 > x^*\);
- \([B]\) is verified, then, \(x^* \geq x^0\), where \(x^0 = \{x^{SL}, x^{NR}\}\).

**Proof:** The proof comes from the combination of propositions 1 and 2 ■

Theorem 1 means that under assumption 1, the injurer chooses a socially non-efficient level of care. This choice is done according his degree of optimism \(\alpha\). Furthermore, the level of care, whatever the liability regimes is the same as shows it corollary 1:
Corollary 1: Given the injurer’s level of optimism, given lemma 1 and propositions 1 and 2, the level of care determined by the injurer is the same whatever the responsibility regime, i.e. $x^0 = x^{NR} = x^{SL} > x^\star$, for [A] of lemma 1, or $x^0 = x^{NR} = x^{SL} \leq x^\star$, for [B] of the same lemma.

Proof: See appendix 2 ■

This result confirms Shavell’s results (Shavell (1980) and (1987) but only partially. Indeed, the determined level of care is identical under both regimes (negligence rule and strict liability), however, the similarity ends there because this care level is not optimal.

Corollary 2: Under the conditions described by assumption 1, lemma 1 and propositions 1 and 2, neither a liability regime nor a negligence rule can enforce the first best level of care.

Proof: The proof comes directly from theorem 1 and corollary 1 ■

Corollaries 1 and 2 mean that under uncertainty the liability regime does not influence the care level. This result is counterintuitive because in most well accepted literature to each liability rule is associated a specific equilibrium level of care.

III. AMBIGUOUS ACCIDENT PROBABILITIES AND MAJOR DAMAGES

Is the previous approach robust enough to integrate Teitelbaum (2007)’s model? This author considers that the tortfeasor’s beliefs bear on the accident probability distribution that is to say on $\pi(x)$ and $(1 - \pi(x))$. Hence, combining both models leads the injurer to face two kinds of uncertainty and, thus two kinds of beliefs. The first one is about the probability distribution of a major accident (Teitelbaum), the second one concerns the scale of damages (our model). In the Teitelbaum’s model, the injurer produces an income $k$, the degree of aversion for ambiguity is $\beta$ and the level of pessimism is $\gamma$. Let be $f$ the set of potential

11
results, \( \Theta \) the level of damage. The worst outcome for the injurer is: \( \inf f = k - x - \Theta \), and the best one: \( \sup f = k - x \). The injurer’s program defines as:

\[
\begin{align*}
\max_{x \geq 0} & \left\{ y \beta \{\inf f\} + (1 - y) \beta \{\sup f\} + (1 - \beta) \{\pi(x) \{\inf f\} + (1 - \pi(x)) \{\sup f\}\}\right\} \\
= & \max_{x \geq 0} \left\{ y \beta (k - x - \Theta) + (1 - y) \beta (k - x) + (1 - \beta) \{\pi(x)(k - x - \Theta) + (1 - \pi(x))(k - x)\}\right\} \text{ or, after simplifying:}
\end{align*}
\]

(11) \( \min_{x \geq 0} \{x + y \beta \Theta + (1 - \beta) \pi(x) \Theta\} \)

From the first order conditions, the solution for the strict liability case \( x^{SL} \) writes as:

(12) \( -\pi'(x^{SL}) \Theta = \frac{1}{1 - \beta} \) (with \( \beta < 1 \)).

For a given \( \Theta \), Teitelbaum shows that the injurer’s optimal level of care is less than the optimal level of care. This can be checked considering that when \( \beta = 0 \), the program becomes:

(13) \( \min_{x \geq 0} \{x + \pi(x) \Theta\} \)

Where (14) is the expression of the social cost, then:

(14) \( -\pi'(x^*) \Theta = 1 \)

Under ambiguity, the solutions for both the injurer and the regulator do not meet each other and \( x^{SL} < x^* \), this is contrary to the standard case where \( x^{SL} = x^* \). A similar argument applies to negligence, but here the injurer can comply with the regulator’s will. This compliance depends on the degree of ambiguity (too little care if the agent is optimistic), the degree of optimism and the magnitude of the accident losses according the degree of local convexity of the probability distribution \( -\left(\frac{\pi''(x)}{\pi'(x)}\right) \).

Assume now that the injurer forms beliefs about the probability distribution of the foremost damages, and, as in section 1, this corresponds to the Choquet integral of the major accident costs:

(15) \( \Theta_p = V_p(A/p, \delta, \alpha) = \delta \alpha d + \delta (1 - \alpha) \tilde{D} + (1 - \delta) E_p(\alpha) \).

While, for the risk-neutral regulator, this value remains the same ( i.e. \( \Theta = E_p(\alpha) \) where \( E_p(\alpha) \) is the expectation of the cost of a major damage as expressed in equation (1)).
Replacing $\Theta$ by $\Theta_p$ in $-\pi'(x^{SL}) \Theta = \frac{1}{1-\beta}$ considering the tortfeasor:

$$-\pi'(x^{SL})[\delta \alpha a + \delta (1-\alpha)\overline{D} + (1-\delta)E_p(a)] = \frac{1}{1-\beta}$$

Thus, the regulator and the injurer diverge about the optimal level of care. Hence, I study the conditions for having either $E_p(a) > V_p(A/p, \delta, \alpha)$ or the reverse as in the previous part. The following proposition insues:

**Proposition 3:** Let us consider strict liability and Knightian uncertainty characterized by the injurer’s beliefs on major damage and accident occurrence. Under assumption 1 and 2, a Choquet cost-minimizer injurer will tend to care too much or not enough compared to the first best level of care. More precisely:

1. $E_p(a) < V_p(A/p, \delta, \alpha)(1-\beta)$ for $1 \geq \alpha > \frac{E_p(a)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)}$ and this relationship is true if and only if $E_p(a) > \frac{(1-\beta)}{\left(\frac{\beta}{\delta}+1-\beta\right)}D$ for $1-\beta > 0$ and $\delta > 0$.

Consequently, this involves that $x^{SL} > x^*$.

2. $E_p(a) > V_p(A/p, \delta, \alpha)$ for $\alpha \leq \frac{E_p(a)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)}$ which is true if and only if $E_p(a) \leq \frac{(1-\beta)}{\left(\frac{\beta}{\delta}+1-\beta\right)}D$, consequently, $x^{SL} \leq x^*$.

**Proof:** See appendix 2 ■

Under strict liability, Proposition 3 shows that proposition 1 is robust to the extension to the Teitelbaum’s analysis. Furthermore, this issue enlarges to negligence. To see this, let us consider that, under negligence, the expected cost of accident express as:

$$EC^{NR}_p = \begin{cases} x^\mu & \text{if } x^\mu \geq x^* \\ x + \gamma \beta \Theta_p + (1-\beta)\pi(x)\Theta_p & \text{if } x < x^* \end{cases}$$

Where, as previously, $\Theta_p = \delta \alpha d + \delta (1-\alpha)\overline{D} + (1-\delta)E_p(a)$

As Shavell (1987 p.36) recalls it, strict liability involves that courts observe only the level of losses ($\Phi = \{f : E \to A\}$) while, under negligence, the courts needs also to check if the
injurer brought the optimal level of care. The wrongdoer should comply with the judge’s opinion, but his beliefs submit to a double screening: a first one bears on the probability of an accident and a second one on the accident scale. Consequently, as proposition 4 shows it, this does not lead to an efficient care level.

**Proposition 4:** Under negligence and Knightian uncertainty, under assumption 1, the optimum level of care chosen by the injurer will be either higher or lower than the first best level of care.

**Proof:** See appendix 2 ■

Taking into account proposition 3 and 4, theorem 2 extends from theorem 1:

**Theorem 2:** By lemma 1, proposition 3 and 4 and assumptions 1 and 2, when:

- \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \) is verified, then, \( x^{NR} > x^* \).

- \( E_p(\alpha) > V_p(A/p, \delta, \alpha) \) is verified, then, \( x^* > x^{NR} \).

This result is independent from the enforced responsibility regime (strict liability regime or negligence rule).

**Proof:** It is sufficient to gather proposition 3 and 4 to get the result ■

As theorem 1, theorem 2 means that under extended uncertainty (that spans both the probability distribution of an accident and the level of maximum damage), the optimum level of care diverges from the injurer’s equilibrium level of safety. This result is more general than theorem 1 because here uncertainty is broader. As a result, neither strict liability nor the negligence can reach the socially optimal level of care. Now, corollary 3 shows that, whatever the enforced liability regime, the injurer’s equilibrium level of care remains identical. It depends on the injurer’s optimism degree and his degree of ambiguity aversion.

**Corollary 3:** Given the conditions described by assumption 1 and propositions 3 and 4, the level of care determined by the injurer is the same whatever the responsibility regime
put into force by the regulator. Hence, \( x^{NR} = x^{SL} > x^* \), for \( 1 \geq \alpha > \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D \delta}{\delta (d - D)(1 - \beta)} \),

and \( x^{NR} = x^{SL} \leq x^* \), for \( \alpha \leq \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D \delta}{\delta (d - D)(1 - \beta)} \).

**Proof:** See appendix 2 ■

Propositions 3 and 4, theorem 2 and corollary 3 confirm and extend corollary 2. This last one shows that neither strict liability nor negligence can enforce the optimal level of care. Furthermore, there exists no argument that can induce the superiority of one regime compared to the other one. Then, modifying corollary 2 to extend the uncertainty field, we get.

**Corollary 4:** Under propositions 3 and 4, neither a liability regime nor a negligence rule can enforce the first-best level of care.

**Proof:** This results from theorem 2 and corollary 3 ■

Under a high degree of uncertainty, the first best level of care remains unattainable. Corollary 4 generalizes to a larger uncertainty concept the result got under corollary 2 (uncertainty on the probability distribution of accident and, the uncertainty about the range of the major harm).

**IV. DIVERGENT VIEW BETWEEN COURT AND REGULATOR**

Usually, because of the legal separation of powers between regulator and Court, the judge assesses independently the level of due care brought by the injurer. Considering that the regulator disposes of more technical and scientific means than the Court, the literature deems that the judge makes mistake when his estimation diverges from the planner’s view (Diamond (1974), Cooter (1984), Shavell (1987 pp. 79-83 and 93-97) or still Cooter and Ulen (2003 pp.337-342). The causes of errors are several and depends on circumstances as Craswell and Calfee (1984) and (1986), Shavell (2004, pp. 227-230) showed it. These contributions share the common feature that the judges fail when assessing the range of responsibility.
Accordingly, the injurers supply either excessive care (for instance Danzon (1985) for defensive medicine) or not enough. Kahan (1989) analyses this point considering the biases induced by the judge’s assessment. Then the injurers take for granted that the Court can mistake and, consequently, they allocate a no null probability to it.

Under negligence, the operator tends to forecast the judge’s beliefs. This modifies the injurer’s system of credence. Then, the components of the wrongdoer’s uncertainty are twofold. First, uncertainty bears on the level of damages: the polluter ignores the true scale of the major harm. Second, the possibility of being indicted even after having complied with the necessary due care cannot be dismissed.

1. Court’s errors and impact on the injurer’s beliefs

For standard theory, under negligence, the judge checks the consistency of prevention efforts with the regulator’s requirement. However, in concrete life, the judge can consider as insufficient the set of measures taken by the tortfeasor, even if this last one complied with the first-best level of care. This is particularly the case under administrative Courts. In most countries, these courts exert an ex-post control over the administrative standard. In several countries, administrative courts are separated from general courts and are organized in local administrative court, court of Appeal and Supreme Administrative Court (France, Italy and in most European Countries). In the United States, administrative law judges belong to several federal agencies as for instance the Environmental Protection Agency (EPA). Hence, the administrative judge can modify the administrative decision by substituting his own rules. For Desai (2002, p.187) “Nevertheless, administrative courts play an important role in environmental policy and conflicts. They exercise comprehensive judicial control over administrative actions, (.), and they are often mobilized by third parties in the course of licensing or planning procedures, with the aim of achieving tighter environmental standards or stopping projects or operating plants”.

http://services.bepress.com/feem/paper828
Against the above argument, one may consider that administrative law is little concerned with negligence that belongs to the field civil law. Caroll (2007) shows the difficulty involving authorities in negligence. However, heuristic reasons explain the reference to administrative courts. These courts can correct the regulator’s assessment. Hence, under negligence, the courts can investigate and acquire new ex post information. Consequently, the operator is never sure of having fully fulfilled his obligations concerning the socially required level of prevention.

2. Building the system of beliefs under negligence

The Courts assess the injurer’s level of due care by taking into account the set of information given by their own investigation procedure and, consequently, the result may be random. Let \( \vartheta \) be the probability that the court confirms the injurer choice (first best), and, conversely, let \( 1 - \vartheta \) be the probability that he did not invest enough in safety.

Consequently, the injurer dedicates only \( x^\mu \) in prevention investment if the court agrees with him (with a probability of \( \vartheta \)) and he will have to pay \( x^\mu + V_p^C \pi(x^\mu) \) in the opposite case (with a probability of \( 1 - \vartheta \)). Therefore, for the compliance case, the effective expected cost will be:

\[
EC_{NR-p} = x \vartheta + (1 - \vartheta)(x + V_p^C \pi(x)) = x + (1 - \vartheta)V_p^C \pi(x)
\]

This new factor of uncertainty can increase the injurer’s ambiguity aversion. Indeed after developing:

\[
EC_{NR-p} = x + (1 - \vartheta)V_p^C \pi(x)
\]

\[
= x + (1 - \vartheta)[\delta \alpha d + \delta(1 - \alpha)\overline{D} + (1 - \delta)E_p(a)] \pi(x^\nu)
\]

Then, I introduce the Court error factor in the bracket:

\[
x + [\delta \alpha d(1 - \vartheta) + \delta(1 - \alpha)(1 - \vartheta)\overline{D} + (1 - \delta)(1 - \vartheta)E_p(a)] \pi(x)
\]
This factor increases uncertainty and modifies the degrees of optimism/pessimism and aversion for ambiguity. Hence, considering $V_{\theta}^{NR}$ the agent’s Choquet integral under negligence:

\begin{equation}
V_{\theta}^{NR} = (1 - \vartheta)V_p^{C} = \\
= \delta \alpha d(1 - \vartheta) + \delta(1 - \alpha)(1 - \vartheta)\bar{D} + (1 - \delta)(1 - V_{\theta}^{NR})E_p(\alpha)
\end{equation}

Then, how could we integrate courts’ errors in the injurer’s beliefs? This involves determining $\psi^* > 0$ and $\varphi^* > 0$, i.e. the expression of the level of optimism and ambiguity aversion such that:

\begin{equation}
V_{\theta}^{NR} = \psi^* \varphi^* d + (1 - \psi^*)\varphi^*\bar{D} + (1 - \varphi^*)E_p(\alpha)
\end{equation}

As a result\footnote{To reach this result, it sufficient to compute: $\left\{ \begin{array}{l} \delta \alpha(1 - \vartheta) = \psi \varphi \\
\delta(1 - \alpha)(1 - \vartheta) = (1 - \psi)\varphi \\
(1 - \delta)(1 - \vartheta) = 1 - \varphi \end{array} \right.$}:

\begin{equation}
\psi^* = \alpha \text{ and } \varphi^* = \delta(1 - \vartheta)
\end{equation}

Let us check that $\psi^* \geq 0$ and $1 \geq \varphi^* \geq 0$. Consequently, the following proposition ensues:

**Proposition 5:** Under Knightian uncertainty, when Court and regulator diverge about the optimum social safety level, then the injurer’s ambiguity aversion increases compared to the case in which both agree, while the optimism degree remains constant.

**Proof:** Concerning the level of optimism $\psi^* = \alpha$, the results come from the above argument. To see how the ambiguity aversion increases, it is sufficient to check that,

$\varphi^* = \delta(1 - \vartheta) < 1$, because $\delta \leq 1$ and $1 - \vartheta < 1$ when the Court is supposed to mistake. Then $\varphi^* < \delta$ and the higher $\delta$ is, the lesser the aversion for ambiguity. $\varphi^* < \delta$, means that the weight allocated to $E_p(\alpha)$ is decreasing. This involves that the injurer is less confident in this value. Hence, his level of aversion to ambiguity increases.
As a result, modifying the injurer’s beliefs leads to conceive his Choquet integral expected cost function:

\[
(22') \quad V_{\delta}^{NR} = \alpha \varphi^* d + (1 - \alpha)\varphi^* \tilde{D} + (1 - \varphi^*)E_p(\alpha).
\]

I keep \( \alpha \) rather than \( \psi^* \) in order to make easier the comparison with the strict liability case. Let us note that the expected cost \( V_{\alpha, \varphi}^{NR} \) does not lead to define a level of care equal to the social first best. This corresponds to proposition 6:

**Proposition 6:** Under uncertainty and negligence, considering an independent court when an injurer is endowed with a Choquet expected cost function \( V_{\delta}^{NR} \) and a regulator with a Savage expected cost function, then the injurer’s equilibrium level of care is inefficient compared to the first best level of care.

**Proof:** See appendix 2 ■

Proposition 6 establishes that, in the judge’s opinion regarding the optimum safety level, true uncertainty covers both the major harm level and the likelihood of error. Then, a negligence rule leads to an inefficient level of care. Hence, as before, the liability regime does not issue “naturally” on a first-best solution: the optimism degree involves either a too high level of care or an insufficient one.

**COMPARING STRICT LIABILITY AND NEGLIGENCE RULE**

To make easier the comparison between both regimes, let us consider that:

\[
V_p(A/p, \delta, \alpha) = V_{\delta}^{SL} = \delta \alpha d + \delta (1 - \alpha) \tilde{D} + (1 - \delta) E_p(\alpha)
\]

\( V_{\delta}^{SL} \) is the Choquet integral of the expected cost of the injurer under a strict liability regime. Relaxing assumption 1 leads to consider that \( V_{\delta}^{NR} \neq V_{\delta}^{SL} \). The question to know is whether \( V_{\delta}^{NR} > V_{\delta}^{SL} \) or the reverse. The conditions for having \( V_{\delta}^{NR} > V_{\delta}^{SL} \) and \( V_{\delta}^{NR} < V_{\delta}^{SL} \) are analyzed.

- \( V_{\delta}^{NR} > V_{\delta}^{SL} \) involves that:
\[ \alpha \varphi^* d + (1 - \alpha)\varphi^* \bar{D} + (1 - \varphi^*) E_p(a) > \delta \alpha d + \delta(1 - \alpha) \bar{D} + (1 - \delta) E_p(a) \]

or, after developing:\[[3] \frac{\bar{D} - E_p(a)}{\bar{D} - d} > \alpha\]

- \[ V_{\Theta}^{NR} < V^{SL} \] involves that:

\[ \alpha \varphi^* d + (1 - \alpha)\varphi^* \bar{D} + (1 - \varphi^*) E_p(a) < \delta \alpha d + \delta(1 - \alpha) \bar{D} + (1 - \delta) E_p(a) \]

Hence, \[ \frac{\bar{D} - E_p(a)}{\bar{D} - d} < \alpha \]

It is striking to see that, formally, we find the conditions defined in lemma 1 and proposition 1. These issues on the following proposition:

**Proposition 7:** \( V_{\Theta}^{NR} > V^{SL} \) involves that \[ \frac{\bar{D} - E_p(a)}{\bar{D} - d} > \alpha \], and, consequently, \( x^{SL} < x^{NR} \).

In the opposite then, \[ \frac{\bar{D} - E_p(a)}{\bar{D} - d} < \alpha \] then, \( V_{\Theta}^{NR} < V^{SL} \) and strict liability involves a higher level of care compared to negligence, i.e. \( x^{SL} > x^{NR} \).

**Proof:** The proof is obvious. It borrows the same demonstration scheme than for proposition 1 and 6.

Proposition 7 says that the degree of optimism/pessimism is a determinant factor in the definition of the equilibrium level of care. This is similar to proposition 1 and 2. Strict liability supplies a higher safety than negligence according the optimism degree. This makes the regime to enforce indeterminate. Besides, gathering propositions 1, 6 and 7, we deduce the following proposition. It establishes, first, that, whatever the level of uncertainty and the nature of the legal responsibility regime, neither of them can implement the first best social optimal level of care. Second, neither the liability regime nor the rule of negligence is more efficient than the other. The degree of protection depends on the level of optimism.

**Proposition 8:** In an uncertain world as in propositions, 5, 6 and 7, when the level of optimism is such that:

\[ \text{It is easy to develop the inequality: } \alpha(\varphi^* - \delta)d + (1 - \alpha)(\varphi^* - \delta)\bar{D} - (\varphi^* - \delta)E_p(a) > 0, \]

simplifying by(\( \varphi^* - \delta \)) gives: \( \alpha d + (1 - \alpha)\bar{D} - E_p(a) > 0 \).
\[-\frac{\delta - E_p(a)}{\delta - d} > \alpha, \text{ then, the injurer will tend to over-invest in care and} \]
\[x^{NR} > x^{SL} > x^*, \]
\[-\frac{\delta - E_p(a)}{\delta - d} < \alpha, \text{ then, the injurer will tend to over-invest in care and} \]
\[x^* > x^{SL} > x^{NR}. \]

**Proof:** Proposition 8 results from propositions 1, 6 and 7. The following table summarizes the results:

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Consequences</th>
<th>Level of care</th>
<th>Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\frac{\delta - E_p(a)}{\delta - d} &gt; \alpha]</td>
<td>[V_p &gt; E_p(a)]</td>
<td>[x^{SL} &gt; x^*]</td>
<td><strong>Situation 1</strong></td>
</tr>
<tr>
<td>Proposition 1</td>
<td>Proposition 6</td>
<td>Proposition 7</td>
<td>[x^{NR} &gt; x^{SL} &gt; x^*]</td>
</tr>
<tr>
<td>[\frac{\delta - E_p(a)}{\delta - d} &lt; \alpha]</td>
<td>[V_p &lt; E_p(a)]</td>
<td>[x^* &gt; x^{SL}]</td>
<td><strong>Situation 2</strong></td>
</tr>
<tr>
<td>Proposition 1</td>
<td>Proposition 6</td>
<td>Proposition 7</td>
<td>[x^* &gt; x^{SL} &gt; x^{NR}]</td>
</tr>
</tbody>
</table>

**Table 1: Comparison of care performance when the regulator and the Court disagree**

When \(\alpha\) lies below \(\frac{\delta - E_p(a)}{\delta - d}\), the injurer over-invests in safety and the care level is higher under negligence than under strict liability. The complete reverse is true, \(\alpha\) higher than \(\frac{\delta - E_p(a)}{\delta - d}\) induces a higher level of care under strict liability than under negligence.\]
**Remark 1:** Is it possible to draw conclusion when considering the variations of \( \bar{D} \) or \( d \) in a comparative static framework? Other things being equal, note that the higher \( \bar{D} \) is, the nearest is \( \frac{\bar{D} - E_p(a)}{\bar{D} - d} \) from 1. Hence, in most cases, it exceeds \( \alpha \) and the situation 1 (\( (x^{NR} > x^{SL} > x^*) \)) should prevail. This is particularly the case for situations with potential high level of damage. Then, it is strongly probable that \( \frac{\bar{D} - E_p(a)}{\bar{D} - d} > \alpha \). Hence, even if the equilibrium is inefficient, negligence seems to be a better regime than strict liability. Indeed, the tendency would be to over-invest with \( x^{NR} > x^{SL} > x^* \). Hence, even if seductive, this view, cannot be followed because it seems unrealistic to consider that changes in \( \bar{D} \) would not involve changes in the whole spectrum of probability distribution.

**Remark 2:** As a result, from proposition 8, I deduce that the definition of an “ad hoc” liability regime is neither a necessary condition nor a sufficient one to lead potential injurers to supply the first best optimal level of care. Furthermore, these results involve that no responsibility regime can be considered as “better” or more efficient than another one.

**CONCLUSION**

Despite appearances, this article brings no negative inference about the importance of liability regimes as regulatory factors and prevention of major hazard. It shows only that the performances comparison as the minimization of the social cost of care and the maximization of the security costs is not a relevant criteria to discriminate between liability regimes. In fact, the model only shows that in the context of radical uncertainty, polluters tend to either over-invest or spend not enough money in safety. Consequently, uncertainty prevents to reach the first best in terms of prevention costs. Then, it follows that the mutual comparison of their relative performance cannot preside to the enforcing of a responsibility regime.
I proved the impossibility of choosing among liability regimes using a twofold argument. First, when the court complies with the regulator about the optimal social level of care, then strict liability and negligence are perfect substitutes as in the standard accident model. However, the injurer’s equilibrium level of care is not equal to the socially required level.

Second, weakening the assumption of compliance between the judge and the regulator leads to different issue when regarding the equilibrium level of care. This depends on the nature of the enforced liability regime. However, neither strict liability nor negligence allows reaching the social first best. Both depend on the degree of optimism compared to the ratio composed, first, by the difference between the highest level of damage and its expected cost and, second, the difference between the highest level of damage and the lowest one. Hence, this result prevents to make any conclusion about the best liability regime to enforce. Indeed, the degree of optimism of the polluter is private knowledge and defining a revelation mechanism is impossible. As a conclusion, under a regime of “true uncertainty”, the a priori efficiency of a responsibility regime does not constitute the ground for its enforcement. The injurer’s degree of optimism is not rock-solid enough to build a theory or a liability regime on it.

Consequently, a liability regime is not that decisive factor that would induce the injurers to supply the socially first best level of care. Indeed, facing uncertainty, the choice of a responsibility regime remains deeply unspecified. However, the fact that agents are led to over-invest or under invest in safety involves that this factor is not neutral as a prevention instrument. Then, this raises the question of what kind of liability to enforce and on what grounds implementing it. This is the further step for future researches.
Bibliography


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Appendix 1

Neo-additive capacity and Choquet utility function

I do not propose here a full formal mathematical presentation. The interested lector may refer to the clear exposition of Chateauneuf, Eichberger and Grant (2007) (CEG(2007) in the following).

A capacity is an extension of a probability. It is a function $\tau(p)$ that assigns real numbers to events $E$, where $E$ is the set built from the set $S$ of the states of nature. To be a capacity the following two conditions should be fulfilled. First, for all $E, F \in E$, and $E \subseteq F$, then $\tau(E) \leq \tau(F)$ as monotonicity condition and, second, as normalization conditions, $\tau(\emptyset) = 0$ and $\tau(S) = 1$.

The best way to integrate capacities is the Choquet integral. To do that, it is assumed that exists a simple function of finite range $f$ that takes values $\mu_1 \geq \mu_2 \geq \mu_n$. A Choquet integral of a simple function $f$ with respect to a capacity $\mu(.)$ is defined as:

$$V(f/\tau) = \sum_{\mu \in f(S)} \mu[\tau(\{s/f \geq \mu\}) - \tau(\{s/f > \mu\})]$$

Through the concept of neo-additive capacity the Choquet integral overweight high outcomes if the capacity is concave or overweight low income if the capacity is convex. Convexity of a capacity is verified by the following relationships:

$$\tau(E \cup F) \geq \tau(E) + \tau(F) - \tau(E \cap F)$$

Applying this to the model, the polluter and the society cannot assess with certainty the exact value of a maximum damage. Let be $E$ the finite set of states to which correspond the catastrophic events $A$ ($\sigma$-algebra of $E$). Consider a finite set of outcomes ($A \subseteq R$) and let $\Phi = \{f: E \rightarrow A\}$ be a set of simple functions from states to outcomes which correspond to simple acts and takes on values $a_1 \geq a_2 \cdots \geq a_n$.

The polluter is gifted with a Choquet objective function which corresponds here to an expected cost function. His beliefs on the level of damage correspond to a neo-additive capacity ($\mu$) based on ($p$). Hence, the operator will form beliefs about the level of the damage. This is a supplementary uncertainty. One can define now the neo-additive capacity. To do that let us consider that the $\sigma$-algebra $A$ is partitioned in three subsets (for a more complete information see CFG (2002, 3):

- The set of null events $N$, where $\emptyset \in N$ and for $G \subseteq H$, and $G \in N$ if $H \in N$. 
- The set of “universal events” $\mathcal{W}$, in which an event is certain to occur, (complement of each member of the set $\mathcal{N}$).
- The set of essential events, $\mathcal{A}^*$, in which events are neither impossible nor certain. This set is composed of the following:

$$\mathcal{A}^* = \mathcal{A} - (\mathcal{N} \cup \mathcal{W})$$

Before going further, let us define the following capacities $\nu$ (see appendix):

$$\nu_0(A) = 1 \text{ if } A \in \mathcal{W} \text{ and 0 otherwise and } \nu_1(A) = 0 \text{ for } A \in \mathcal{N} \text{ and } \nu_1(A) = 1 \text{ otherwise.}$$

Furthermore, let be a finite additive probability $p(.)$ such that $p(A) = 0$, if $A \in \mathcal{N}$ and 1 otherwise.

**Definition 1:** Let $\lambda, \gamma$ be values that belong to a simplex $\Delta$ in $\mathbb{R}^2$, $(\Delta:= \{(\alpha, \beta) / \alpha \geq 0, \beta \geq 0, \alpha + \beta \leq 1 \})$, a neo-additive capacity $\mu$ based on the distribution of probability $p(.)$ is defined as:

$$\mu(A / p, \lambda, \gamma) = \begin{cases} 0 & f or A = \emptyset \\ \lambda \nu_0(A) + \gamma \nu_1(A) + (1 - \gamma - \lambda)p(A) & f or \emptyset \subset A \subset \mathcal{E} \end{cases}$$

One can check that a neo-additive capacity is additive on non-extreme outcomes, $p$ corresponds to the probability of a major accident of a given scale. This is a common belief and $(1 - \gamma - \lambda)$ represents the degree of confidence of the agent in this belief. A Choquet integral is a weighted sum of the minimum, the maximum and the expectation of a simple function $f$: $\mathcal{E} \to \mathbb{R}$ as the following relationship:

$$V( f / p, \lambda, \gamma) = \lambda . \inf(f) + \gamma . \sup(f) + (1 - \gamma - \lambda)E_p(f)$$

Where $E_p(f)$ is the expected value of the expected costs of a major accident, and from the linearity of the Choquet integral with respect to the capacity, $V(f / \nu_0(.)) = \inf(f)$ and $V(f / \nu_1(.)) = \sup(f)$, (proof see CFG(2002, 3) and CFG(2007, 3).

Then for $e \in \mathcal{E}, f(e) = a$, we put, $f(e_1) = \sup(f) = a_1 = \overline{D}$ and $f(e_n) = \inf(f) = a_n = d$. As, $p(.)$ is a finitely additive probability distribution on $\mathcal{A}$, $E_p(f)$ is defined as:

$$E_p(f) = E_p(a) = \int_0^{\overline{D}} a \ p(a) \ da$$

Taking into account these factors, the Choquet integral writes now:

$$V_p = \lambda \cdot d + \gamma \overline{D} + (1 - \gamma - \lambda)E_p(a)$$

Then, when $\gamma = \lambda = 0$, we find the usual expected utility. With $1 \geq \gamma > 0, \lambda = 0$, the subject is waiving between the lowest value and the expected value of the function. That
corresponds to pessimism because the operator cannot consider that $\overline{D}$ occurs with sufficiently high probability. Then, optimism is induced by $\gamma = 0$, $1 \geq \lambda > 0$.

However, to keep a correspondence with the analysis of Teitelbaum (2007) I make the following change of variable that corresponds to the treatment of CEG (2007):

$\lambda = \delta \alpha$, $\gamma = \delta (1 - \alpha)$, then check that $1 - \gamma - \lambda = 1 - \delta$ with $\delta, \alpha \in (0, 1)$

The neo-additive capacity is then:

$$\mu (A / p, \delta, \alpha) = \begin{cases} 
\delta \alpha \nu_0 (A) + \delta (1 - \alpha) \nu_1 (A) + (1 - \delta) p (A) & \text{for } \emptyset \subsetneq A \subsetneq \mathcal{E} \\
0 & \text{for } A = \emptyset \\
1 & \text{for } A = \mathcal{E}
\end{cases}$$

(6A)

Or, still, for $\emptyset \subsetneq A \subsetneq \mathcal{E}$

$$\mu (\cdot) = \delta \alpha d + \delta (1 - \alpha) \overline{D} + (1 - \delta) p (A)$$

(7A)

Then the Choquet Integral of the neocapacity is:

$$V_p = \delta \alpha d + \delta (1 - \alpha) \overline{D} + (1 - \delta) E_p (a)$$

(8A)

The precise meaning of the weight $\delta$ (aversion for ambiguity) and $\alpha$ (degree of optimism) is given in the argument of the paper.
Appendix 2

Proof of the propositions

**Proposition 1:** Under a strict liability regime, with a Choquet expected cost-minimizer injurer, by lemma 1 and under assumptions 1 and 2:

When \([A]\) is verified, then the injurer over-invests in care, i.e. \(x^{SL} > x^*\)

When \([B]\) is verified, then the injurer under-invests in care: \(x^{SL} \leq x^*\).

**Proof:** We consider the first order conditions for the expected social cost function

\[ ECS' = 0 \Rightarrow \exists x^*: \pi'(x^*) = -\frac{1}{E_p(a)} \]

And, for the expected cost function of the injurer:

\[ EC'_p = 0 \Rightarrow \exists x^0: \pi'(x^{SL}) = -\frac{1}{\nu_p(A/p, \delta, \alpha)} = -\frac{1}{\delta ad+\delta(1-\alpha)B+(1-\delta)E_p(a)} \]

By lemma 1 if \([A]\) is verified, i.e., if \(\frac{\delta-E_p(a)}{B-d} > \alpha\) then \(\nu_p(A/p, \delta, \alpha) > E_p(a)\) or, if \([B]\) is verified, then, if \(\frac{\delta-E_p(a)}{B-d} < \alpha\) and \(\nu_p(A/p, \delta, \alpha) \leq E_p(a)\).

The consequence of having \(\nu_p(A/p, \delta, \alpha) > E_p(a)\) or \(\nu_p(A/p, \delta, \alpha) \leq E_p(a)\) is that \(\pi'(x^*)\) may be higher or lower than \(\pi'(x^{SL})\). Because of the continuity of \(\pi'(x)\), and because it is an increasing function, (by assumption, the second derivative \(\pi''(x)\) is positive) (see figure 1 for a geometrical representation) then:

- By lemma 1, when \([A]\) is verified (i.e. if \(\frac{\delta-E_p(a)}{B-d} > \alpha\) then

\[ \nu_p(A/p, \delta, \alpha) > E_p(a) \]. Consequently, \(-\frac{1}{\nu_p(A/p, \delta, \alpha)} > -\frac{1}{E_p(a)}\), or still, \(\pi'(x^{SL}) > \pi'(x^*)\), then, because \(\pi'(x)\) is increasing, \(x^{SL} > x^*\).

- By the same argument applied to \([B]\), (i.e. if \(\frac{\delta-E_p(a)}{B-d} < \alpha\) then, then \(\nu_p(A/p, \delta, \alpha) \leq E_p(a)\) and, as a consequence \(x^{SL} \leq x^*\).
Fig 1: Representation of $\pi'(x)$
**Proposition 2:** Under a rule of negligence, with a Choquet expected cost-minimizer injurer, by lemma 1 and under assumptions 1 and 2:

When \([A]\) is verified, the injurer will tend to over-invest in safety. Hence, if \(x^{NR}\) is the optimum level of safety for the injurer, then \(x^{NR} > x^*\).

When \([B]\) is verified, then he will under-invest \(x^{NR} \leq x^*\).

**Proof:** The starting point is similar to proposition 1, that means that we consider the first order conditions for the regulator and the injurer. Then, we define the program of the injurer submitted to a negligence rule. He will expect to pay:

\[
EC_{P}^{NR} = \begin{cases} 
  x^\mu & \text{if } x^\mu \geq x^* \\
  (x^S + \pi(x^S))V_p(A/p, \delta, \alpha) & \text{if } x^S < x^* 
\end{cases}
\]

The injurer looks for minimizing his expected cost function in a similar way to proposition 1. The argument is then identical to it:

- We know by lemma 1, that when \([A]\) is verified then, the solution of his program \(x^{NR}\) will be such that \(x^{NR} > x^*\);

- And, when \([B]\) is verified, then, then \(x^{NR} \leq x^*\).
**Corollary 1:** Given the injurer’s level of optimism, given lemma 1 and propositions 1 and 2, the level of care determined by the injurer is the same whatever the responsibility regime, i.e. \( x^0 = x^{NR} = x^{SL} > x^* \), for [A] of lemma 1, or \( x^0 = x^{NR} = x^{SL} \leq x^* \), for [B] of the same lemma.

**Proof:**

We give the proof for situation [A]. When [A] is verified, then, under a strict liability regime, the optimum level of care of the injurer, \( x^{SL} \), is deduced from the cost function \( x + V(.)\pi(x) \), and, as shown by proposition 1, \( x^{SL} > x^* \). Under a negligence rule, it is only if \( x^{NR} \geq x^* \), that the injurer escape any liability. \( x^{NR} \) is calculated from the cost function \( x + V(.)\pi(x) \). This cost function is strictly identical to the one used under a strict liability regime, then \( x^{NR} \) is the solution and \( x^{NR} > x^* \). Then, \( x^{NR} = x^{SL} \).

(For situation [B] in lemma 1 the argument is its strictly symmetric) ■
Proposition 3: Let us consider strict liability and Knightian uncertainty characterized by the injurer’s beliefs on major damage and accident occurrence. Under assumption 1 and 2, a Choquet cost-minimizer injurer will tend to care too much or not enough compared to the first best level of care. More precisely:

1. \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \) for \( 1 \geq \alpha > \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} \) and this relationship is true if and only if \( E_p(\alpha) > \frac{(1 - \beta)}{\left(\frac{\beta}{\delta} + 1 - \beta\right)} D \) for \( 1 - \beta > 0 \) and \( \delta > 0 \).

Consequently, this involves that \( x^{SL} > x^* \).

2. \( E_p(\alpha) > V_p(A/p, \delta, \alpha) \) for \( \alpha \leq \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} \) which is true if and only if

\[
E_p(\alpha) \leq \frac{(1 - \beta)}{\left(\frac{\beta}{\delta} + 1 - \beta\right)} D, \text{ consequently, } x^{SL} \leq x^*.
\]

Proof:

1) Proof that \( x^{SL} > x^* \) when \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \).

The case for which \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \) involves that:

\[
1 \geq \alpha > \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)}
\]

This relationship is right when the following conditions are met:

i) If \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} < 0 \), obviously the condition is verified. Is this relationship economically plausible? We can see that the denominator is negative ((\( d - D \) < 0 for \( 1 - \beta > 0 \) and \( \delta > 0 \) positive). Then, what are the conditions for having the numerator positive? This is true for:

\[
E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta > 0 \text{ i.e. for } E_p(\alpha) > \frac{(1 - \beta)}{\left(\frac{\beta}{\delta} + 1 - \beta\right)} D. \text{ This is potentially true because } \frac{(1 - \beta)}{\left(\frac{\beta}{\delta} + 1 - \beta\right)} < 1, \frac{\beta}{\delta} > 0.
\]

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ii) We can check immediately that, if \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} > 0 \), the condition cannot be fulfilled indeed, not only \( E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta \) should be negative or null which is true for \( E_p(\alpha) \leq \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} D \), but also, the expression \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} \) should be less than one because \( 1 \geq \alpha \).

Hence, \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} \leq 1 \Rightarrow E_p(\alpha) \leq \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} d \), we know that 
\( d = \text{Inf}(f) \), hence, by definition, \( E_p(\alpha) \in [\text{Inf}(f), \text{Sup}(f)] \) then, because 
\( \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} \leq 1 \), \( \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} d < E_p(\alpha) \), (indeed \( 0 \leq \frac{\beta}{\delta} \) then \( E_p(\alpha) \leq \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} d \) is impossible.

As a conclusion, it is only if \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} < 0 \), that \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \) which is economically plausible.

Then, it is sufficient to follow the proof of proposition 1, to show that \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1 - \beta) \) involves \( x^{SL} > x^* \).

2) \( E_p(\alpha) > V_p(A/p, \delta, \alpha) \) involves that \( x^{SL} > x^* \)

By developing, \( E_p(\alpha) > V_p(A/p, \delta, \alpha) \), then
\[
\alpha \leq \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)}
\]

Here, we have to check only that \( \frac{E_p(\alpha)(\beta + \delta - \delta \beta) - (1 - \beta)D\delta}{\delta(d - D)(1 - \beta)} > 0 \), then, how this condition can be fulfilled? We know that \((d - D) < 0\) and \((1 - \beta) > 0\), \( \delta > 0 \), consequently, the denominator is negative. As \( \alpha > 0 \), we have to look for the conditions that insure that the numerator, is negative. We have seen above that is true for \( E_p(\alpha) \leq \frac{(1 - \beta)}{\frac{\delta}{\delta + 1 - \beta}} D \). Hence, we can find values of \( \beta \) or \( \delta \), such that the inequality is respected.

As previously, we follow proposition one, and it results from above that \( x^{SL} > x^* \).

**Proposition 4:** Under negligence and Knightian uncertainty, under assumption 1, the optimum level of care chosen by the injurer will be either higher or lower than the first best level of care.

**Proof:** Here again, the starting point is similar to proposition 1, that means that we consider the first order conditions for the regulator and the injurer. Then, we define the program of the injurer submitted to a negligence rule. He will expect to pay:

\[
\begin{align*}
\{ x^\mu & \text{ if } x^\mu \geq x^* \\
(x + \gamma \beta \Theta) + (1 - \beta)p(x) [\delta a d + \delta (1 - \alpha)D + (1 - \delta)E_p(a)] & \text{ if } x^S < x^*
\end{align*}
\]

The injurer looks for minimizing his expected cost function in a similar way to proposition 1. The argument is then identical to it, then after having define the foc:

- We know by proposition 3 that when \( E_p(a) < V_p(A/p, \delta, \alpha)(1 - \beta) \) is verified for

\[
1 \geq \alpha > \frac{E_p(a)(\beta + \delta - \delta \beta)(1 - \beta)D \delta}{\delta(d - D)(1 - \beta)}
\]

this involves that \( x^{SL} \geq x^* \) is verified then, the solution of his program \( x^{NR} \) will be such that \( x^{NR} > x^* \);

- And, when \( E_p(a) > V_p(A/p, \delta, \alpha) \) for \( \alpha \leq \frac{E_p(a)(\beta + \delta - \delta \beta)(1 - \beta)D \delta}{\delta(d - D)(1 - \beta)} \) is verified, then

\( x^{SL} < x^* \)
**Corollary 3:** Given the conditions described by assumption 1 and propositions 3 and 4, the level of care determined by the injurer is the same whatever the responsibility regime put into force by the regulator. Hence, \( x^{NR} = x^{SL} > x^* \), for \( 1 \geq \alpha > \frac{E_p(\alpha)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)} \), and \( x^{NR} = x^{SL} \leq x^* \), for \( \alpha \leq \frac{E_p(\alpha)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)} \).

**Proof:**

We give the proof for the following situation:

\[
1 \geq \alpha > \frac{E_p(\alpha)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)}.
\]

When this relationship is verified, then, \( E_p(\alpha) < V_p(A/p, \delta, \alpha)(1-\beta) \) and, under a strict liability regime, the optimum level of care of the injurer is \( x^{SL} \). From proposition 3 we know that \( x^{SL} > x^* \). As for the proof of corollary 1, under a negligence rule, it is only if \( x^{NR} \geq x^* \) that the injurer escape any liability. The question to know is if \( x^{NR} = x^{SL} \). We have to consider that for any \( x^0 \), such that \( x^0 > x^{NR} \), the cost function \( x + y \beta \Theta + (1-\beta)\pi(x)[\delta ax + \delta(1-\alpha)D + (1-\delta)E_p(\alpha)] \) is not minimized for \( x^0 \), even if \( x^0 = x^* \). As for the strict liability regime, the relevant value that minimizes accident costs is \( x^{NR} \) and, then, \( x^{SL} = x^{NR} \).

- The proof for \( \alpha \leq \frac{E_p(\alpha)(\beta+\delta-\delta\beta)-(1-\beta)D\delta}{\delta(d-D)(1-\beta)} \) follows the same argument and is not developed here.■
Proposition 6: Under uncertainty and negligence, considering an independent court when an injurer is endowed with a Choquet expected cost function $V^{NR}_{a,\varphi^*}$, and a regulator with a Savage expected cost function, then the injurer’s equilibrium level of care is inefficient compared to the first best level of care.

Proof: It is sufficient to note that in its structure the cost function of the injurer is similar in structure of the cost function of proposition 1, $V^{NR}_{a,\varphi^*} = \alpha \varphi^* d + (1 - \alpha) \varphi^* D + (1 - \varphi^*) E_p(\alpha)$. Consequently, the same argument applies.

Hence, let us consider

i) $V^{NR}_{a,\varphi^*} > E_p(\alpha)$ if $\frac{D - E_p(\alpha)}{D - d} > \alpha$ and,

ii) $V^{NR}_{a,\varphi^*} < E_p(\alpha)$ if $\frac{D - E_p(\alpha)}{D - d} < \alpha$

These relationships come from the comparison of $E_p(\alpha)$ and $V^{NR}_{a,\varphi^*}$ as for establishing the lemma 1. As $V^{NR}_{a,\varphi^*} = \alpha \varphi^* d + (1 - \alpha) \varphi^* D + (1 - \varphi^*) E_p(\alpha)$ is similar in structure to $V_p(A/p, \delta, \alpha)$, consequently, the same argument as used for proving proposition 1 applies and we deduce then:

- When $\frac{D - E_p(\alpha)}{D - d} > \alpha$) then $V^{NR}_{a,\varphi^*} > E_p(\alpha))$. Consequently $\frac{1}{V^{NR}_{a,\varphi^*}} > -\frac{1}{E_p(\alpha)}$, or still, $\pi'(x^{NR}) > \pi'(x^*)$, then, because $\pi'(x)$ is increasing, $x^{NR} > x^*$.

- By the same argument, when $\frac{D - E_p(\alpha)}{D - d} < \alpha$ then $V^{NR}_{a,\varphi^*} < E_p(\alpha)$ and, consequently, $x^{NR} < x^*$.